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Design and Implementation of a Complex Binary Adder

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Abstract: To represent complex number as single-unit binary number, a complex binary number utilizing base (-1+j) has been proposed in the scientific literature. In this study, we have designed a nibble-size adder based on this number system using the traditional truth table/Kmap approach and implemented it on Xilinx Virtex FPGAs. We have compared this design with the minimum-delay nibble-size complex binary adders and base-2 binary adders designed using decoder and ripple-carry principle. This research work leads us to the conclusion that the complex binary is a viable number system for designing Arithmetic and Logic Unit (ALU) of today's microprocessors.

Key words: Binary complex numbers, FPGA, complex binary adder, Logism Software, microprocessors, utilizing

INTRODUCTION

The role of the complex numbers in the field of engineering cannot be overstated but despite their wide usage across an extensive range of applications, the hardware used to deal with arithmetic operations involving this type of numbers is very rudimentary and old-fashioned. In today's computers, a complex number a+jb is segmented along real (a) and imaginary lines (jb) and then the arithmetic operation is carried out on pairs of real components and imaginary components of the numbers independently. Thus, a simple addition of two complex numbers a+jb and c+jd becomes two separate additions of (a+c) and (b+d). This is due to the fact that the traditional base-2 binary number system, used in today's computers, does not allow a complex number to be represented as single string of bits. Efforts to devise binary number system with base other than 2 include work by Jamil and Ali (2014), Herd (2017), Ito and Sadahiro (2009) and Varghese *et al*. (2016). In this study, we have designed a nibble-size binary adder based on Complex Binary Number System (CBNS) with base (-1+j) using the traditional approach of truth table/Kmaps and obtaining simplified Boolean expressions for the outputs.

MATERIALS AND METHODS

Complex binary number system: The value of an n-bit binary number with base (-1+j) can be written in the form of a power series as follows: $a_{n-1}(-1+j)^{n-1}+a_{n-2}(-1+j)^{n-2}+, ..., + a_1(-1+j)^1+a_0\ (-1+j)^0$ where the coefficients $a_{n-1}, a_{n-2}, ..., a_1, a_0$ are binary (either 0 or 1) (Jamil *et al*., 2000).

Conversion algorithms: The algorithms for conversion of various types of numbers to CBNS have been presented by Jamil *et al*. (2003) and are described as follows:

To represent a given positive integer N in CBNS, the following steps are followed: Express N in terms of power of 4 using repeated division process. Convert the base 4 number ($q_5q_4q_3q_2q_1q_0$) to base -4 by replacing each digit in odd location (q_1, q_3, q_{5Y}) with its negative to get (Y-q_5, q_4, -q_3, q_2, -q_1, q_0). Normalize the new number (i.e., get each digit in the range 0-3) by repeatedly adding 4 to the negative digits and adding one to the digit on its left. If the digit is 4, replace it by a zero and subtract a one from the digit on its left. Now replace each digit in base -4 representation with the corresponding four-bit sequence (0→0000; 1→0001; 2→1100; 3→1101). To convert a negative integer number into CBNS representation, we simply multiply the representation of the corresponding positive integer number with 11101 (equivalent to $-1_{base\ -1+j}$) according to the multiplication algorithm.

To obtain the CBNS representation of imaginary numbers, we simply multiply the CBNS representation of corresponding positive number with 11 (equivalent to (+j) base 10) or 111 (equivalent to (-j) base 10) according to the multiplication algorithm.

To represent a fraction F in CBNS, we first express the fraction in terms of powers of 1/2 = 2-1 such that F = r0 = f1. 2-1+f2, 2-2 +f3, 2-3+f4, 2-4+... to the machine limit. The coefficients fi and the remainders ri are given as follows: Initially, if 2r0-1<0 then f1 = 0 and set r1 = 2r0 or if 2r0-1≥0 then f1 = 1 and set r1 = 2r0-1.

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Then, if 2ri-1<0 then fi+1 = 0 and ri+1 = 2 ri or if 2ri-1≥0 then fi+1 = 1 and ri+1 = 2ri-1. We continue this process until ri = 0 or the machine limit has been reached. Then, for ∀fi = 1, we replace its associated 2-i according to the sequence (2-1→1.11, 2-2→1.1101, 2-3→0.000011, 2-4→0.00000001) [for i>4, refer to [x]).

To represent a floating-point number in CBNS which consists of both an integer and a fraction numbers, CBNS representation is obtained by adding the CBNS representation of each individual part according to the addition algorithm. All rules for obtaining negative integer and positive/negative imaginary number representations as discussed previously are equally applicable for obtaining negative floating point and positive/negative imaginary floating point representations in CBNS.

To represent a complex number in CBNS, we simply add the CBNS representation of real part with the CBNS representation of the imaginary part according to the addition algorithm.

By applying these algorithms, we can convert a given complex number into CBNS as illustrated by the following examples:

- $2012_{10} = (-1.3, -3.1, -3.0)_{Base-4} =$ $(1, 3, 4, 1, 2, 1, 0) = (1, 2, 0, 1, 2, 1, 0)_{Normalized} =$ $1110000000000111000000010000_{Base}$ (-1+j)
- $-2012_{10} =$ 1110000000000111000000010000×11101 = $11000000000001101111010000_{Base}$ (-1+j)
- $+j2012_{10} =$ 11000000000001110000010000×11 = $1000000000000110000_{Base}$ (-1+j)
- $-j2012_{10} =$ 11000000000001110000010000×111 = $111010000000011101000111 0000_{Base}$ (-1+j)
- $2012_{10}+j2012_{10} =$ $11000000000001110000010000_{Base}$ (-1+j)+ $10000000000001000011 0000_{Base}$ (-1+j) = $11101000000001110100011100000_{Base}$ (-1+j)

Arithmetic algorithms: As developed by Jamil *et al.* (2003), the procedure to perform the arithmetic operations (add, subtract, multiply, divide) is described as follows:

To add two complex numbers represented in CBNS format, the truth table is given as follows: 0 0 = 0; 0+1 = 1; 1+0 = 1; 1+1 = 1100. When two numbers with 1s in position n are added, this results in 1s in positions n+3 and n+2 and 0s in positions n+1 and n in the sum. Similar to the ordinary computer rule where 1+111... (to limit of machine) = 0, we have 11+111 = 0 [Zero Rule] in CBNS,

To subtract two complex binary numbers, the truth table followed is: 0-0 = 0; 0-1 = *; 1-0 = 1; 1-1 = 0. For the case where 1 is subtracted from 0 (the * case in the rules), the following algorithm is applied: Given our minuend is: $a_n a_{n-1} a_{n-2}$, ..., $a_{k+4} a_{k+3} a_{k+2} a_{k+1} a_k 0 a_{k-1}$, ..., $Y a_3 a_2 a_1 a_0$ and subtrahend is: $b_n b_{n-1} b_{n-2}$, ..., $b_{k+4} b_{k+3} b_{k+2} b_{k+1} 1 b_{k-1} Y b_3 b_2 b_1 b_0$. Then, the result of subtracting 1 from 0 is obtained by changing: $a_k \rightarrow a_k+1$, $a_{k+1} \rightarrow a_{k+1}$ (unchanged), $a_{k+2} \rightarrow a_{k+2}+1$, $a_{k+3} \rightarrow a_{k+3}+1$, $a_{k+4} \rightarrow a_{k+4}+1$ and $b_k \rightarrow 0$.

To multiply two complex binary numbers, we follow the same method that we use for traditional binary numbers, except that while adding intermediate summands, addition algorithm outlined previously in this sub-section is used. The zero rule plays an important role in speeding up the result of the multiplication operation.

To perform the division of two complex numbers represented in CBNS, we take the reciprocal of the denominator and multiply it with the numerator as per algorithm described above. The reciprocal of the complex number is estimated using the following algorithm: given that $z = w^{-1}$, we start with our initial approximation of z by setting $z_0 = (-1+j)^{-k}$ where k is obtained from the representation of w such that:

$$w = \sum_{i=-\infty}^{k} a_i(-1+j)^{-1}$$

in which a_k/1 and a_i/0 for i>k. The successive approximations are then obtained by $z_{i+1} = z_i\,(2\text{-}wz_1)$. If the values of z do not converge, we correct our initial approximation of z by setting $z_0 = j(-1+j)^{-k}$ which will definitely converge (Jamil, 2012, 2001). Some examples of operations completed by application of these arithmetic algorithms are given as follows:

- $(1+j)+(2-j2) = 1110_{Base\,(-1+j)}+111000_{Base\,(-1+j)} =$ $111010110_{Base\,(-1+j)} = 3$
- $2-(1+j3) = 1100_{Base\,(-1+j)}-1010_{Base\,(-1+j)} =$ $0100_{Base(-1+j)}-0010_{Base\,(-1+j)} = 111110_{Base\,(-1+j)}-000000_{Base\,(-1+j)} =$ $111110_{Base\,(-1+j)} = 1-j3$
- $(1+j2)\times(2-j) = (1110101)_{Base\,(-1+j)}\times(111011)_{Base\,(-1+j)} =$ $(1100111)_{Base\,(-1+j)} = (4+j3)$
- $1+j2/1+j3 = (1+j2)\times(1+j3)^{-1} =$ $(1110101)_{Base\,(-1+j)}\times$ $(0.001111001011110010111100...)_{Base\,(-1+j)} =$ $(1.11111001011110010111100101 11...)_{Base(-1+j)} = 0.7-j0.1$

Complex binary adder design: A nibble-size CBNS adder design based on minimum-delay decoder and ripple-carry designs has been previously published in literature (Anonymous, 2018). In this study, we have presented the design of the nibble-size adder circuit using traditional truth table/Kmap simplification approach and implemented the design on Xilinx FPGAs. The truth table of a nibble-size adder with augend bits ABCD, addend bits EFGH and result bits JKLMPRSTUWYZ is given in Table 1 and Fig. 1.

