

## Method for Optimal Design Electromagnets of High-Precision Systems for Positioning Objects in a Horizontal Plane

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**Abstract:** The study is proposed of the method for optimal design electromagnets of high-precision systems for positioning objects in a horizontal plane, based on the solving conditionally-correct inverse problems and the transformation of constraints on the tractive force, flux density in the gap of the electromagnet of high-precision system and temperature of the coil in the criteria (objective functions). Minimization of these objective functions is performed by gradient descent numerically. In this case is solved sequence of the direct tasks of calculation of magnetic and heat fields. The fourth criterion is the mass of the electromagnet. Minimizing mass is performed analytically. Objective functions are ranked by importance. Method is a modification of the method for solving multicriteria problems the method of lexicographic ordering. Modification takes into account the features of the designed object; each objective function depends on a limited number of parameters of the object. The formula for calculating the optimal parameters are given. The study shows the results of experimental research. The proposed method is more effective than the known methods for example, the method of the main criterion and the method of penalty functions. The method is highly efficient and can be used in optimal design of a variety of electrotechnical devices.

**Key words:** High-precision systems, positioning, multiobjective optimization, inverse problems, system and temperature, minimizing

### INTRODUCTION

The considered high-precision system for positioning objects includes a platform on which are fixed relocatable objects, XY-coordinate electric actuator with linear stepper motors, enabling the movement of objects along X and Y coordinates, the three systems of magnetic levitation platform with electromagnets shell type, drive control system and the stabilization gap  $\delta$  between the platform and electromagnets (Fig. 1).

Such systems are used for investigating samples by the electron microscope in technological processes of manufacturing integrated microcircuits, etc. (Kim and Maheshwari, 2002; Kovalev *et al.*, 1997; Jansen *et al.*, 2008; Tzeng and Wang, 1994; Williams *et al.*, 1993).

Application of the magnetic suspension eliminates mechanical contacts of the platform with other parts of the device, provides high accuracy of fixing objects on the plane and the ability to research in different environments and vacuum which is impossible with other types of suspension (Nikitenko, 2007).

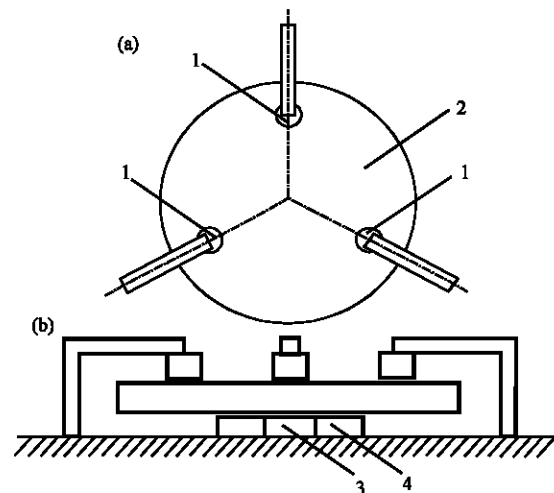


Fig. 1: System for positioning objects in a horizontal plane with magnetic levitation: a) Top view and b) Side view; 1 electromagnet, 2 platform, 3 linear motor which is carrying out the movement by Oy direction, 4 linear motor which is carrying out the movement by Ox direction

## MATERIALS AND METHODS

**Statement of the problem and solution method:** The optimal design problem of the technical system and its components in a general way is formulated as follows. The optimality criterion (objective function) and number of restrictions are known. Is required to determine the parameters of the object.

This study is devoted to the development of the method for optimal design of system considered and its elements using a methodology based on the solution of inverse problems. This methodology is promising area of research of physical processes and optimization of the technical objects and technologies (Alifanov, 1994; Bahvalov *et al.*, 2014; Bakhvalov *et al.*, 2015, 2017; Korovkin *et al.*, 2007; Shaykhutdinov *et al.*, 2016; Shurygin *et al.*, 2016). Next are considered the inverse boundary value problems which proved the uniqueness of solutions (Alifanov *et al.*, 1995); stability is provided by search of the solution  $\phi$  in the class of bounded functions with the norm:

$$\|\phi\| \leq N$$

where,  $N$  is positive constant. Such problems are called the conditionally-correct inverse problems or correct according to Tikhonov (Samarskiy and Vabishchevich, 2009).

