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Kinematic Analysis and Simulation of Three Link (Open Chain) Robot Manipulator with Six DOF

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Abstract: The kinematics of manipulator is a central problem in robot control. AR2 robot is adopted as a case study which has six Degree of Freedom (DOF), three links with revolute joints and spherical wrist. Danivat Hartenberg (DH) convention is based to predicate the forward kinematics position and orientation of the robot end effector. The calculated forward kinematics has been verified and simulated with MATLAB robotics toolbox while the Inverse Kinematics (IK) problem is divided into two sub problem, arm IK (position solving) and wrist IK (orientation solving). Both geometrical and numerical approaches are presented to solve the problems which lead to eight different configurations four for arm and two for wrist and many sets of joints angles. The presented approaches can be applicable to solve the kinematics problem of other similar kinds of robot manipulators and achieve a suitable solution for tracking trajectories.

Key words: DH parameters, forward kinematics, inverse kinematics (geometrical and numerical appraochs), applicable, suitable, trajectories

INTRODUCTION

Robotics is a relatively young field of modern technology that crosses traditional engineering boundaries. Understanding the complexity of robot and their application required knowledge of mechanical structure, sensory system, automatic control system, mathematics and computer science.

This study concerned with the kinematics of the robot manipulator. The problem of kinematics modeling is usually categorized into two sub-problem, first is the forward or direct kinematics which is the transformation between joint space and the cartesian space to solve the position and orientation of a robot end effector, the second is the inverse or backward kinematics which is the computation of joint variable using the given information of a robot's end effector position and orientation. In case of open chain robotic arm, Inverse Kinematics (IK) problem is more complex than the forward kinematics problem because the high non linearity and multiple solutions existence (Spong *et al*., 2005).

There is a large amount of literatures which discuss the robot kinematics modeling and analysis, especially industrials robots. In general almost the researchers are modeling the forward kinematics based on Denavit-Hartenberg (DH) convention and using the MATLAB as software program because it's capabilities in

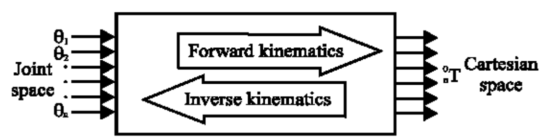

Fig. 1: The schematic representation of forward and inverse kinematic

simulations. Where Koyuncu and Guzel (2007) developed a software for both forward and inverse kinematics of Lynx6 educational robot. Qassem *et al*. (2010) develop a visual software for AL5B with five DOF as educational experimental tool in practical aspect of robot manipulator. Abbas (2013) presented the direct modeling of five Degree of Freedom (DOF) stationary articulated robot arm where a Labvolt R5150 arm has been taken as case study. Mohammed and Sunar (2015) presented the kinematic model of RA02 arm with 4 DOF and simulated with robotics toolbox of MATLAB.

In this research, forward kinematic model will be achieved by DH convention for 6 DOF (AR2) robotic arm as shown in Fig. 2 and inverse kinematic solutions will be presented in both geometrical and numerical approaches. This model would achieve provision of real time solution and suitable for tracking trajectories. The study is organized sequentially as follow: robot description, forward kinematic, inverse kinematic, result and discussion and conclusions.

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Fig. 2: AR2 robotic arm designed by Chris Annin https://github.com/Chris-Annin

Forward kinematics: Denavit Hartenberg convention is based on attaching coordinate frame system at each joint and specifying the four parameter of DH: α_{i-1}, a_{i-1}, θ_i and di where (Spong *et al*., 2005; Jazar, 2010):

- a_{i-1}: (link length) is the distance between z_{i-1} and z_i axes along the x_i axis
- α_{i-1}: (link twist) is the required rotation of z_{i-1} to z_i axes about the x_i axis
- d_i: (joint offset) is the distance between x_{i-1} and x_i axes along the z_i axis
- θ_i: (joint angle) is the required rotation of x_{i-1} to x_i axes about the $z_i$axis