Table 1: Truth table for nibble-size complex binary adder

Variables	Augend				Addend				Results											
m#	A	B	C	D	E	F	G	H	J	K	L	M	P	R	S	T	U	W	Y	Z
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
2	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0
3	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1
4	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	1
6	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0
7	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1
8	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
9	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1
10	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	1	0
11	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	1	0	1	1
12	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0
13	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	1	1	0	1
14	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0
15	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1
16	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
17	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0
18	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1
19	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	1	1	1	0
20	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1
21	0	0	0	1	0	1	0	1	0	0	0	0	0	0	1	1	1	0	0	0
22	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1
23	0	0	0	1	0	1	1	1	0	0	0	0	0	0	1	1	1	0	1	0
24	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
25	0	0	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0
26	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	1	0	1	1
27	0	0	0	1	1	0	1	1	0	0	0	0	0	1	1	0	0	1	1	0
28	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	1
29	0	0	0	1	1	1	0	1	0	0	0	1	1	1	0	1	0	0	0	0
30	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1
31	0	0	0	1	1	1	1	1	0	0	0	1	1	1	0	1	0	0	1	0
32	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
33	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1
34	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0
35	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0	0	1
36	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0
37	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	1	1
38	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	1	1	1	0	0
39	0	0	1	0	0	1	1	1	0	0	0	0	0	0	0	1	1	1	0	1
40	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
41	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	1	1
42	0	0	1	0	1	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0
43	0	0	1	0	1	0	1	1	0	0	0	0	0	1	1	1	0	0	0	1
44	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0
45	0	0	1	0	1	1	0	1	0	0	0	0	0	0	0	0	1	1	1	1
46	0	0	1	0	1	1	1	0	0	0	0	0	0	1	1	1	0	1	0	0
47	0	0	1	0	1	1	1	1	0	0	0	0	0	1	1	1	0	1	0	1
48	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
49	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0
50	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	1
51	0	0	1	1	0	0	1	1	0	0	0	0	0	1	1	1	0	1	0	0
52	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1
53	0	0	1	1	0	1	0	1	0	0	0	0	0	0	1	1	1	0	1	0
54	0	0	1	1	0	1	1	0	0	0	0	0	0	0	0	1	1	1	0	1
55	0	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
56	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1
57	0	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	1	0
58	0	0	1	1	1	0	1	0	0	0	0	0	0	1	1	1	0	0	0	1
59	0	0	1	1	1	0	1	1	0	0	0	0	0	1	1	1	1	1	0	1
60	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1
61	0	0	1	1	1	1	0	1	0	0	0	1	1	1	0	1	0	0	1	0
62	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	1	0	1
63	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	0	0	0

Table 1: Continue

Variables	Augend				Addend				Results											
m#	A	B	C	D	E	F	G	H	J	K	L	M	P	R	S	T	U	W	Y	Z
64	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
65	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	1	0
66	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0
67	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	1	1
68	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0
69	0	1	0	0	0	1	0	1	0	0	0	0	0	0	1	1	0	0	0	1
70	0	1	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0	0	1	0
71	0	1	0	0	0	1	1	1	0	0	0	0	0	0	1	1	0	0	1	1
72	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
73	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	1	0	1
74	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	1	1	0
75	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	1	1	1	1
76	0	1	0	0	1	1	0	0	0	0	0	0	0	0	1	1	1	0	0	0
77	0	1	0	0	1	1	0	1	0	0	0	0	0	0	1	1	1	0	0	1
78	0	1	0	0	1	1	1	0	0	0	0	0	0	0	1	1	1	0	1	0
79	0	1	0	0	1	1	1	1	0	0	0	0	0	0	1	1	1	0	1	1
80	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
81	0	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1	1
82	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	1
83	0	1	0	1	0	0	1	1	0	0	0	0	0	0	1	1	1	0	1	0
84	0	1	0	1	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	1
85	0	1	0	1	0	1	0	1	0	0	0	0	0	0	1	1	1	1	0	0
86	0	1	0	1	0	1	1	0	0	0	0	0	0	0	1	1	0	0	1	1
87	0	1	0	1	0	1	1	1	0	0	0	0	0	0	1	1	1	1	1	0
88	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1
89	0	1	0	1	1	0	0	1	0	0	0	1	1	1	0	1	0	0	0	0
90	0	1	0	1	1	0	1	0	0	0	0	0	0	0	0	0	1	1	1	1
91	0	1	0	1	1	0	1	1	0	0	0	1	1	1	0	1	0	0	1	0
92	0	1	0	1	1	1	0	0	0	0	0	0	0	0	1	1	1	0	0	1
93	0	1	0	1	1	1	0	1	0	0	0	1	1	1	0	1	0	1	0	0
94	0	1	0	1	1	1	1	0	0	0	0	0	0	0	1	1	1	0	1	1
95	0	1	0	1	1	1	1	1	0	0	0	1	1	1	0	1	0	1	1	0
96	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
97	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	1
98	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0	0
99	0	1	1	0	0	0	1	1	0	0	0	0	0	0	0	1	1	1	0	1
100	0	1	1	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	1	0
101	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	1	0	0	1	1
102	0	1	1	0	0	1	1	0	0	0	0	0	1	1	1	0	1	0	0	0
103	0	1	1	0	0	1	1	1	0	0	0	0	1	1	1	0	1	0	0	1
104	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0
105	0	1	1	0	1	0	0	1	0	0	0	0	0	0	0	0	1	1	1	1
106	0	1	1	0	1	0	1	0	0	0	0	0	0	1	1	1	0	1	0	0
107	0	1	1	0	1	0	1	1	0	0	0	0	0	1	1	1	0	1	0	1
108	0	1	1	0	1	1	0	0	0	0	0	0	0	0	1	1	1	0	1	0
109	0	1	1	0	1	1	0	1	0	0	0	0	0	0	1	1	1	0	1	1
110	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
111	0	1	1	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1
112	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
113	0	1	1	1	0	0	0	1	0	0	0	0	0	0	1	1	1	0	1	0
114	0	1	1	1	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0	1
115	0	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
116	0	1	1	1	0	1	0	0	0	0	0	0	0	0	1	1	0	0	1	1
117	0	1	1	1	0	1	0	1	0	0	0	0	0	0	1	1	1	1	1	0
118	0	1	1	1	0	1	1	0	0	0	0	0	1	1	1	0	1	0	0	1
119	0	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0
120	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
121	0	1	1	1	1	0	0	1	0	0	0	1	1	1	0	1	0	0	1	0
122	0	1	1	1	1	0	1	0	0	0	0	0	0	1	1	1	0	1	0	1
123	0	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0
124	0	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	0	1	1
125	0	1	1	1	1	1	0	1	0	0	0	1	1	1	0	1	0	1	1	0
126	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1
127	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
128	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

Table 1: Continue

Variables	Augend				Addend				Results											
m#	A	B	C	D	E	F	G	H	J	K	L	M	P	R	S	T	U	W	Y	Z
129	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1
130	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	0
131	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	1	1
132	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0
133	1	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	1	1	0	1
134	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0
135	1	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1
136	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
137	1	0	0	0	1	0	0	1	0	0	0	0	0	1	1	0	0	0	0	1
138	1	0	0	0	1	0	1	0	0	0	0	0	0	1	1	0	0	0	1	0
139	1	0	0	0	1	0	1	1	0	0	0	0	0	1	1	0	0	0	1	1
140	1	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0	0	1	0	0
141	1	0	0	0	1	1	0	1	0	0	0	0	0	1	1	0	0	1	0	1
142	1	0	0	0	1	1	1	0	0	0	0	0	0	1	1	0	0	1	1	0
143	1	0	0	0	1	1	1	1	0	0	0	0	0	1	1	0	0	1	1	1
4 bits Adder truth table for complex binary number system																				
144	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
145	1	0	0	1	0	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0
146	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	1
147	1	0	0	1	0	0	1	1	0	0	0	0	0	1	1	0	0	1	1	0
148	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0	1
149	1	0	0	1	0	1	0	1	0	0	0	1	1	1	0	1	0	0	0	0
150	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1
151	1	0	0	1	0	1	1	1	0	0	0	1	1	1	0	1	0	0	1	0
152	1	0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1
153	1	0	0	1	1	0	0	1	0	0	0	0	0	1	1	0	1	1	0	0
154	1	0	0	1	1	0	1	0	0	0	0	0	0	1	1	0	0	0	1	1
155	1	0	0	1	1	0	1	1	0	0	0	0	0	1	1	0	1	1	1	0
156	1	0	0	1	1	1	0	0	0	0	0	0	0	1	1	0	0	1	0	1
157	1	0	0	1	1	1	0	1	0	0	0	1	1	1	0	1	1	0	0	0
158	1	0	0	1	1	1	1	0	0	0	0	0	0	1	1	0	0	1	1	1
159	1	0	0	1	1	1	1	1	0	0	0	1	1	1	0	1	1	0	1	0
160	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
161	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	1
162	1	0	1	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0
163	1	0	1	0	0	0	1	1	0	0	0	0	0	1	1	1	0	0	0	1
164	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0
165	1	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	1	1	1	1
166	1	0	1	0	0	1	1	0	0	0	0	0	0	1	1	1	0	1	0	0
167	1	0	1	0	0	1	1	1	0	0	0	0	0	1	1	1	0	1	0	1
168	1	0	1	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0
169	1	0	1	0	1	0	0	1	0	0	0	0	0	1	1	0	0	0	1	1
170	1	0	1	0	1	0	1	0	0	0	0	0	0	1	1	1	1	0	0	0
171	1	0	1	0	1	0	1	1	0	0	0	0	0	1	1	1	1	0	0	1
172	1	0	1	0	1	1	0	0	0	0	0	0	0	1	1	0	0	1	1	0
173	1	0	1	0	1	1	0	1	0	0	0	0	0	1	1	0	0	1	1	1
174	1	0	1	0	1	1	1	0	0	0	0	0	0	1	1	1	1	1	0	0
175	1	0	1	0	1	1	1	1	0	0	0	0	0	1	1	1	1	1	0	1
176	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1
177	1	0	1	1	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	0
178	1	0	1	1	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	1
179	1	0	1	1	0	0	1	1	0	0	0	0	0	1	1	1	1	1	0	0
180	1	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1
181	1	0	1	1	0	1	0	1	0	0	0	1	1	1	0	1	0	0	1	0
182	1	0	1	1	0	1	1	0	0	0	0	0	0	1	1	1	0	1	0	1
183	1	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	1	0	0	0
184	1	0	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1
185	1	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	1	1	1	0
186	1	0	1	1	1	0	1	0	0	0	0	0	0	1	1	1	1	0	0	1
187	1	0	1	1	1	0	1	1	1	1	1	0	1	0	0	1	0	1	0	0
188	1	0	1	1	1	1	0	0	0	0	0	0	0	1	1	0	0	1	1	1
189	1	0	1	1	1	1	0	1	0	0	0	1	1	1	0	1	1	0	1	0
190	1	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	0	1
191	1	0	1	1	1	1	1	1	0	0	0	0	0	1	1	0	0	0	0	0
192	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0

