

## Deflections and Frequencies of Natural Oscillations of Systems of Cross Beams with Different Cell Sizes on the Triangular Plan Taking into Account the Pliability of Nodal Connections

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**Abstract:** This study considers the relationship between the fundamental frequency of free transverse oscillations  $\omega$  of the systems of cross-beams with different sizes of cells on a triangular plan and their maximum deflections  $W_0$  under the action of evenly distributed loads depending on the stiffness characteristics of nodal connections. We constructed dependences of deflections and frequencies of transverse oscillations depending on the rigidity of nodal connections for the systems of cross beams with different cell sizes. We proved that for the systems with cross-beams of the cell sizes  $a/l < 0.17$  ( $a$  is a cell size,  $l$  is a size in terms of the system of cross-beams) the coefficient  $K$  with an accuracy to 7.01% corresponds to the theoretical value for the isotropic triangular plates of the constant.

**Key words:** System of cross-beams, stiffness of nodal connections, natural frequency, maximum deflection, triangular systems, action

### INTRODUCTION

For elastic isotropic plates of constant thickness and arbitrary shape, Korobko (1989, 1997) revealed a correlation which states: regardless of the boundary conditions, the product of the maximum deflection  $W_0$  from the action of a evenly distributed load  $q$  on the surface of the fundamental frequency of oscillation of the beam in the no-load state  $\omega$  with an accuracy to the dimension of the multiplier of  $q/m$  is a constant and corresponds to the patterns:

$$W_0 \times \omega^2 = K \frac{q}{m} \quad (1)$$

where,  $m$  is a plate mass which is evenly distributed on the surface.

A large number of works is devoted to the design and static calculation of cross-beams systems (Ignatev, 1973; Labudin, 1978; Hisamov, 1977) but studies of the relationship of static and dynamic parameters of the construction have not been conducted till present. In works (Turkov and Makarov, 2013a-d, 2014a, b; Turkov *et al.*, 2016a, b; Newman, 1995), the dependence (Eq. 1) was considered in relation to the system of cross-beams on a square and a rectangular plan. The researchers of this study tried to identify the

validity of this dependence on the cross-beams in a triangular plan with a different cell size when changing the pliability of construction nodal connections.

### MATERIALS AND METHODS

The system of cross-beams in the form of a right triangle with sides 18 m (Fig. 1) was used for calculating. The sides of cells  $a$  were 1.0, 1.5, 2.0, 3.0, 4.5 and 6.0 m.

The elements in the nodes are connected with steel angles and steel pins (bolts, studs). The scheme

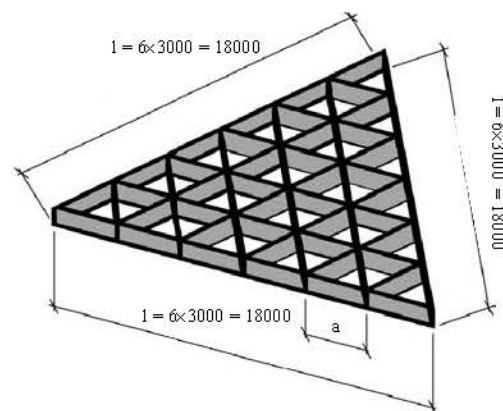


Fig. 1: The scheme of cross-beams in a triangular plan

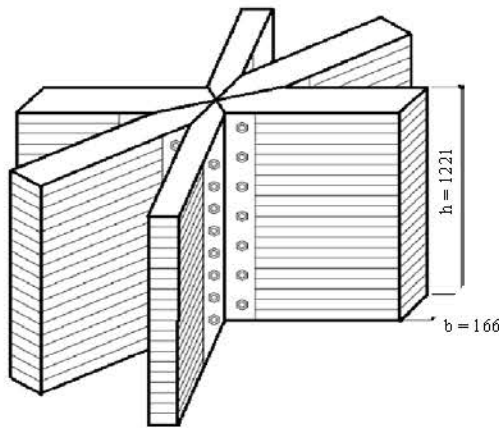


Fig. 2: The node of adjunction of elements

nodes is shown in Fig. 2. On the contour, the system of cross-beams was based on hinged supports in contour knots. A constant load is received from the cover (a plywood plate)  $q_{\text{cover}} = 0.156 \text{ kH/m}^2$  and from the weight of the cross-beams system (the cross-section of glulam elements of the system is taken constant  $b \times h = 160 \times 1221 \text{ mm}$ )  $q_{\text{weight}} = 0.187 \text{ kH/m}^2$ . The calculating snow load was  $S_g = 1.8 \text{ kH/m}^2$ . The total evenly distributed static load was  $q = 2.143 \text{ kH/m}^2$ . To determine the oscillation frequencies, in the system nodes the concentrated loads were attached and their intensity was calculated according to the actual structure's own weight and was  $G = 0.51 \text{ kH}$ . The module of elasticity of wood in the calculations was taken according to SNiP II-25-80  $E = 10000 \text{ MPa}$ .