DH parameters of AR2 are defined for the assigned frames in Table 1. The transformation matrix frame one frame to previous frame is:

$$T_i^{i-1} = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_{i-1} & s\theta_i s\alpha_{i-1} & a_{i-1}c\theta_i \\ s\theta_i & c\theta_i c\alpha_{i-1} & -c\theta_i s\alpha_{i-1} & a_{i-1}s\theta_i \\ 0 & s\alpha_{i-1} & c\alpha_{i-1} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The first three columns matrix represent rotation matrix that leads to orientation and the fourth column represent the origin position. The complete transformation of end effector to base coordinate frame system T_6^0 is can be calculated from:

$$^0_6T = {}^0_1T \ {}^1_2T \ {}^2_3T \ {}^3_4T \ {}^4_5T \ {}^5_6T \quad (2)$$

Table 1: DH parameters of AR2

i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	90	0	θ_1
2	a_2	0	0	θ_2
3	0	90	0	θ_3
4	0	-90	d_4	θ_4
5	0	90	0	θ_5
6	0	0	d_6	θ_6

$$T_6^0 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$n_x = c_1(c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6) + s_1(s_4c_5c_6 + c_4s_6)$$

$$n_y = s_1(c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6) - c_1(s_4c_5c_6 + c_4s_6)$$

$$n_z = s_{23}(c_4c_5c_6 - s_4s_6) + c_{23}s_5c_6$$

$$o_x = c_1(-c_{23}(c_4c_5s_6 + s_4c_6) + s_{23}s_5s_6) + s_1(-s_4c_5s_6 + c_4s_6)$$

$$o_y = s_1(-c_{23}(c_4c_5s_6 + s_4c_6) + s_{23}s_5s_6) - c_1(-s_4c_5s_6 + c_4s_6)$$

$$o_z = -s_{23}(c_4c_5s_6 + s_4c_6) - c_{23}s_5s_6$$

$$a_x = c_1(c_{23}c_4s_5 + s_{23}c_5) + s_1s_4s_5$$

$$a_y = s_1(c_{23}c_4s_5 + s_{23}c_5) - c_1s_4s_5$$

$$a_z = s_{23}c_4s_5 - c_{23}c_5$$

$$p_x = a_2c_1c_2 + d_4c_1s_{23} + d_6(c_1(c_{23}c_4s_5 + s_{23}c_5) + s_1s_4s_5)$$

$$p_y = a_2s_1c_2 + d_4s_1s_{23} + d_6(s_1(c_{23}c_4s_5 + s_{23}c_5) - c_1s_4s_5)$$

$$p_z = a_2s_2 - d_4c_{23} + d_6(s_{23}c_4s_5 - c_{23}c_5)$$

Inverse kinematics: There are different ways to solve the IK; geometrical, analytical and numerical approaches; Geometrical more efficient from a point of view when possible because it's related to structure kinematics (Craig, 2008), analytical required algebraic techniques and mathematical expression related to kinematic model, geometrical and analytic leads to closed form solutions. The difficulty is how to choose the right solution in order the robot manipulator to proceed the trajectory. Here, the numerical approach is preferred because it's gives the