Table 1: Continue

Variables	Augend				Addend				Results										
m#	A	B	C	D	E	F	G	H	J	K	L	M	P	R	S	T	U	W	Y	Z
193	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0	1
194	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	1	0
195	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	1	1	1
196	1	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0	0	0
197	1	1	0	0	0	1	0	1	0	0	0	0	0	0	1	1	1	0	0	1
198	1	1	0	0	0	1	1	0	0	0	0	0	0	0	1	1	1	0	1	0
199	1	1	0	0	0	1	1	1	0	0	0	0	0	0	1	1	1	0	1	1
200	1	1	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0
201	1	1	0	0	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	1
202	1	1	0	0	1	0	1	0	0	0	0	0	0	1	1	0	0	1	1	0
203	1	1	0	0	1	0	1	1	0	0	0	0	0	1	1	0	0	1	1	1
204	1	1	0	0	1	1	0	0	0	0	0	1	1	1	0	1	0	0	0	0
205	1	1	0	0	1	1	0	1	0	0	0	1	1	1	0	1	0	0	0	1
206	1	1	0	0	1	1	1	0	0	0	0	1	1	1	0	1	0	0	1	0
207	1	1	0	0	1	1	1	1	0	0	0	1	1	1	0	1	0	0	1	1
208	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1
209	1	1	0	1	0	0	0	1	0	0	0	1	1	1	0	1	0	0	0	0
210	1	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	1	1	1
211	1	1	0	1	0	0	1	1	0	0	0	1	1	1	0	1	0	0	1	0
212	1	1	0	1	0	1	0	0	0	0	0	0	0	0	1	1	1	0	0	1
213	1	1	0	1	0	1	0	1	0	0	0	1	1	1	0	1	0	1	0	0
214	1	1	0	1	0	1	1	0	0	0	0	0	0	0	1	1	1	0	1	1
215	1	1	0	1	0	1	1	1	0	0	0	1	1	1	0	1	0	1	1	0
216	1	1	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1
217	1	1	0	1	1	0	0	1	0	0	0	1	1	1	0	1	1	0	0	0
218	1	1	0	1	1	0	1	0	0	0	0	0	0	1	1	0	0	1	1	1
219	1	1	0	1	1	0	1	1	0	0	0	1	1	1	0	1	1	0	1	0
220	1	1	0	1	1	1	0	0	0	0	0	1	1	1	0	1	0	0	0	1
221	1	1	0	1	1	1	0	1	0	0	0	1	1	1	0	1	1	1	0	0
222	1	1	0	1	1	1	1	0	0	0	0	1	1	1	0	1	0	0	1	1
223	1	1	0	1	1	1	1	1	0	0	0	1	1	1	0	1	1	1	1	0
224	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0
225	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1	1
226	1	1	1	0	0	0	1	0	0	0	0	0	0	1	1	1	0	1	0	0
227	1	1	1	0	0	0	1	1	0	0	0	0	0	1	1	1	0	1	0	1
228	1	1	1	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0	1	0
229	1	1	1	0	0	1	0	1	0	0	0	0	0	0	1	1	1	0	1	1
230	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
231	1	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1
232	1	1	1	0	1	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0
233	1	1	1	0	1	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1
234	1	1	1	0	1	0	1	0	0	0	0	0	0	1	1	1	1	1	0	0
235	1	1	1	0	1	0	1	1	0	0	0	0	0	1	1	1	1	1	0	1
236	1	1	1	0	1	1	0	0	0	0	0	1	1	1	0	1	0	0	1	0
237	1	1	1	0	1	1	0	1	0	0	0	1	1	1	0	1	0	0	1	1
238	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0
239	1	1	1	0	1	1	1	1	0	0	0	0	0	0	0	0	1	0	0	1
240	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
241	1	1	1	1	0	0	0	1	0	0	0	1	1	1	0	1	0	0	1	0
242	1	1	1	1	0	0	1	0	0	0	0	0	0	1	1	1	0	1	0	1
243	1	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0
244	1	1	1	1	0	1	0	0	0	0	0	0	0	0	1	1	1	0	1	1
245	1	1	1	1	0	1	0	1	0	0	0	1	1	1	0	1	0	1	1	0
246	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1
247	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
248	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0	1	1	1
249	1	1	1	1	1	0	0	1	0	0	0	1	1	1	0	1	1	0	1	0
250	1	1	1	1	1	0	1	0	0	0	0	0	0	1	1	1	1	1	0	1
251	1	1	1	1	1	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0
252	1	1	1	1	1	1	0	0	0	0	0	1	1	1	0	1	0	0	1	1
253	1	1	1	1	1	1	0	1	0	0	0	1	1	1	0	1	1	1	1	0
254	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	1
255	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	0	0	1	0	0

	$\overline{E}.\overline{F}.\overline{G}.\overline{H}$	$\overline{E}.\overline{F}.\overline{G}.H$	$\overline{E}.\overline{F}.G.H$	$\overline{E}.\overline{F}.G.\overline{H}$	$\overline{E}.F.\overline{G}.\overline{H}$	$\overline{E}.F.\overline{G}.H$	$\overline{E}.F.G.H$	$\overline{E}.F.G.\overline{H}$	$E.F.\overline{G}.\overline{H}$	$E.F.\overline{G}.H$	$E.F.G.H$	$E.F.G.\overline{H}$	$E.\overline{F}.\overline{G}.\overline{H}$	$E.\overline{F}.\overline{G}.H$	$E.\overline{F}.G.H$	$E.\overline{F}.G.\overline{H}$
$\overline{A}.\overline{B}.\overline{C}.\overline{D}$	0	1	3	2	4	5	7	6	12	13	15	14	8	9	11	10
$\overline{A}.\overline{B}.\overline{C}.D$	16	17	19	18	20	21	23	22	28	29	31	30	24	25	27	26
$\overline{A}.\overline{B}.C.D$	48	49	51	50	52	53	55	54	60	61	63	62	56	57	59	58
$\overline{A}.\overline{B}.C.\overline{D}$	32	33	35	34	36	37	39	38	44	45	47	46	40	41	43	42
$\overline{A}.B.\overline{C}.\overline{D}$	64	65	67	66	68	69	71	70	76	77	79	78	72	73	75	74
$\overline{A}.B.\overline{C}.D$	80	81	83	82	84	85	87	86	92	93	95	94	88	89	91	90
$\overline{A}.B.C.D$	112	113	115	114	116	117	119	118	124	125	127	126	120	121	123	122
$\overline{A}.B.C.\overline{D}$	96	97	99	98	100	101	103	102	108	109	111	110	104	105	107	106
$A.B.\overline{C}.\overline{D}$	192	193	195	194	196	197	199	198	204	205	207	206	200	201	203	202
$A.B.\overline{C}.D$	208	209	211	210	212	213	215	214	220	221	223	222	216	217	219	218
$A.B.C.D$	240	241	243	242	244	245	247	246	252	253	255	254	248	249	251	250
$A.B.C.\overline{D}$	224	225	227	226	228	229	231	230	236	237	239	238	232	233	235	234
$A.\overline{B}.\overline{C}.\overline{D}$	128	129	131	130	132	133	135	134	140	141	143	142	136	137	139	138
$A.\overline{B}.\overline{C}.D$	144	145	147	146	148	149	151	150	156	157	159	158	152	153	155	154
$A.\overline{B}.C.D$	176	177	179	178	180	181	183	182	188	189	191	190	184	185	187	186
$A.\overline{B}.C.\overline{D}$	160	161	163	162	164	165	167	166	172	173	175	174	168	169	171	170

Fig. 1: Standard Kmap with minterms for an 8-variable Boolean function

	$\overline{E}.\overline{F}.\overline{G}.\overline{H}$	$\overline{E}.\overline{F}.\overline{G}.H$	$\overline{E}.\overline{F}.G.H$	$\overline{E}.\overline{F}.G.\overline{H}$	$\overline{E}.F.\overline{G}.\overline{H}$	$\overline{E}.F.\overline{G}.H$	$\overline{E}.F.G.H$	$\overline{E}.F.G.\overline{H}$	$E.F.\overline{G}.\overline{H}$	$E.F.\overline{G}.H$	$E.F.G.H$	$E.F.G.\overline{H}$	$E.\overline{F}.\overline{G}.\overline{H}$	$E.\overline{F}.\overline{G}.H$	$E.\overline{F}.G.H$	$E.\overline{F}.G.\overline{H}$
$\overline{A}.\overline{B}.\overline{C}.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\overline{A}.\overline{B}.\overline{C}.D$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\overline{A}.\overline{B}.C.D$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\overline{A}.\overline{B}.C.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\overline{A}.B.\overline{C}.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\overline{A}.B.\overline{C}.D$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\overline{A}.B.C.D$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\overline{A}.B.C.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A.B.\overline{C}.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A.B.\overline{C}.D$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A.B.C.D$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A.B.C.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A.\overline{B}.\overline{C}.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A.\overline{B}.\overline{C}.D$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A.\overline{B}.C.D$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
$A.\overline{B}.C.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Fig. 2: Kmap for adder outputs J, K and L; J = K = L = $\sum$ (187); $J = K = L = A.\overline{B}.C.D.E.\overline{F}.G.H$

	$\overline{E}.\overline{F}.\overline{G}.\overline{H}$	$\overline{E}.\overline{F}.\overline{G}.H$	$\overline{E}.\overline{F}.G.H$	$\overline{E}.\overline{F}.G.\overline{H}$	$\overline{E}.F.\overline{G}.\overline{H}$	$\overline{E}.F.\overline{G}.H$	$\overline{E}.F.G.H$	$\overline{E}.F.G.\overline{H}$	$E.F.\overline{G}.\overline{H}$	$E.F.\overline{G}.H$	$E.F.G.H$	$E.F.G.\overline{H}$	$E.\overline{F}.\overline{G}.\overline{H}$	$E.\overline{F}.\overline{G}.H$	$E.\overline{F}.G.H$	$E.\overline{F}.G.\overline{H}$
$\overline{A}.\overline{B}.\overline{C}.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\overline{A}.\overline{B}.\overline{C}.D$	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
$\overline{A}.\overline{B}.C.D$	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
$\overline{A}.\overline{B}.C.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\overline{A}.B.\overline{C}.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\overline{A}.B.\overline{C}.D$	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0
$\overline{A}.B.C.D$	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
$\overline{A}.B.C.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A.B.\overline{C}.\overline{D}$	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
$A.B.\overline{C}.D$	0	1	1	0	0	1	1	0	1	1	1	1	0	1	1	0
$A.B.C.D$	0	1	0	0	0	1	0	0	1	1	0	0	0	1	0	0
$A.B.C.\overline{D}$	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
$A.\overline{B}.\overline{C}.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A.\overline{B}.\overline{C}.D$	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0	0
$A.\overline{B}.C.D$	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0
$A.\overline{B}.C.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Fig. 3: Kmap for adder output M

Outputs J and K and L (Kmap, Boolean expression): Figure 2 discussed the output J, K and L.

Output M (Kmap, specific minterm's groups for simplification, Boolean expression): Figure 3 and 4 discussed the output M.

Output P (Kmap, specific minterm's groups for simplification, Boolean expression): Figure 5 and 6 discussed Kmap.

Output R (Kmap, specific minterm's groups for simplification, Boolean Expression): Figure 7 and 8 discussed the output R.