To assess the degree of influence of the nodal connections pliability, the finite element with inserts at the ends was developed (Fig. 3). The length of each of this finite element depends on the size of the cell and it is  $a = 1, 1.5, 2, 3, 4.5$  and  $6 \text{ m}$ , the length of the insertion is  $0.01 a$ . Flexural rigidity of the element cor-3 responded to the stiffness of glulam cross-section  $b \times h = 160 \times 1221 \text{ mm}$  (the cross-section height is of 37 plates of 33 mm thickness) and flexural rigidity of the insertions varied in the range  $EI_{\text{insert}}/EI_{\text{el}}$  from 0-1 (in practical calculations, the accepted ratio is 0.0001). It is obvious that the ratio of the rigidity of the insertion to the stiffness of the element  $EI_{\text{insert}}/EI_{\text{el}} = 0$ , in the nodes of the cross-beam system a hinge is formed and when  $EI_{\text{insert}}/EI_{\text{el}} = 1$  there is no pliability in nodes.

For the static calculation, the loads were attached in the nodes. The research was carried out by finite element method. The calculation was performed in software complex SCAD. As a result of the calculation, the maximum deflection and the fundamental frequency of the transverse oscillations of the cross-beam system were determined.

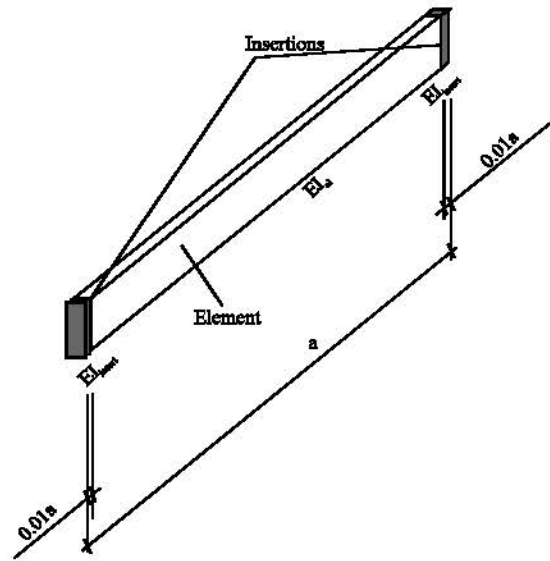


Fig. 3: The scheme of the element with insertions

## RESULTS AND DISCUSSION

The results of the calculation of hinged along the contour structure with different cell sizes are shown in Fig. 4-6. They represent the graphs of deflections and frequencies as well as  $K_{\text{cal}}$  coefficient depending on the ratio of the flexural rigidity of the insertion to the stiffness of the element  $EI_{\text{insert}}/EI_{\text{el}}$  (Fig. 4-6). The deviation of the actual values of  $C$  coefficient from the theoretical one was calculated by the equation:

$$\Delta = \frac{K_{\text{theor}} - K}{K_{\text{theor}}} \times 100\% \quad (2)$$

$K_{\text{theor}}$  coefficient for triangular plates, hinged along the contour is  $K = 1.603$  (Table 1).

The analysis of the data shows that with a decrease in the cell size, the system of cross-beams tends to the plates and as a consequence, the coefficient  $K_{\text{cal}}$  tends to the value  $K_{\text{theor}}$  for the triangular plates. It was founded that the change of the ratio of the insertion stiffness to the element stiffness has little effect on the change of the coefficient  $K$ . For the systems of cross-beams in a triangular plan with the cell size  $a/l < 0.17$  ( $a$  is a cell size,  $l$  is a size in terms of the system of cross-beams) the coefficient  $K$  with an accuracy to 7.01% corresponds to the theoretical value for isotropic triangular plates with constant thickness, regardless of the ratio of the insertion stiffness to the element stiffness. For large ratios  $a/l$  the functional dependence (Eq. 1) gives a significant error: the deviation coefficient  $K$  from the theoretical one when  $0.17 < a/l = 0.25$  is to 35.67% and  $0.25 < a/l = 0.33$  is up to 37.62%.

Table 1: Values of  $K_{ca}$  coefficient for different cell sizes depending on the flexural rigidity of the insertions