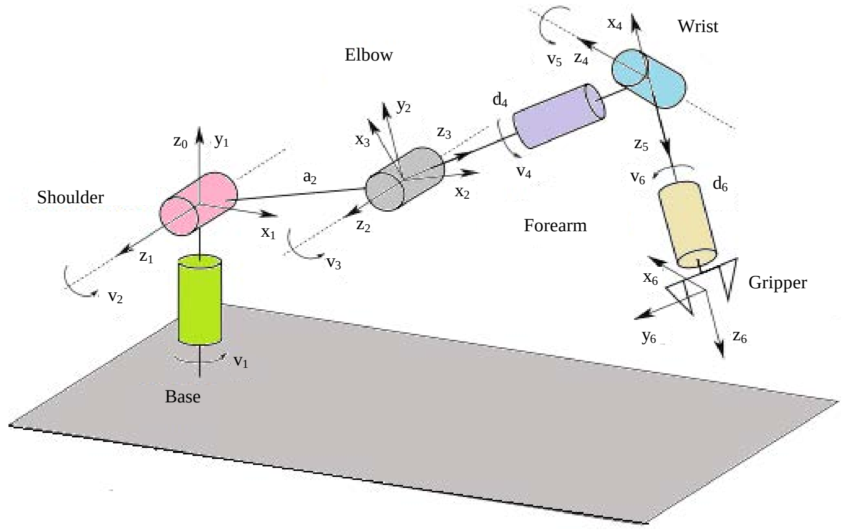

Fig. 3: Illustrates the attached coordinate frame systems

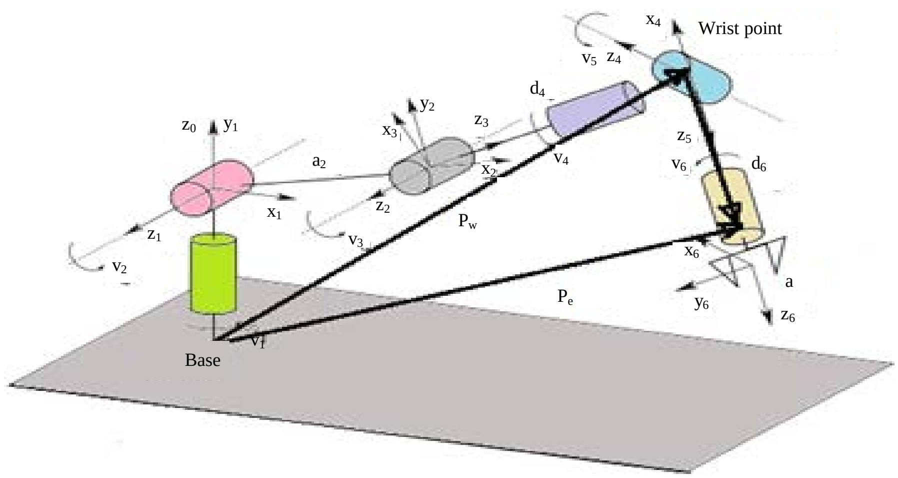

Fig. 4: Illustrates the wrist point position

nearest solution but not all numerical solutions are right, it must be verified by the forward kinematics before applying. Since, the numerical solution of nonlinear equation depends on the differentiated functions to closing the zero of error line where the intersections point are considered the solutions (Newton-Raphson method) (Jazar, 2010).

Geometrical approach: Many industrial manipulators have a kinematically decoupled structure for which it possible to decompose the problem into IK for position (Wrist point) and end effector orientation (Craig, 2008) (Fig. 3 and 4):

$$P_w = P_e - d_e\{a\} \tag{4}$$

To solve arm joint variable or wrist point position, it may solving the direct forward equations from base to wrist point by neglecting the wrist joints where DH parameter in Table 2. Then:

Table 2: DH of arm

i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	90	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

$$ {}^0_3T = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & c_1(a_2c_2+a_3c_{23}) \\ s_1c_{23} & -s_1s_{23} & -c_1 & s_1(a_2c_2+a_3c_{23}) \\ s_{23} & c_{23} & 0 & a_3s_{23}+a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

From geometry:

$$\theta_1 = a\tan 2(p_y, p_x) \tag{6}$$

By working with x_1-y_1 plane where:

Table 3: DH of wrist

i	a_{i-1}	α_{i-1}	d_i	θ_I
3	0	-90	0	θ_4
4	0	90	0	θ_5
5	0	0	d_6	θ_6

$$c_3 = \frac{p_x^2 + p_Y^2 + p_z^2 - a_2^2 - a_3^2}{2a_2 a_3}$$

$$s_3 = \pm\sqrt{1 - c_3^2}$$

$$\theta_3 = a\tan 2(s_3, c_3) \quad (7)$$

(two solutions $\theta_{3(1)}$, $\theta_{3(2)}$). Moreover, by geometrical arguments, it's possible to state that:

$$\theta_2 = a\tan 2(p_z, \sqrt{p_x^2 + p_y^2}) - a\tan 2(a_3 s_3, a_2 + a_3 c_3) \quad (8)$$