(29,31,93,95,157,159,221,223)	$\overline{C}.D.E.F.H$
(29,61,93,125,157,189,221,253)	$D.E.F.\overline{G}.H$
(89,91,93,95,217,219,221,223)	$B.\overline{C}.D.E.H$
(89,93,121,125,217,221,249,253)	$B.D.E.\overline{G}.H$
(149,151,157,159,213,215,221,223)	$A.\overline{C}.D.F.H$
(149,157,181,189,213,221,245,253)	$A.D.F.\overline{G}.H$
(204,205,206,207,220,221,222,223)	$A.B.\overline{C}.E.F$
(204,205,220,221,236,237,252,253)	$A.B.E.F.\overline{G}$
(209,211,213,215,217,219,221,223)	$A.B.\overline{C}.D.H$
(209,213,217,221,241,245,249,253)	$A.B.D.\overline{G}.H$

Fig. 4: Specific minterms groups for simplification; $M = \overline{C}.D.E.F.H + D.E.F.\overline{G}.H + B.\overline{C}.D..H + B.D.E.\overline{G},H + A.\overline{C}.D.F.H + A.B.\overline{C}.E.F + A.B.E.F.\overline{G} + A.B.\overline{C}.D.H + A.B.D.\overline{G}.H$

	$\bar{E}.\bar{F}.\bar{G}.\bar{H}$	$\bar{E}.\bar{F}.\bar{G}.H$	$\bar{E}.\bar{F}.G.H$	$\bar{E}.\bar{F}.G.\bar{H}$	$\bar{E}.F.\bar{G}.\bar{H}$	$\bar{E}.F.\bar{G}.H$	$\bar{E}.F.G.H$	$\bar{E}.F.G.\bar{H}$	$E.F.\bar{G}.\bar{H}$	$E.F.\bar{G}.H$	$E.F.G.H$	$E.F.G.\bar{H}$	$E.\bar{F}.\bar{G}.\bar{H}$	$E.\bar{F}.\bar{G}.H$	$E.\bar{F}.G.H$	$E.\bar{F}.G.\bar{H}$
$\bar{A}.\bar{B}.\bar{C}.\bar{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{A}.\bar{B}.\bar{C}.D$	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
$\bar{A}.\bar{B}.C.D$	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
$\bar{A}.\bar{B}.C.\bar{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{A}.B.\bar{C}.\bar{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{A}.B.\bar{C}.D$	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0
$\bar{A}.B.C.D$	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0
$\bar{A}.B.C.\bar{D}$	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
$A.B.\bar{C}.\bar{D}$	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
$A.B.\bar{C}.D$	0	1	1	0	0	1	1	0	1	1	1	1	0	1	1	0
$A.B.C.D$	0	1	0	0	0	1	0	0	1	1	0	0	0	1	0	0
$A.B.C.\bar{D}$	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
$A.\bar{B}.\bar{C}.\bar{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A.\bar{B}.\bar{C}.D$	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0	0
$A.\bar{B}.C.D$	0	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0
$A.\bar{B}.C.\bar{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Fig. 5: Kmap for adder output P

(29,31,93,95,157,159,221,223)	$\bar{C}.D.E.F.H$
(29,61,93,125,157,189,221,253)	$D.E.F.\bar{G}.H$
(89,91,93,95,217,219,221,223)	$B.\bar{C}.D.E.H$
(89,93,121,125,217,221,249,253)	$B.D.E.\bar{G}.H$
(149,151,157,159,213,215,221,223)	$A.\bar{C}.D.F.H$
(149,157,181,189,213,221,245,253)	$A.D.F.\bar{G}.H$
(204,205,206,207,220,221,222,223)	$A.B.\bar{C}.E.F$
(204,205,220,221,236,237,252,253)	$A.B.E.F.\bar{G}$
(209,211,213,215,217,219,221,223)	$A.B.\bar{C}.D.H$
(209,213,217,221,241,245,249,253)	$A.B.D.\bar{G}.H$
(102,103)	$\bar{A}.B.C.\bar{D}.\bar{E}.F.G$
(102,118)	$\bar{A}.B.C.\bar{E}.F.G.\bar{H}$
(187)	$A.\bar{B}.C.D.E.\bar{F}.G.H$

Fig. 6: Specific minterms groups for simplification; $\bar{P} = \bar{C}.D.E.F.H + D.E.F.\bar{G}.H + B.\bar{C}.D..H + B.D.E.\bar{G},H + A.\bar{C}.D.F.H + A.D.F.\bar{G}.H + A.B.\bar{C}.E.F + A.B.E.F.\bar{G} + A.B.\bar{C}.D.H + A.B.D.\bar{G}.H + \bar{A}.B.C.\bar{D}.\bar{E}.F.G + \bar{A}.B.C.\bar{E}.F.G.\bar{H} + A.\bar{B}.C.D.E.\bar{F}.G.H$

	$\bar{E}.\bar{F}.\bar{G}.\bar{H}$	$\bar{E}.\bar{F}.\bar{G}.H$	$\bar{E}.\bar{F}.G.H$	$\bar{E}.\bar{F}.G.\bar{H}$	$\bar{E}.F.\bar{G}.\bar{H}$	$\bar{E}.F.\bar{G}.H$	$\bar{E}.F.G.H$	$\bar{E}.F.G.\bar{H}$	$E.F.\bar{G}.\bar{H}$	$E.F.\bar{G}.H$	$E.F.G.H$	$E.F.G.\bar{H}$	$E.\bar{F}.\bar{G}.\bar{H}$	$E.\bar{F}.\bar{G}.H$	$E.\bar{F}.G.H$	$E.\bar{F}.G.\bar{H}$
$\bar{A}.\bar{B}.\bar{C}.\bar{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{A}.\bar{B}.\bar{C}.D$	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0
$\bar{A}.\bar{B}.C.D$	0	0	1	0	0	0	0	0	0	1	0	1	0	1	1	1
$\bar{A}.\bar{B}.C.\bar{D}$	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1
$\bar{A}.B.\bar{C}.\bar{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{A}.B.\bar{C}.D$	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0
$\bar{A}.B.C.D$	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	1
$\bar{A}.B.C.\bar{D}$	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1
$A.B.\bar{C}.\bar{D}$	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
$A.B.\bar{C}.D$	0	1	1	0	0	1	1	0	1	1	1	1	1	1	1	1
$A.B.C.D$	0	1	0	1	0	1	0	0	1	1	1	0	1	1	1	1
$A.B.C.\bar{D}$	0	0	1	1	0	0	0	0	1	1	0	0	1	1	1	1
$A.\bar{B}.\bar{C}.\bar{D}$	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
$A.\bar{B}.\bar{C}.D$	0	1	1	0	0	1	1	0	1	1	1	1	1	1	1	1
$A.\bar{B}.C.D$	0	1	1	1	0	1	0	1	1	1	1	1	1	1	0	1
$A.\bar{B}.C.\bar{D}$	0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1

Fig. 7: Kmap for adder output R

Output S (Kmap, specific minterm's groups for simplification, Boolean expression): Figure 9 and 10 discussed the output S.

Output T (Kmap, specific minterm's groups for simplification, Boolean expression): Figure 11 and 12 discussed the output T.

Output U (Kmap, specific minterm's groups for simplification, Boolean expression): Figure 13 and 14 discussed the output U.

Output W (Kmap, specific minterm's groups for simplification, Boolean expression): Figure 15 and 16 discussed the output W.

(136,137,138,139,140,141,142,143,152,153,154,155,156,157,158,159,200,201,202,203,204,205,206,207,216,217,218,219,220,221,222,223)	$A.\bar{C}.E$
(136,137,140,141,152,153,156,157,168,169,172,173,184,185,188,189,200,201,204,205,216,217,220,221,232,233,236,237,248,249,252,253)	$A.E.\bar{G}$
(25,27,29,31,89,91,93,95,153,155,157,159,217,219,221,223)	$\bar{C}.D.E.H$
(25,29,57,61,89,93,121,125,153,157,185,189,217,221,249,253)	$D.E.\bar{G}.H$
(145,147,149,151,153,155,157,159,209,211,213,215,217,219,221,223)	$A.\bar{C}.D.H$
(145,149,153,157,177,181,185,189,209,213,217,221,241,245,249,253)	$A.D.\bar{G}.H$
(200,201,202,203,216,217,218,219,232,233,234,235,248,249,250,251)	$A.B.E.\bar{F}$
(42,43,46,47,170,171,174,175)	$\bar{B}.C.\bar{D}.E.G$
(42,43,106,107,170,171,234,235)	$C.\bar{D}.E.\bar{F}.G$
(42,46,58,62,170,174,186,190)	$\bar{B}.C.E.G.\bar{H}$
(42,58,106,122,170,186,234,250)	$C.E.\bar{F}.G.\bar{H}$
(157,159,189,191,221,223,253,255)	$A.D.E.F.H$
(162,163,166,167,170,171,174,175)	$A.\bar{B}.C.\bar{D}.G$
(162,163,170,171,226,227,234,235)	$A.C.\bar{D}.\bar{F}.G$
(162,166,170,174,178,182,186,190)	$A.\bar{B}.C.G.\bar{H}$
(162,170,178,186,226,234,242,250)	$A.C.\bar{F}.G.\bar{H}$
(145,147,177,179)	$A.\bar{B}.D.\bar{E}.\bar{F}.H$
(51,59)	$\bar{A}.\bar{B}.C.D.\bar{F}.G.H$
(102,103)	$\bar{A}.B.C.\bar{D}.\bar{E}.F.G$
(102,118)	$\bar{A}.B.C.\bar{E}.F.G.\bar{H}$

Fig. 8: Specific minterms groups for simplification;

$$R = A.\bar{C}.E+A.E.\bar{G}+C.D.E.H+D.E.\bar{G}.H+A.\bar{C}.D.H+A.D.\bar{G}.H+A.B.E.\bar{F}.+\bar{B}.C.\bar{D}.E.G+C.\bar{D}.E.\bar{F}.G+\bar{B}.C.E.G.\bar{H}+C.E.\bar{F}.G.\bar{H}+A.D.E.F.H+A.\bar{B}.C.\bar{D}.G+A.C.\bar{D}.\bar{E}.G+A.\bar{B}.C.G.\bar{H}+A.C.\bar{F}.G.\bar{H}+A.\bar{B}.D.\bar{E}.\bar{F}.H+\bar{A}.\bar{B}.C.D.\bar{F}.G.H+\bar{A}.B.C.\bar{D}.\bar{E}.F.G+\bar{A}.B.C.\bar{E}.F.G.\bar{H}$$