$EI_{insert}/EI_d$	The cell size (m) (Coefficient $K_{ca}$ , Deviation Kcal from $K_{theor}$ (%))					
	1.0×1.0×1.0	1.5×1.5×1.5	2.0×2.0×2.0	3.0×3.0×3.0	4.5×4.5×4.5	6.0×6.0×6.0
0.0001	1.600/0.19	1.558/2.83	1.536/4.15	1.491/7.01	1.031/35.67	1.000/37.62
0.001	1.601/0.13	1.558/2.81	1.537/4.11	1.492/6.91	1.037/35.28	1.000/37.62
0.005	1.605/-0.12	1.559/2.77	1.539/3.97	1.498/6.54	1.062/33.77	1.000/37.62
0.008	1.607/-0.25	1.563/2.52	1.541/3.90	1.502/6.31	1.077/32.79	1.000/37.62
0.01	1.608/-0.33	1.565/2.37	1.541/3.85	1.504/6.18	1.087/32.20	1.000/37.62
0.03	1.616/-0.78	1.579/1.47	1.552/3.17	1.518/5.31	1.153/28.10	1.000/37.62
0.05	1.619/-0.98	1.586/1.06	1.561/2.60	1.525/4.85	1.190/25.76	1.000/37.62
0.1	1.622/-1.19	1.593/0.63	1.571/1.98	1.534/4.31	1.238/22.70	1.000/37.62
0.2	1.624/-1.33	1.597/0.35	1.578/1.57	1.545/3.61	1.275/20.47	1.000/37.62
0.3	1.625/-1.39	1.599/0.24	1.580/1.42	1.551/3.23	1.290/19.51	1.000/37.62
0.5	1.626/-1.43	1.601/0.15	1.582/1.28	1.556/2.91	1.304/18.65	1.000/37.61
0.6	1.626/-1.44	1.601/0.12	1.583/1.25	1.558/2.82	1.308/18.42	1.000/37.60
0.8	1.626/-1.46	1.602/0.09	1.584/1.20	1.559/2.72	1.313/18.12	1.000/37.60
1	1.626/-1.47	1.602/0.07	1.584/1.17	1.561/2.65	1.315/17.94	1.0004/37.59

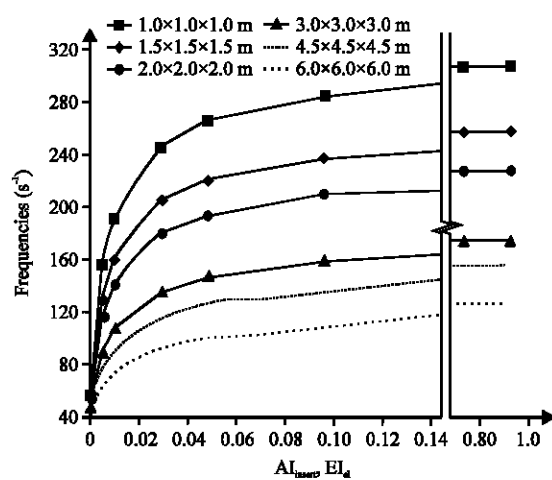


Fig. 4: The frequency change of the natural oscillations depending on the ratio of the insertion stiffness to the element stiffness

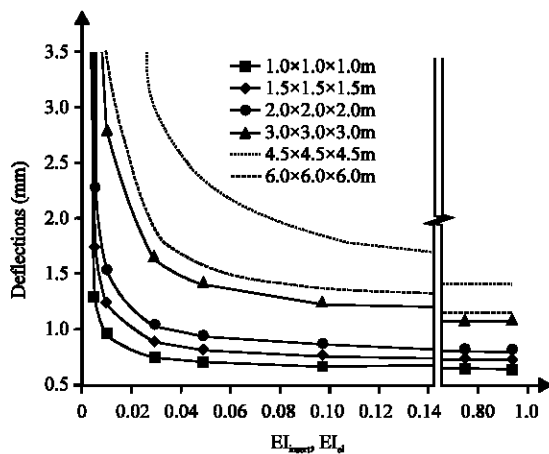


Fig. 5: The change of the deflections depending on the ratio of the insertion stiffness to the element stiffness at different cell sizes

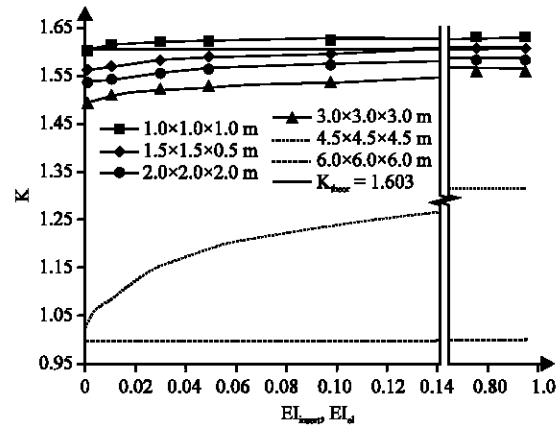


Fig. 6: Change in  $K_{ca}$  coefficient depending on the ratio of the insertion stiffness to the element stiffness at different cell sizes

## CONCLUSION

The studies found that the dependence Eq. 1 applies to the systems of cross-beams in the triangular plan and it identified the limits of this dependence applicability, depending on the pliability of nodal connections and the cell size of the system; it is shown that with the decrease of the cell size coefficient  $K_{ca}$  approaches the analytical value of the ratio. It was also found that the pliability of the connections in the nodes of the cross-beam system insignificantly affects the values of the coefficient  $K$ .

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