(two solutions $\theta_{2(1)}$, $\theta_{2(2)}$), also, the solutions of back side or shadow are valid:

$$\theta_{1(2)} = \theta_{1(1)} + \pi \text{ or } \theta_{1(2)} = \theta_{1(1)} - \pi \quad (9)$$

$$\theta_{2(3)} = \theta_{2(1)} + \pi \text{ or } \theta_{2(3)} = \theta_{2(1)} - \pi \quad (10)$$

$$\theta_{2(4)} = \theta_{2(2)} + \pi \text{ or } \theta_{2(4)} = \theta_{2(2)} - \pi \quad (11)$$

$$\theta_{3(3)} = \theta_{3(1)} + \pi \text{ or } \theta_{3(3)} = -\theta_{3(1)} - \pi \quad (12)$$

$$\theta_{1(4)} = \theta_{3(2)} + \pi \text{ or } \theta_{3(4)} = -\theta_{3(2)} - \pi \quad (13)$$

Figure 5 shows the four configuration of arm (shoulder right-elbow up, shoulder right-elbow down, shoulder left-elbow up and shoulder left-elbow down). To solve the spherical wrist independly as shown in Fig. 6 the DH parameters are listed in Table 3. Then:

$$ {}^3_6T = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (14)$$

where, ${}^3_6R$ can be obtained from:

$${}^3_6R = {}^0_3R^{T\,0}_{\ \ 6}R \quad (15)$$

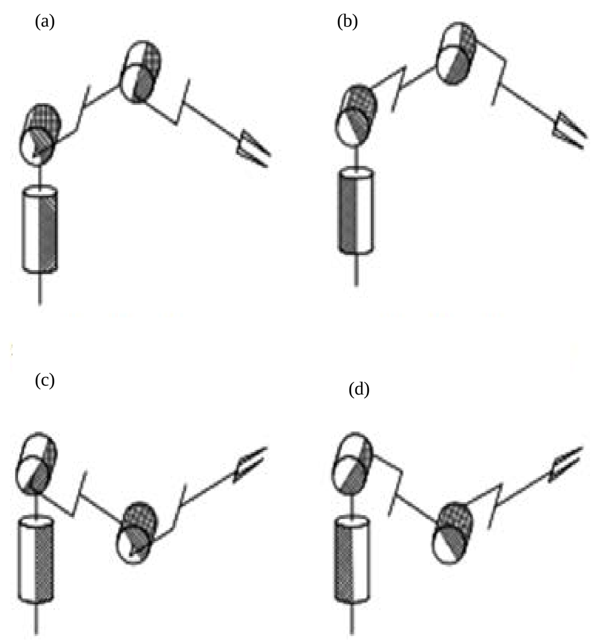

Fig. 5: Illustrates the four arm solutions: a) Shoulder right-elbow up; b) Shoulder left-elbow up; c) Shoulder right-elbow down and d) Shoulder left-elbow down

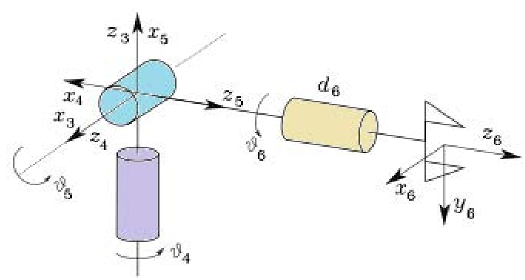

Fig. 6: Illustrates the wrist coordinate frames systems

The solution is computed as ZYZ Euler angles Spong *et al*. (2005) as follow:

When, $\theta_s \in [0, \pi]$:

$$\theta_4 = a\tan 2(a_y, a_x) \quad (16)$$

$$\theta_5 = a\tan 2\left(\sqrt{a_x^2 + b_y^2}, a_z\right) \quad (17)$$

$$\theta_6 = a\tan 2(o_z, n_z) \quad (18)$$

When, $\theta_s \in [-\pi, 0]$:

$$\theta_4 = a\tan 2(-a_y, -a_x) \quad (19)$$

$$\theta_5 = a\tan 2\left(-\sqrt{a_x^2 + a_y^2},\, a_z\right) \tag{20}$$

$$\theta_6 = a\tan 2\left(-o_z, n_z\right) \tag{21}$$

Numerical approach: The most common method for solving a set of nonlinear algebraic equations Newton-Raphson method, iteration based method. The solution Eq. 22:

$$q_{i+1} = q_i - J^{-1}(q_i) f(q_i) \tag{22}$$

Where:

I = Arbitrary counter

q = $[q_1, q_2, ..., q_n]^T$

f(q) = $[f_1(q), f_2(q), ..., f_2(q)]^T$

where is the position vector of transformation matrix (called jacobain matrix):

$$J = \begin{bmatrix} \frac{\partial f_1(q)}{\partial q_1} & \frac{\partial f_1(q)}{\partial q_2} & . & \frac{\partial f_1(q)}{\partial q_n} \\ \frac{\partial f_2(q)}{\partial q_1} & . & . & . \\ . & . & . & . \\ \frac{\partial f_n(q)}{\partial q_1} & . & . & \frac{\partial f_n(q)}{\partial q_n} \end{bmatrix}$$

RESULTS AND DISCUSSION

A programs have been developed to embrace the theoretical work, the programs are able to calculate the forward kinematics, inverse kinematics in both approaches, the results show a full agreement with MATLAB robotics toolbox. Where the input data of matlab are listed in Table 4. And the output results of the transformation matrix ${}^0_6T$ for theta [pi/9, pi/6, pi/3, pi/18, pi/2, 0] listed in Table 5 and the configuration is shown in Fig. 7:

Table 4: Input data to MATLAB toolbox

j	Theta	d (cm)	a (cm)	Alpha (rad)	Offset
1	q_1	0	0	1.5708	0
2	q_2	0	20	0	0
3	q_3	0	0	1.5708	0
4	q_4	35	0	-1.5708	0
5	q_5	0	0	1.5708	0
6	q_6	0	0	0	0

Table 5: Input DH of to MATLAB toolbox ${}^0_2T$

j	Theta	d (cm)	a (cm)	Alpha (rad)	Offset
1	q_1	0	0	1.5708	0
2	q_2	0	20	0	0
3	q_3	0	35	0	0

According to DH parameter in Tables 1 and 2, There is an offset rotation in joint 3 by $\pi/2$ therefor the input DH as listed in Table 5 and the output transformation matrix ${}^0_6T$ listed in Table 6 and the configration is shown in Fig. 8. Table 5 forward transformation matrix ${}^0_6T$:

$$\text{MATLAB Toolbox}: \begin{matrix} -0.9397 & 0.3368 & 0.0594 & 49.17 \\ -0.3420 & -0.9254 & -0.1632 & 17.89 \\ 0 & -0.1736 & 0.9848 & 10 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$$\text{Own program}: \begin{matrix} -0.9397 & 0.3368 & 0.0594 & 49.17 \\ -0.3420 & -0.9254 & -0.1632 & 17.89 \\ 0 & -0.1736 & 0.9848 & 10 \\ 0 & 0 & 0 & 0 \end{matrix}$$

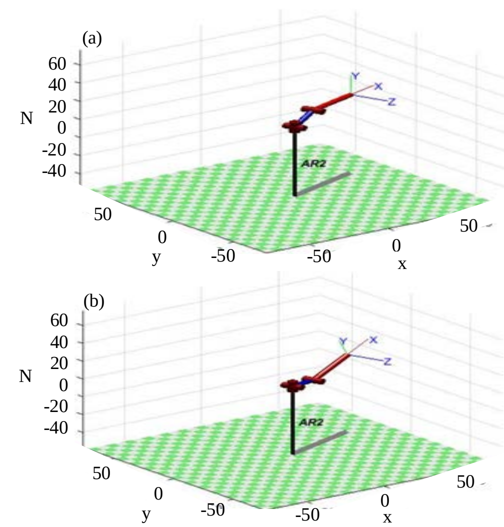

Fig. 7: a) Illustrate the [pi/9, pi/6, pi/3-pi/2, 0] and b) [pi/9, pi/6, pi/3-pi/2, 0] angles configuration