	$\bar{E}.\bar{F}.\bar{G}.\bar{H}$	$\bar{E}.\bar{F}.\bar{G}.H$	$\bar{E}.\bar{F}.G.H$	$\bar{E}.\bar{F}.G.\bar{H}$	$\bar{E}.F.\bar{G}.\bar{H}$	$\bar{E}.F.\bar{G}.H$	$\bar{E}.F.G.H$	$\bar{E}.F.G.\bar{H}$	$E.F.\bar{G}.\bar{H}$	$E.F.\bar{G}.H$	$E.F.G.H$	$E.F.G.\bar{H}$	$E.\bar{F}.\bar{G}.\bar{H}$	$E.\bar{F}.\bar{G}.H$	$E.\bar{F}.G.H$	$E.\bar{F}.G.\bar{H}$
$\bar{A}.\bar{B}.\bar{C}.\bar{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{A}.\bar{B}.\bar{C}.D$	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0
$\bar{A}.\bar{B}.C.D$	0	0	1	0	0	1	0	0	0	0	0	1	0	1	1	1
$\bar{A}.\bar{B}.C.\bar{D}$	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1
$\bar{A}.B.\bar{C}.\bar{D}$	0	1	0	0	1	1	1	1	1	1	1	1	0	0	0	0
$\bar{A}.B.\bar{C}.D$	0	0	1	0	1	1	1	1	1	0	0	1	0	0	0	0
$\bar{A}.B.C.D$	0	1	0	0	1	1	0	1	1	0	0	0	0	0	0	1
$\bar{A}.B.C.\bar{D}$	0	0	0	0	1	1	1	1	1	1	0	0	0	0	1	1
$A.B.\bar{C}.\bar{D}$	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
$A.B.\bar{C}.D$	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	1
$A.B.C.D$	0	0	0	1	1	0	0	0	0	0	1	0	1	0	1	1
$A.B.C.\bar{D}$	0	0	1	1	1	1	0	0	0	0	0	0	1	1	1	1
$A.\bar{B}.\bar{C}.\bar{D}$	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
$A.\bar{B}.\bar{C}.D$	0	1	1	0	0	0	0	0	1	0	0	1	1	1	1	1
$A.\bar{B}.C.D$	0	1	1	1	0	0	0	1	1	0	1	1	1	1	0	1
$A.\bar{B}.C.\bar{D}$	0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1

Fig. 9: Kmap for adder output S

(136,137,138,139,140,141,142,143,168,169,170,171,172,173,174,175)	$A.\bar{B}.\bar{D}.E$
(136,137,138,139,168,169,170,171,200,201,202,203,232,233,234,235)	$A.\bar{D}.E.\bar{F}$
(136,138,140,142,152,154,156,158,168,170,172,174,184,186,188,190)	$A.\bar{B}.E.\bar{H}$
(136,138,152,154,168,170,184,186,200,202,216,218,232,234,248,250)	$A.E.\bar{F}.\bar{H}$
(42,43,46,47,170,171,174,175)	$\bar{B}.C.\bar{D}.E.G$
(42,43,106,107,170,171,234,235)	$C.\bar{D}.E.\bar{F}.G$
(42,46,58,62,170,174,186,190)	$\bar{B}.C.E.G.\bar{H}$
(42,58,106,122,170,186,234,250)	$C.E.\bar{F}.G.\bar{H}$
(68,69,70,71,76,77,78,79)	$\bar{A}.B.\bar{C}.\bar{D}.F$
(68,69,70,71,100,101,102,103)	$\bar{A}.B.\bar{D}.\bar{E}.F$
(68,69,70,71,196,197,198,199)	$B.\bar{C}.\bar{D}.\bar{E}.F$
(68,69,76,77,100,101,108,109)	$\bar{A}.B.\bar{D}.F.\bar{G}$
(68,69,100,101,196,197,228,229)	$B.\bar{D}.\bar{E}.F.\bar{G}$
(68,70,76,78,84,86,92,94)	$\bar{A}.B.\bar{C}.F.\bar{H}$
(68,70,84,86,100,102,116,118)	$\bar{A}.B.\bar{E}.F.\bar{H}$
(68,70,84,86,196,198,212,214)	$B.\bar{C}.\bar{E}.F.\bar{H}$
(68,76,84,92,100,108,116,124)	$\bar{A}.B.F.\bar{G}.\bar{H}$
(68,84,100,116,196,212,228,244)	$B.\bar{E}.F.\bar{G}.\bar{H}$
(136,137,138,139,152,153,154,155)	$A.\bar{B}.\bar{C}.E.\bar{F}$
(136,137,152,153,168,169,184,185)	$A.\bar{B}.E.\bar{F}.\bar{G}$
(162,163,166,167,170,171,174,175)	$A.\bar{B}.C.\bar{D}.G$
(162,163,170,171,226,227,234,235)	$A.C.\bar{D}.\bar{F}.G$
(162,166,170,174,178,182,186,190)	$A.\bar{B}.C.G.\bar{H}$
(162,170,178,186,226,234,242,250)	$A.C.\bar{F}.G.\bar{H}$
(21,23,85,87)	$\bar{A}.\bar{C}.D.\bar{E}.F.H$
(21,53,85,117)	$\bar{A}.D.\bar{E}.F.\bar{G}.H$
(25,27,57,59)	$\bar{A}.\bar{B}.D.E.\bar{F}.H$
(145,147,177,179)	$A.\bar{B}.D.\bar{E}.\bar{F}.H$
(234,235,250,251)	$A.B.C.E.\bar{F}.G$
(51,59)	$\bar{A}.\bar{B}.C.D.\bar{F}.G.H$
(65,69)	$\bar{A}.B.\bar{C}.\bar{D}.\bar{E}.\bar{G}.H$
(83,87)	$\bar{A}.B.\bar{C}.D.\bar{E}.G.H$
(113,117)	$\bar{A}.B.C.D.\bar{E}.\bar{G}.H$
(191,255)	$A.C.D.E.F.G.H$

Fig. 10: Specific minterms groups for simplification;

$$S = A.\bar{B}.\bar{D}.E+A.\bar{D}.E.\bar{F}+A.\bar{B}.E.\bar{H}+A.E.\bar{F}.\bar{H}+\bar{B}.C.\bar{D}.E.G+C.\bar{D}.E.\bar{F}.G+\bar{B}.C.E.G.\bar{H}+C.E.\bar{F}.G.\bar{H}+\bar{A}.B.\bar{C}.\bar{D}.F+\bar{A}.B.\bar{D}.\bar{E}.F+B.\bar{C}.\bar{D}.\bar{E}.F+\bar{A}.B.\bar{D}.F.\bar{G}+B.\bar{D}.\bar{E}.F.\bar{G}+\bar{A}.B.\bar{C}.F.\bar{H}+\bar{A}.B.\bar{E}.F.\bar{H}+B.\bar{C}.\bar{E}.F.\bar{H}+\bar{A}.B.F.\bar{G}.\bar{H}+B.\bar{E}.F.\bar{G}.\bar{H}+A.\bar{B}.\bar{C}.E.\bar{F}+A.\bar{B}.E.\bar{F}.\bar{G}+A.\bar{B}.C.\bar{D}.G+A.C.\bar{D}.\bar{F}.G+A.\bar{B}.C.G.\bar{H}+A.C.\bar{F}.G.\bar{H}+\bar{A}.\bar{C}.D.\bar{E}.F.H+\bar{A}.D.\bar{E}.F.\bar{G}.H+\bar{A}.\bar{B}.D.E.\bar{F}.H+A.\bar{B}.D.\bar{E}.\bar{F}.H+A.B.C.E.\bar{F}.G+\bar{A}.\bar{B}.C.D.\bar{F}.G.H+\bar{A}.B.\bar{C}.\bar{D}.\bar{E}.\bar{G}.H \; \bar{A}.B.\bar{C}.D.\bar{E}.G.H+\bar{A}.B.C.D.\bar{E}.\bar{G}.H+\bar{A}.C.D.E.F.G.H$$

	$\overline{E}.\overline{F}.\overline{G}.\overline{H}$	$\overline{E}.\overline{F}.\overline{G}.H$	$\overline{E}.\overline{F}.G.H$	$\overline{E}.\overline{F}.G.\overline{H}$	$\overline{E}.F.\overline{G}.\overline{H}$	$\overline{E}.F.\overline{G}.H$	$\overline{E}.F.G.H$	$\overline{E}.F.G.\overline{H}$	$E.F.\overline{G}.\overline{H}$	$E.F.\overline{G}.H$	$E.F.G.H$	$E.F.G.\overline{H}$	$E.\overline{F}.\overline{G}.\overline{H}$	$E.\overline{F}.\overline{G}.H$	$E.\overline{F}.G.H$	$E.\overline{F}.G.\overline{H}$
$\overline{A}.\overline{B}.\overline{C}.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\overline{A}.\overline{B}.\overline{C}.D$	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0	0
$\overline{A}.\overline{B}.C.D$	0	0	1	1	0	1	0	1	0	1	0	1	0	0	1	1
$\overline{A}.\overline{B}.C.\overline{D}$	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
$\overline{A}.B.\overline{C}.\overline{D}$	0	1	0	0	1	1	1	1	1	1	1	1	0	0	0	0
$\overline{A}.B.\overline{C}.D$	0	0	1	0	1	1	1	1	1	1	1	1	0	1	1	0
$\overline{A}.B.C.D$	0	1	0	1	1	1	0	0	1	1	0	0	0	1	0	1
$\overline{A}.B.C.\overline{D}$	0	0	1	1	1	1	0	0	1	1	0	0	0	0	1	1
$A.B.\overline{C}.\overline{D}$	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
$A.B.\overline{C}.D$	0	1	1	0	1	1	1	1	1	1	1	1	0	1	1	0
$A.B.C.D$	0	1	0	1	1	1	0	0	1	1	0	0	0	1	0	1
$A.B.C.\overline{D}$	0	0	1	1	1	1	0	0	1	1	0	0	0	0	1	1
$A.\overline{B}.\overline{C}.\overline{D}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A.\overline{B}.\overline{C}.D$	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0	0
$A.\overline{B}.C.D$	0	0	1	1	0	1	0	1	0	1	0	1	0	0	1	1
$A.\overline{B}.C.\overline{D}$	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1

Fig. 11: Kmap for adder output T

(68,69,70,71,76,77,78,79,84,85,86,87,92,93,94,95,196,197,198,199,204,205,206,207,212,213,214,215,220,221,222,223)	$B.\overline{C}.F$
(68,69,76,77,84,85,92,93,100,101,108,109,116,117,124,125,196,197,204,205,212,213,220,221,228,229,236,237,244,245,252,253)	$B.F.\overline{G}$
(21,23,29,31,85,87,93,95,149,151,157,159,213,215,221,223)	$\overline{C}.D.F.H$
(21,29,53,61,85,93,117,125,149,157,181,189,213,221,245,253)	$D.F.\overline{G}.H$
(34,35,38,39,42,43,46,47,162,163,166,167,170,171,174,175)	$\overline{B}.C.\overline{D}.G$
(34,35,42,43,50,51,58,59,162,163,170,171,178,179,186,187)	$\overline{B}.C.\overline{F}.G$
(34,35,42,43,98,99,106,107,162,163,170,171,226,227,234,235)	$C.\overline{D}.\overline{F}.G$
(34,38,42,46,50,54,58,62,162,166,170,174,178,182,186,190)	$\overline{B}.C.G.\overline{H}$
(34,42,50,58,98,106,114,122,162,170,178,186,226,234,242,250)	$C.\overline{F}.G.\overline{H}$
(83,87,91,95,211,215,219,223)	$B.\overline{C}.D.G.H$
(89,91,93,95,217,219,221,223)	$B.\overline{C}.D.E.H$
(113,117,121,125,241,245,249,253)	$B.C.D.\overline{G}.H$
(209,211,213,215,217,219,221,223)	$A.B.\overline{C}.D.H$
(65,69)	$\overline{A}.B.\overline{C}.\overline{D}.\overline{E}.\overline{G}.H$