The solution of inverse problems is reduced to the solution of a sequence of the direct tasks and application of optimization at each step of the iteration algorithm. Herewith, it is possible to apply the method of lagrange multipliers (Nocedal and Wright, 2006). However, this method leads to the need to solve difficult non-linear system of equations.

It is proposed a simpler approach which consists in the following. Restrictions are converted into objective function containing a small number of unknowns. All objective functions, including optimality criterion are ranked by importance. Next, is applied the modification of lexicographical ordering method (Harzheim, 2005). Sequential minimization of the objective functions is carried out using a gradient method or analytically with the use of necessary and sufficient conditions for the existence of extrema (minimizes mass).

Consider the application of the proposed method for optimal design electromagnets of system for positioning objects in a horizontal plane (Fig. 2).

The mass of electromagnet (excluding the masses of the platform) is selected as an objective function (optimality criterion). For the considered design (Fig. 2) the mass of the electromagnet  $M_{em}$  is determined by the Eq. 1:

$$M_{em} = [2S_p h + \pi R_2^2 h_1] \rho_{st} + \pi (R_2^2 - R_1^2) h \rho_{cu} \quad (1)$$

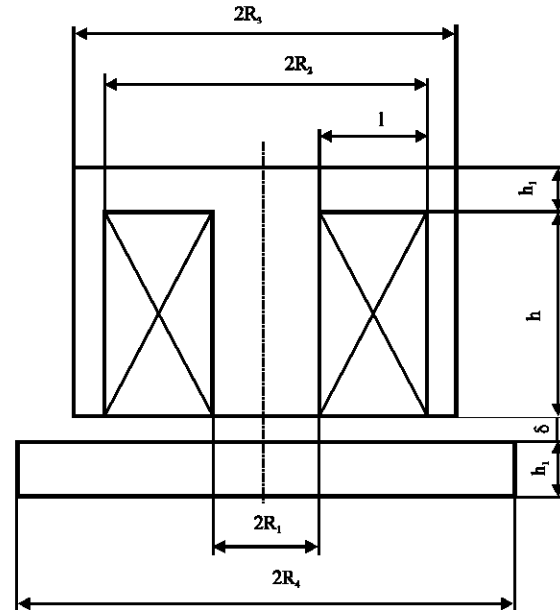


Fig. 2: Graphs of the strength of rocks in compression

Where:

$S_p$  = The square pole of the electromagnet

$\rho_{st}, \rho_{cu}$  = Density of ferromagnetic (steel) and copper

In order to ensure efficiency of performance of the magnetic suspension system is necessary to select the parameters of the electromagnet with which are performed the following conditions:

- Tractive force of the electromagnet should be no less than the required force of levitation
- Magnetic system is not saturated
- Maximum temperature of the insulation of the windings must not exceed the admissible

Additional relations for the parameters under which the magnetic flux density is identical in all cross sections of the electromagnet elements are obtained on the basis (Fig. 2):

$$\begin{aligned} S_p &= \pi R_1^2 = \pi (R_3^2 - R_2^2); h_1 = 0.5 R_1; R_2 = R_1 + l; \\ \pi R_3^2 &= S_{p2} + \pi R_2^2; \pi (R_2^2 - R_1^2) = \pi (2 R_1 l + l^2) \end{aligned} \quad (2)$$

The area of the coil window  $S_w$  and the parameter  $l$  is determined by Eq. 3:

$$S_w = i_w / (k_f j) = l h, l = S_w / h \quad (3)$$

Where:

$i_w$  = Magnetomotive force of coil

$k_f$  = Filling factor of the window by the copper

$j$  = Current density in the coil

Equation 1 are mathematically transformed using the obtained ratios has the form:

$$M_{em} = \left[ 2S_p h + \left( 2S_p + 2\pi R_1 \frac{S_w}{h} + \pi \frac{S_w^2}{h^2} \right) \frac{R_1}{2} \right] \rho_{st} + \pi \left( 2R_1 S_w + \frac{S_w^2}{h} \right) \rho_{cu} \quad (4)$$

Where:

$$R_1 = \sqrt{S_p / \pi}$$

From Eq. 4 follows, if known  $S_p$ ,  $S_w$  and mass of the electromagnet  $M_{em}$  depends only on  $h$ . Necessary and sufficient condition for the extremum of the function  $M_{em}$  is Bronshtein and Semendyaev (1981):

$$\frac{dM_{em}}{dh} = 0, \quad \frac{d^2M_{em}}{dh^2} > 0 \quad (5)$$

Formation the objective functions, based on the above conditions and ranking them by importance:

$$J_1(S_p) = [F^{(n)} - (1 + \epsilon_F) F^*]^2$$

$$J_2(i_w) = [B^{(n)} - (1 + \epsilon_B) B_\delta^*]^2$$

$$J_3(j) = [T^{(n)} - (1 + \epsilon_T) T^*]^2$$

$$J_4(h) = M_{em}$$

Where:

- $F^*$  = Required levitation force
- $\epsilon_F$  = Allowable relative error in determining the Force
- $B_\delta^*$  = Average flux density in the gap
- $\epsilon_B$  = Allowable relative error in determining the flux density
- $T^*$  = Allowable temperature, determined by the selected insulation class of coils
- $\epsilon_T$  = Allowable relative error in determining the temperature
- $F^{(n)}, B_\delta^{(n)}$  and  $T^{(n)}$  = Calculated value at the  $n$ -th step of the algorithm for solving inverse problems

Assume, that the magnetic permeability of steel is infinite  $\mu_{st} = \infty$ , current density  $j^{(0)}$  is known. Initial values of the parameters are defined from the approximate (Eq. 6):

$$i_w = \frac{B_\delta^*}{\mu_0} 2\delta, S_w^{(0)} = \frac{i_w^{(0)}}{k_f j^{(0)}}, S_p^{(0)} = \frac{\mu_0 F^*}{B_\delta^2 k_b}, R_1^{(0)} = \sqrt{\frac{S_p}{\pi}} \quad (6)$$

Where:

$\mu_0$  = Magnetic constant  $\mu_0 = 4\pi \cdot 10^{-7}$  (H/m)

$k_b$  = Coefficient of buckling of the magnetic field in the gap

$k_f$  = Filling factor of the window by the copper

$h^{(0)}$  is received, solving the Eq. 5. Other parameters are determined using relations Eq. 2 and 3. If initial values of the parameters were obtained then they are refined by solution of inverse problems of the stationary magnetic field theory and heat transfer as well as using a consistent minimization of functionals. The following conditions are checked at each step of the iteration algorithm:

$$J_1(S_p) \leq [\epsilon_F F^*]^2, J_2(i_w) \leq [\epsilon_B B_\delta^*]^2, J_3(j) \leq [\epsilon_T T^*]^2$$

## RESULTS AND DISCUSSION

Required to determine the parameters of the electromagnet of the system for positioning objects in a horizontal plane (Fig. 1) that provides performance of the following requirements with minimum mass:  $F^* = 23$  N;  $B_\delta^* = 0.5$  T;  $T^* = 180^\circ\text{C}$ ;  $\delta = 0.5 \cdot 10^{-3}$  material of the ferromagnetic is steel brand Steel 1010;  $j = 3.10^6$  A/m<sup>2</sup>;  $\rho_{st} = 7800$  kg/m<sup>3</sup>;  $\rho_{cu} = 8900$  kg/m<sup>3</sup>;  $k_b = 1.5$ ;  $k_f = 0.7$ ;  $\epsilon_F = \epsilon_B = \epsilon_T = 0.01$ . Initial values of the parameters are defined from Eq. 2, 3 and 6:

$$i_w^{(0)} = 398 \text{ A};$$

$$S_w^{(0)} = 190 \cdot 10^{-6} \text{ m}^2;$$

$$S_p^{(0)} = 77 \cdot 10^{-6} \text{ m}^2;$$

$$R_1^{(0)} = 4.95 \cdot 10^{-3} \text{ m};$$

$$h_1^{(0)} = 2.48 \cdot 10^{-3} \text{ m};$$

$$l^{(0)} = 5.87 \cdot 10^{-3} \text{ m};$$

$$R_2^{(0)} = 10.82 \cdot 10^{-3} \text{ m};$$

$$R_3^{(0)} = 11.90 \cdot 10^{-3} \text{ m};$$

$$h^{(0)} = 32.28 \cdot 10^{-3} \text{ m}$$

From the Eq. 8, if initial values of the parameters were obtained then they are refined by solution of inverse problems. At the 14th iteration are received:

$$i_w = 306 \text{ A};$$

$$R_1 = 7.45 \cdot 10^{-3} \text{ m};$$

$$h = 19.13 \cdot 10^{-3} \text{ m};$$

$$h_1 = 3.72 \cdot 10^{-3} \text{ m};$$

$$l = 7.62 \cdot 10^{-3} \text{ m};$$

$$R_2 = 15.06 \cdot 10^{-3} \text{ m};$$

$$R_3 = 16.81 \cdot 10^{-3} \text{ m};$$

$$R_4 = 41.80 \cdot 10^{-3} \text{ m}$$

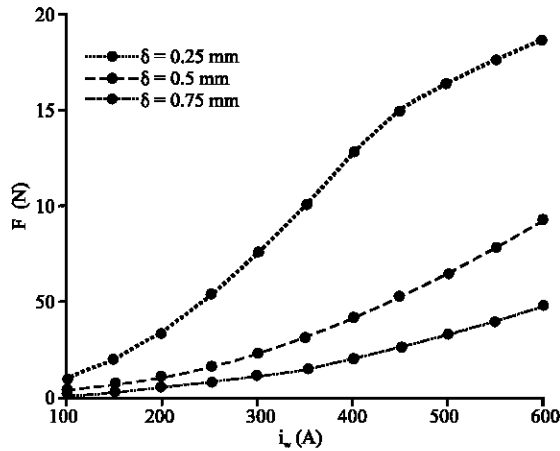


Fig. 3: Dependence of levitation force from the magnetomotive force of the winding

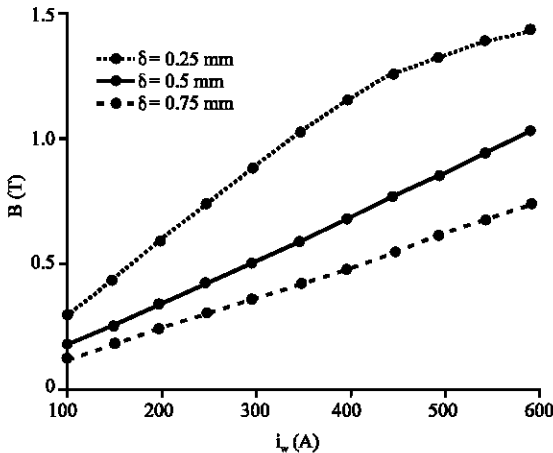


Fig. 4: Dependence of flux density in the gap of electromagnet from the magnetomotive force of the winding

Mass of the electromagnet is  $M_{em} = 0.17$  kg. The solution of the problem is completed. Optimal parameters of the electromagnet providing a given tractive force of the electromagnet and the magnetic flux density in the gap as well as having the minimum volume at the winding temperature does not exceeding the permissible are defined.

It is important to know the dependence of levitation force from the magnetomotive force of the winding (Fig. 3) and flux density in the gap of electromagnet from the magnetomotive force of the winding (Fig. 4).

Dependence analysis shows that for the working gaps of the positioning system ( $\delta = 0.25 \div 0.75$  mm) for  $i_w = 306$  A ferromagnets of the systems are

unsaturated. This makes it possible to realize an effective control of the operation of the system which provides for precise positioning in the horizontal plane.

## CONCLUSION

The study is proposed of the method for optimal design high-precision systems for positioning objects in a horizontal plane and its elements, based on the solving inverse problems and the transformation of constraints in the objective functions. Minimization of objective functions is performed numerically by gradient descent method. Minimizing mass electromagnet of the system is performed analytically. Objective functions are ranked by importance.

Method is a modification of the method for solving multicriteria problems the method of lexicographic ordering (Harzheim, 2005). Modification takes into account the features of the designed object, each objective function depends on a limited number of parameters of the object. The proposed method allows significantly reduce design time, it is more effective than the known methods for example, the method of the main criterion and the method of penalty functions (Nocedal and Wright, 2006). The method is highly efficient and can be used in optimal design of a variety of electrotechnical devices. For example, the objective function is a mass in the optimization of the linear electric motor and restrictions are of the form:

$$F^* \leq F^{(n)} \leq (1 + \epsilon_F) F^*; T_{max}^{(n)} \leq T_{ad}$$

Where:

$F^*$  = Required tractive force

$F^{(n)}$  = Calculated value of the tractive force at the nth computing step

$\epsilon_F$  = Allowable relative error in determining the tractive force

$T_{max}^{(n)}$  = Maximum insulation temperature at the nth computing step

$T_{ad}$  = Allowable temperature defined by the selected class of coil insulation

Geometric dimensions of the rotor and stator of the motor, the dimensions of the coils stator and the number of turns coils it is determine by proposed method.

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