Table 6: Arm inverse kinematics (geometrical approach)

j	Theta 1	Theta 2	Theta 3
1st set	20.000	-8.3592	30.000
2nd set	20.000	30.000	-30.000
3rd set	200	188.3592	330.0
4th set	200	150.0	30.0

Table 7: Some of arm inverse kinematics (numerical approach)

Theta 1	Theta 2	Theta 3	Iterations
20	-8.3592	30	1862
20	351.6408	30	1529
-160	-171.6408	330	1626
-160	188.3592	330	1378
200	150	30	1657
-340	351.6408	30	1379

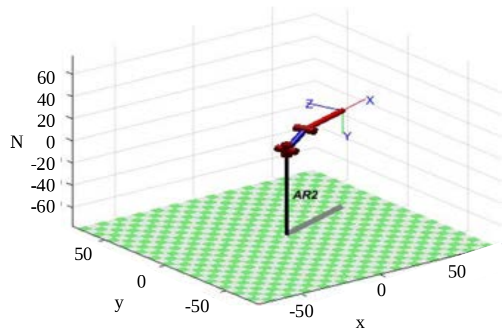

Fig. 8: Illustrates 1st set configuration

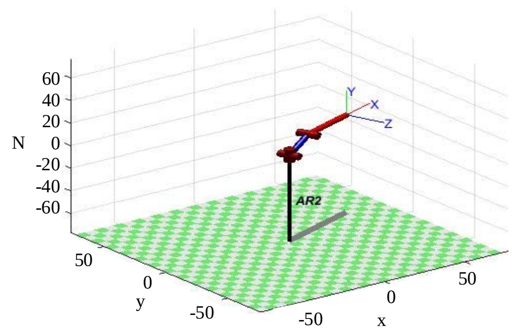

Fig. 9: Illustrates 2rd set configuration

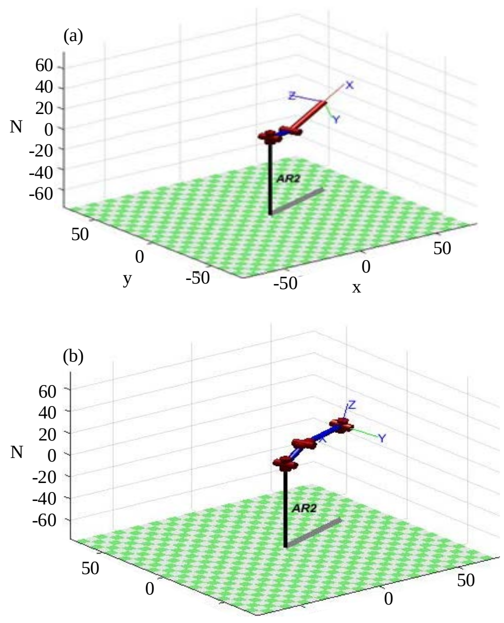

Fig. 10: Illustrates 4th set configuration

For the inverse kinematic, Fig. 8-11 shows the four configuration with a 4 sets of solutions for the position (49.17,17.89, 10) as listed in Table 7, numerically there will be 12 solution more because the deferent reading of the angles, e.g., 20 equal -340 and more and more by adding complete cycle, e.g., 20, 380, etc., that is happened in iterative technique to converge the solution based on the initial guesses as listed in Table 7. The wrist angles are calculated from equations 15-17 in degree are (10.000, 90.000 and 0.000).

CONCLUSION

The proposed model makes it possible to control the manipulator to achieve any reachable position and orientation. Geometrical approaches showed the robot has eight different configuration, four for arm(end effector position) and two for wrist (end effector orientation).

Numerical approach depends on initial guesses to find the solution and it's preferred in trajectories determining by considering the previous point coordinate in an initial guesses for next point and so on. The builded programs are capable to solve similar kinds of robot manipulators.

RECOMMENDATION

Theoretical and experimental trajectories determining and motion controlling of AR2 robot manipulator.

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