Fig. 12: Specific minterms groups for simplification;

$$T = B.\overline{C}.F + B.F.\overline{G} + \overline{C}.D.F.H + D.F.\overline{G}.H + \overline{B}.C.\overline{D}.G + \overline{B}.C.\overline{F}.G + C.\overline{D}.\overline{F}.G + \overline{B}.C.G.\overline{H} + C.\overline{F}.G.\overline{H} + B.\overline{C}.D.G.H + B.\overline{C}.D.E.H + B.C.D.\overline{G}.H + A.B.\overline{C}.D.H + \overline{A}.B.\overline{C}.\overline{D}.\overline{E}.\overline{G}.H$$

	$\overline{E}.\overline{F}.\overline{G}.\overline{H}$	$\overline{E}.\overline{F}.\overline{G}.H$	$\overline{E}.\overline{F}.G.H$	$\overline{E}.\overline{F}.G.\overline{H}$	$\overline{E}.F.\overline{G}.\overline{H}$	$\overline{E}.F.\overline{G}.H$	$\overline{E}.F.G.H$	$\overline{E}.F.G.\overline{H}$	$E.F.\overline{G}.\overline{H}$	$E.F.\overline{G}.H$	$E.F.G.H$	$E.F.G.\overline{H}$	$E.\overline{F}.\overline{G}.\overline{H}$	$E.\overline{F}.\overline{G}.H$	$E.\overline{F}.G.H$	$E.\overline{F}.G.\overline{H}$
$\overline{A}.\overline{B}.\overline{C}.\overline{D}$	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
$\overline{A}.\overline{B}.\overline{C}.D$	0	1	1	0	0	1	1	0	1	0	0	1	1	0	0	1
$\overline{A}.\overline{B}.C.D$	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0
$\overline{A}.\overline{B}.C.\overline{D}$	0	0	1	1	0	0	1	1	1	1	0	0	1	1	0	0
$\overline{A}.B.\overline{C}.\overline{D}$	0	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1
$\overline{A}.B.\overline{C}.D$	0	1	1	0	0	1	1	0	1	0	0	1	1	0	0	1
$\overline{A}.B.C.D$	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0
$\overline{A}.B.C.\overline{D}$	0	0	1	1	0	0	1	1	1	1	0	0	1	1	0	0
$A.B.\overline{C}.\overline{D}$	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
$A.B.\overline{C}.D$	1	0	0	1	1	0	0	1	0	1	1	0	0	1	1	0
$A.B.C.D$	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1
$A.B.C.\overline{D}$	1	1	0	0	1	1	0	0	0	0	1	1	0	0	1	1
$A.\overline{B}.\overline{C}.\overline{D}$	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
$A.\overline{B}.\overline{C}.D$	1	0	0	1	1	0	0	1	0	1	1	0	0	1	1	0
$A.\overline{B}.C.D$	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1
$A.\overline{B}.C.\overline{D}$	1	1	0	0	1	1	0	0	0	0	1	1	0	0	1	1

Fig. 13: Kmap for adder output U

Output Y (Kmap, specific minterm's groups for simplification, Boolean expression): Figure 17 discussed the output Y.

Output Z (Kmap, specific minterm's groups for simplification, Boolean expression): Figure 18-32 discussed the output Z. The logic diagrams of the overall

(8,9,10,11,12,13,14,15,72,73,74,75,76,77,78,79)	$\overline{A}.\overline{C}.\overline{D}.E$
(8,9,12,13,40,41,44,45,72,73,76,77,104,105,108,109)	$\overline{A}.\overline{D}.E.\overline{G}$
(8,10,12,14,24,26,28,30,72,74,76,78,88,90,92,94)	$\overline{A}.\overline{C}.E.\overline{H}$
(8,12,24,28,40,44,56,60,72,76,88,92,104,108,120,124)	$\overline{A}.E.\overline{G}.\overline{H}$
(128,129,130,131,132,133,134,135,192,193,194,195,196,197,198,199)	$A.\overline{C}.\overline{D}.\overline{E}$
(128,129,132,133,160,161,164,165,192,193,196,197,224,225,228,229)	$A.\overline{D}.\overline{E}.\overline{G}$
(128,130,132,134,144,146,148,150,192,194,196,198,208,210,212,214)	$A.\overline{C}.\overline{E}.\overline{H}$
(128,132,144,148,160,164,176,180,192,196,208,212,224,228,240,244)	$A.\overline{E}.\overline{G}.\overline{H}$
(17,19,21,23,81,83,85,87)	$\overline{A}.\overline{C}.D.\overline{E}.H$
(17,21,49,53,81,85,113,117)	$\overline{A}.D.\overline{E}.\overline{G}.H$
(34,35,38,39,98,99,102,103)	$\overline{A}.C.\overline{D}.\overline{E}.G$
(34,38,50,54,98,102,114,118)	$\overline{A}.C.\overline{E}.G.\overline{H}$
(153,155,157,159,217,219,221,223)	$A.\overline{C}.D.E.H$
(153,157,185,189,217,221,249,253)	$A.D.E.\overline{G}.H$
(170,171,174,175,234,235,238,239)	$A.C.\overline{D}.E.G$
(170,174,186,190,234,238,250,254)	$A.C.E.G.\overline{H}$
(59,63,123,127)	$\overline{A}.C.D.E.G.H$
(179,183,243,247)	$A.C.D.\overline{E}.G.H$
(65,73)	$\overline{A}.B.\overline{C}.\overline{D}.F.\overline{G}.H$

Fig. 14: Specific minterms groups for simplification; $U = \overline{A}.\overline{C}.\overline{D}.E + \overline{A}.\overline{D}.E.\overline{G} + \overline{A}.\overline{C}.E.\overline{H} + \overline{A}.E.\overline{G}.\overline{H} + A.\overline{C}.\overline{D}.\overline{E} + A.\overline{D}.\overline{E}.\overline{G} + A.\overline{C}.\overline{E}.\overline{H} + A.\overline{E}.\overline{G}.\overline{H} + \overline{A}.\overline{C}.D.\overline{E}.H + \overline{A}.D.\overline{E}.\overline{G}.H + \overline{A}.C.\overline{E}.G.\overline{H} + A.\overline{C}.D.E.H + A.D.E.\overline{G}.H + A.C.\overline{D}.E.G + A.C.E.G.\overline{H} + \overline{A}.C.D.E.G.H + A.C.D.\overline{E}.G.H + \overline{A}.B.\overline{C}.\overline{D}.F.\overline{G}.H$

	$\overline{E}.\overline{F}.\overline{G}.\overline{H}$	$\overline{E}.\overline{F}.\overline{G}.H$	$\overline{E}.\overline{F}.G.H$	$\overline{E}.\overline{F}.G.\overline{H}$	$\overline{E}.F.\overline{G}.\overline{H}$	$\overline{E}.F.\overline{G}.H$	$\overline{E}.F.G.H$	$\overline{E}.F.G.\overline{H}$	$E.F.\overline{G}.\overline{H}$	$E.F.\overline{G}.H$	$E.F.G.H$	$E.F.G.\overline{H}$	$E.\overline{F}.\overline{G}.\overline{H}$	$E.\overline{F}.\overline{G}.H$	$E.\overline{F}.G.H$	$E.\overline{F}.G.\overline{H}$
$\overline{A}.\overline{B}.\overline{C}.\overline{D}$	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
$\overline{A}.\overline{B}.\overline{C}.D$	0	1	1	0	1	0	0	1	1	0	0	1	0	1	1	0
$\overline{A}.\overline{B}.C.D$	0	1	1	0	1	0	0	1	1	0	0	1	0	1	1	0
$\overline{A}.\overline{B}.C.\overline{D}$	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
$\overline{A}.B.\overline{C}.\overline{D}$	1	0	1	1	0	0	0	0	0	0	0	0	1	1	1	1
$\overline{A}.B.\overline{C}.D$	1	1	0	1	0	1	1	0	0	1	1	0	1	0	0	1
$\overline{A}.B.C.D$	1	0	0	1	0	1	1	0	0	1	1	0	1	0	0	1
$\overline{A}.B.C.\overline{D}$	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1
$A.B.\overline{C}.\overline{D}$	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1
$A.B.\overline{C}.D$	1	0	0	1	0	1	1	0	0	1	1	0	1	0	0	1
$A.B.C.D$	1	0	0	1	0	1	1	0	0	1	1	0	1	0	0	1
$A.B.C.\overline{D}$	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1
$A.\overline{B}.\overline{C}.\overline{D}$	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
$A.\overline{B}.\overline{C}.D$	0	1	1	0	1	0	0	1	0	1	0	1	0	1	1	0
$A.\overline{B}.C.D$	0	1	1	0	1	0	0	1	1	0	0	1	0	1	1	0
$A.\overline{B}.C.\overline{D}$	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0

Fig. 15: Kmap for adder output W

(4,5,6,7,12,13,14,15,36,37,38,39,44,45,46,47,132,133,134,135,140,141,142,143,164,165,166,167,172,173,174,175)	$\overline{B}.\overline{D}.F$
(64,66,72,74,80,82,88,90,96,98,104,106,112,114,120,122,192,194,200,202,208,210,216,218,224,226,232,234,240,242,248,250)	$B.\overline{F}.\overline{H}$
(4,6,12,14,20,22,28,30,36,38,44,46,52,54,60,62)	$\overline{A}.\overline{B}.F.\overline{H}$
(4,6,20,22,36,38,52,54,132,134,148,150,164,166,180,182)	$\overline{B}.\overline{E}.F.\overline{H}$
(6,14,22,30,38,46,54,62,134,142,150,158,166,174,182,190)	$\overline{B}.F.G.\overline{H}$
(17,19,25,27,49,51,57,59,145,147,153,155,177,179,185,187)	$\overline{B}.D.\overline{F}.H$
(36,38,44,46,52,54,60,62,164,166,172,174,180,182,188,190)	$\overline{B}.C.F.\overline{H}$
(66,67,74,75,98,99,106,107,194,195,202,203,226,227,234,235)	$B.\overline{D}.\overline{F}.G$
(72,73,74,75,104,105,106,107,200,201,202,203,232,233,234,235)	$B.\overline{D}.E.\overline{F}$
(85,87,93,95,117,119,125,127,213,215,221,223,245,247,253,255)	$B.D.F.H$
(96,97,98,99,104,105,106,107,224,225,226,227,232,233,234,235)	$B.C.\overline{D}.\overline{F}$
(192,193,194,195,200,201,202,203,224,225,226,227,232,233,234,235)	$A.B.\overline{D}.\overline{F}$
(17,81)	$\overline{A}.\overline{C}.D.\overline{E}.\overline{F}.\overline{G}.H$
(141,157)	$A.\overline{B}.\overline{C}.E.F.\overline{G}.H$

Fig. 16: Specific minterms groups for simplification; $W = \overline{B}.\overline{D}.F + B.\overline{F}.\overline{H} + \overline{A}.\overline{B}.F.\overline{H} + \overline{B}.\overline{E}.F.\overline{H} + \overline{B}.F.G.\overline{H} + \overline{B}.D.\overline{F}.H + \overline{B}.C.F.\overline{H} + B.\overline{D}.\overline{F}.G + B.D.E.F + B.D.F.H + B.C.\overline{D}.\overline{F} + A.B.\overline{D}.\overline{F} + \overline{A}.\overline{C}.D.\overline{E}.\overline{F}.\overline{G}.H + A.\overline{B}.\overline{C}.E.F.\overline{G}.H$

	$\bar{E}.\bar{F}.\bar{G}.\bar{H}$	$\bar{E}.\bar{F}.\bar{G}.H$	$\bar{E}.\bar{F}.G.H$	$\bar{E}.\bar{F}.G.\bar{H}$	$\bar{E}.F.\bar{G}.\bar{H}$	$\bar{E}.F.\bar{G}.H$	$\bar{E}.F.G.H$	$\bar{E}.F.G.\bar{H}$	$E.F.\bar{G}.\bar{H}$	$E.F.\bar{G}.H$	$E.F.G.H$	$E.F.G.\bar{H}$	$E.\bar{F}.\bar{G}.\bar{H}$	$E.\bar{F}.\bar{G}.H$	$E.\bar{F}.G.H$	$E.\bar{F}.G.\bar{H}$
$\bar{A}.\bar{B}.\bar{C}.\bar{D}$	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
$\bar{A}.\bar{B}.\bar{C}.D$	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
$\bar{A}.\bar{B}.C.D$	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
$\bar{A}.\bar{B}.C.\bar{D}$	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
$\bar{A}.B.\bar{C}.\bar{D}$	0	1	1	1	0	0	1	1	0	0	1	1	0	0	1	1
$\bar{A}.B.\bar{C}.D$	0	1	1	1	0	0	1	1	0	0	1	1	0	0	1	1
$\bar{A}.B.C.D$	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
$\bar{A}.B.C.\bar{D}$	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
$A.B.\bar{C}.\bar{D}$	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
$A.B.\bar{C}.D$	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
$A.B.C.D$	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
$A.B.C.\bar{D}$	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
$A.\bar{B}.\bar{C}.\bar{D}$	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
$A.\bar{B}.\bar{C}.D$	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
$A.\bar{B}.C.D$	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
$A.\bar{B}.C.\bar{D}$	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0

Fig. 17: Kmap for adder output Y; $Y = \bar{C}.G. + C.\bar{G}. + \bar{A}.B.\bar{C}.\bar{E}.\bar{F}.H$

	$\bar{E}.\bar{F}.\bar{G}.\bar{H}$	$\bar{E}.\bar{F}.\bar{G}.H$	$\bar{E}.\bar{F}.G.H$	$\bar{E}.\bar{F}.G.\bar{H}$	$\bar{E}.F.\bar{G}.\bar{H}$	$\bar{E}.F.\bar{G}.H$	$\bar{E}.F.G.H$	$\bar{E}.F.G.\bar{H}$	$E.F.\bar{G}.\bar{H}$	$E.F.\bar{G}.H$	$E.F.G.H$	$E.F.G.\bar{H}$	$E.\bar{F}.\bar{G}.\bar{H}$	$E.\bar{F}.\bar{G}.H$	$E.\bar{F}.G.H$	$E.\bar{F}.G.\bar{H}$
$\bar{A}.\bar{B}.\bar{C}.\bar{D}$	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
$\bar{A}.\bar{B}.\bar{C}.D$	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1
$\bar{A}.\bar{B}.C.D$	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1
$\bar{A}.\bar{B}.C.\bar{D}$	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
$\bar{A}.B.\bar{C}.\bar{D}$	0	0	1	0	0	1	1	0	0	1	1	0	0	1	1	0
$\bar{A}.B.\bar{C}.D$	1	1	0	1	1	0	0	1	1	0	0	1	1	0	0	1
$\bar{A}.B.C.D$	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1
$\bar{A}.B.C.\bar{D}$	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
$A.B.\bar{C}.\bar{D}$	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
$A.B.\bar{C}.D$	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1
$A.B.C.D$	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1
$A.B.C.\bar{D}$	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
$A.\bar{B}.\bar{C}.\bar{D}$	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
$A.\bar{B}.\bar{C}.D$	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1
$A.\bar{B}.C.D$	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1
$A.\bar{B}.C.\bar{D}$	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0

Fig. 18: Kmap for adder output Z

(16,18,20,22,24,26,28,30,48,50,52,54,56,58,60,62,80,82,84,86,88,90,92,94,112,114,116,118,120,122,124,126,144,146,148,150,152,154,156,158,176,178,180,182,184,186,188,190,208,210,212,214,216,218,220,222,240,242,244,246,248,250,252,254)	$D.\bar{H}$
(1,3,5,7,9,11,13,15,33,35,37,39,41,43,45,47,129,131,133,135,137,139,141,143,161,163,165,167,169,171,173,175)	$\bar{B}.\bar{D}.H$
(3,7,11,15,35,39,43,47,67,71,75,79,99,103,107,111,131,135,139,143,163,167,171,175,195,199,203,207,227,231,235,239)	$\bar{D}.G.H$
(5,7,13,15,37,39,45,47,69,71,77,79,101,103,109,111,133,135,141,143,165,167,173,175,197,199,205,207,229,231,237,239)	$\bar{D}.F.H$
(9,11,13,15,41,43,45,47,73,75,77,79,105,107,109,111,137,139,141,143,169,171,173,175,201,203,205,207,233,235,237,239)	$\bar{D}.E.H$
(33,35,37,39,41,43,45,47,97,99,101,103,105,107,109,111,161,163,165,167,169,171,173,175,225,227,229,231,233,235,237,239)	$C.\bar{D}.H$
(129,131,133,135,137,139,141,143,161,163,165,167,169,171,173,175,193,195,197,199,201,203,205,207,225,227,229,231,233,235,237,239)	$A.\bar{D}.H$
(80,81)	$\bar{A}.B.\bar{C}.D.\bar{E}.F.\bar{G}$

Fig. 19: Specific minterms groups for simplification

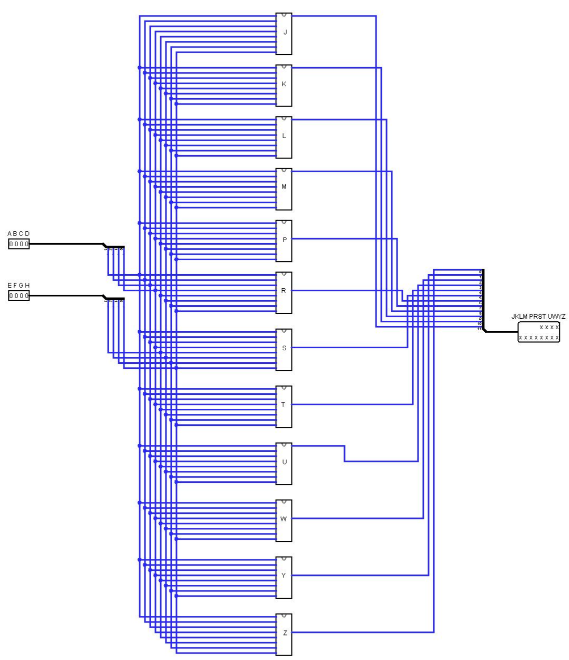

Fig. 20: Nibble-size complex binary adder

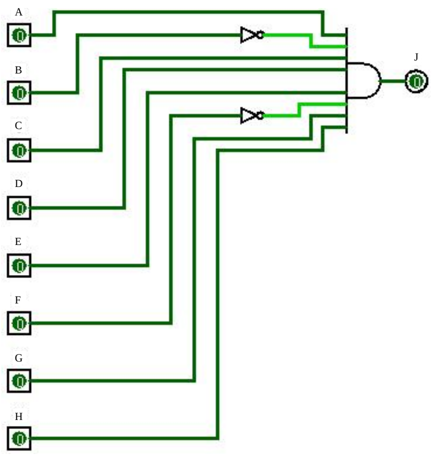

Fig. 21: Logic diagram for output J

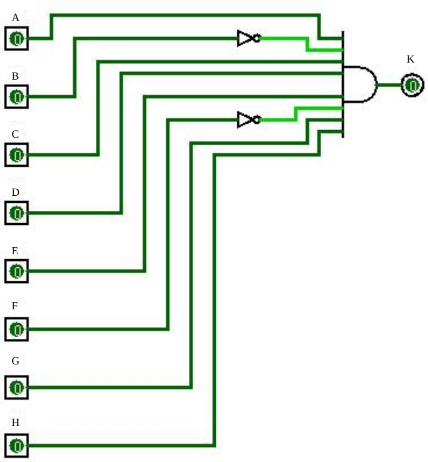

Fig. 22: Logic diagram for output K

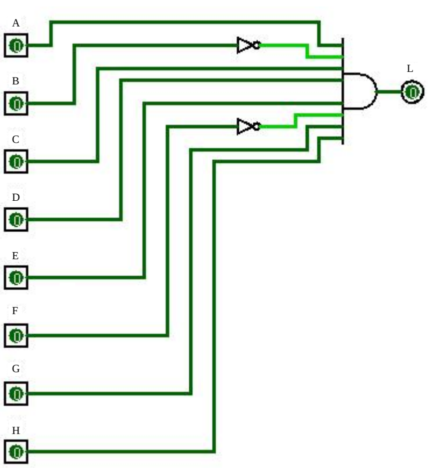

Fig. 23: Logic diagram for output L

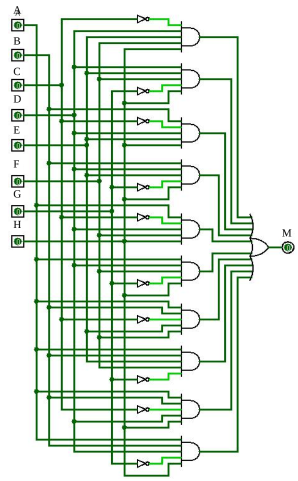

Fig. 24: Logic diagram foroutput M

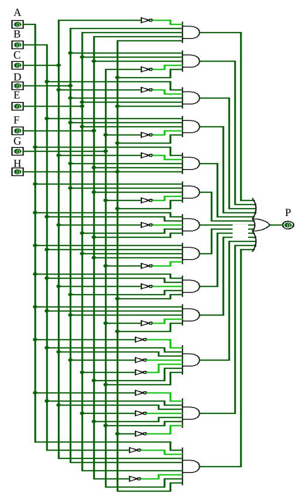

Fig. 25: Logic diagram for output P

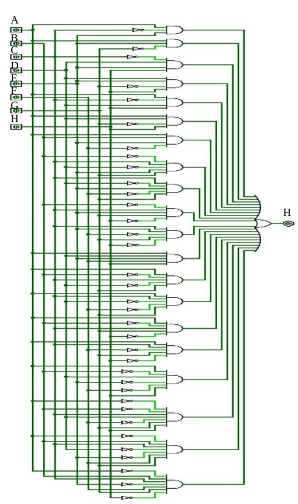

Fig. 26: Logic diagram for output R

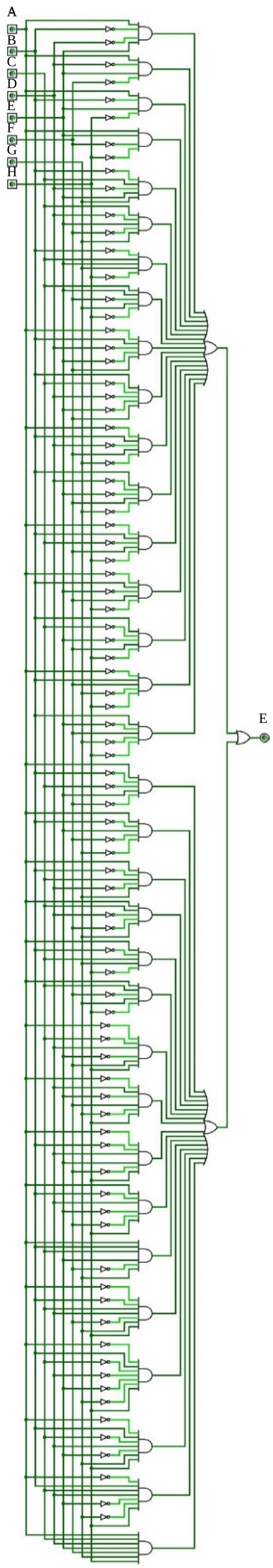

Fig. 27: Logic diagram for output S

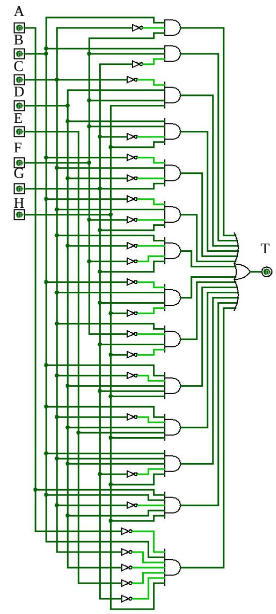

Fig. 28: Logic diagram for output T

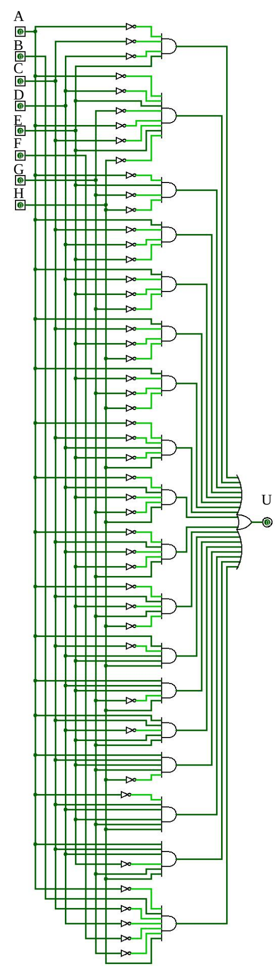

Fig. 29: Logic diagram for output U

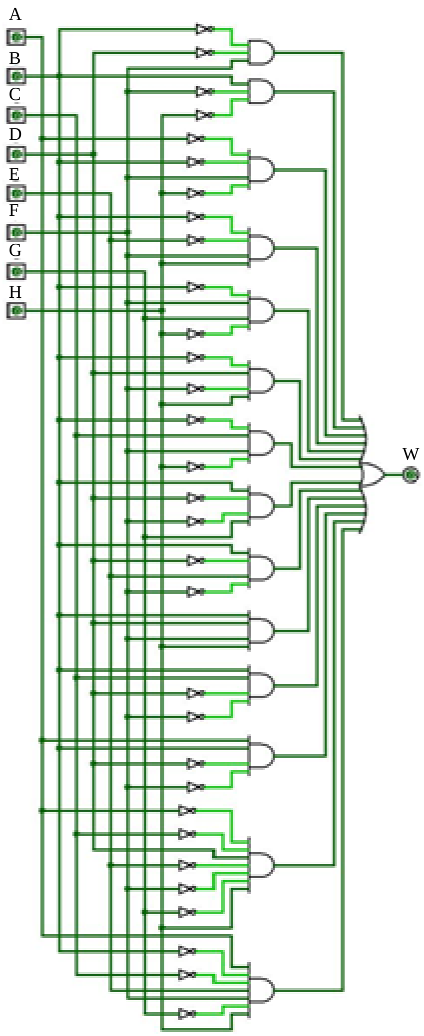

Fig. 30: Logic diagram for output W

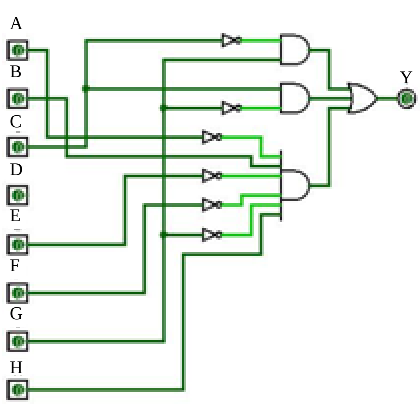

Fig. 31: Logic diagram for output Y

complex binary adder (Fig. 21) and all individual outputs of the result (Fig. 22-32) are presented as follows:

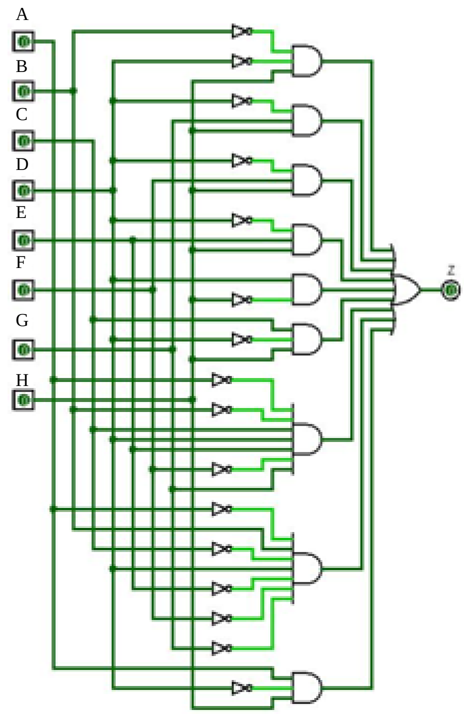

Fig. 32: Logic diagram for output Z

Table 2: Implementation statistics for our designed nibble-size complex binary adder

	Nibble-size minimum-delay complex binary adder using logic gates		
Variables	Virtex5 XC4VLX15	Virtex5 XC5VLX30	Virtex XCV100
Number of external IOBs	20/240 (8%)	20/220 (9%)	20/180 (11%)
Number of Slices	27/6144 (>1%)	25/19200 (1%)	27/1200 (2%)
Number of 4 input LUTs	52/12288 (>1%)	0/27	52/2400 (2%)
Number of bonded IOBs	20/240 (6%)	20/220 (9%)	20/180 (11%)
Gate Count	330	175	330
Maximum net delay (ns)	5.028	2.790	8.856
Maximum combinational delay (ns)	7.827	4.776	17.001

RESULTS AND DISCUSSION

Xilinx FPGAs are frequently used to implement logical expressions of the functions (Singh and Prakash, 2017; Zulfikar *et al*., 2017; Awadalla *et al*., 2017). We have also implemented the adder outputs presented in this paper on Xilinx FPGAs and the achieved results are presented in Table 2-4.

By comparing the achieved results given in Table 2 and 3, we come to the conclusion that the traditional approach of using truth table/Kmaps for designing complex binary adder has resulted in reduction of propagation delay as well as the in the number of gates required for the circuit. This will effectively minimize the

Table 3: Implementation statistics for previously designed nibble-size complex binary adders (Blest and Jamil, 2003; Jamil *et al*., 2003)

	Nibble-size complex binary adder implemented on Virtex V50CS144	
Variables	Minimum-delay using decoder	Ripple-carry
Number of external IOBs	20/94 (21%)	20/94 (21%)
Number of Slices	455/768 (59%)	31/768 (4%)
Number of 4 input LUTs	857/1536 (55%)	59/1536 (3%)
Number of bonded IOBs	20/94 (21%)	20/94 (21%)
Gate Count	5142	354
Maximum net delay (ns)	11.170	4.024
Maximum combinational delay (ns)	32.471	24.839

Table 4: Implementation statistics for previously designed nibble-size base-2 binary adders (Blest and Jamil, 2003; Jamil *et al*., 2003)

	Nibble-size base-2 binary adder implemented on Virtex V50CS144	
Variables	Minimum-delay using decoder	Ripple-carry
Number of external IOBs	13/94 (13%)	13/94 (13%)
Number of Slices	391/768 (50%)	6/768 (1%)
Number of 4 input LUTs	755/1536 (49%)	9/1536 (1%)
Number of bonded IOBs	13/94 (13%)	13/94 (13%)
Gate Count	4530	54
Maximum net delay (ns)	9.207	2.421
Maximum combinational delay (ns)	28.442	15.389

area of the adder circuit in the ALU design and allow for incorporation of additional speed-enhancement features within the microprocessor structure.

CONCLUSION

This study presented a design of a nibble-size adder based on complex binary number system using the traditional truth table/KMap approach and has been implemented on Xilinx Virtex FPGAs. The implementation results and simulation statistics show that the developed system is promising and can be used in the new generation of digital systems.

ACKNOWLEDGEMENTS

We gratefully acknowledge the support provided to us for this research by Sultan Qaboos University through internal research grant: IG/ENG/ECED/16/03. Website: www.32x8.com was used to obtain simplified Boolean expressions for the adder outputs and an open-source software, Logisim was used to obtain their logic diagrams. Xilinx FPGAs with associated proprietary software were used to implement the adder circuit.

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