

## A Monte Carlo Approach to Estimate the Coverage Overlapping Areas in WSNs

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**Abstract:** “Wireless Sensor Networks (WSNs)” represent a set of sensors spatially deployed assisted by other to sense, monitor or track certain zone. Sensors can communicate with base station either directly or through other nodes. Each sensor has an ability to cover certain area usually represented by a circle. This circle radius is equal to the sensor sensing range and its center is the location (coordinates) of the sensor node. In most WSN applications there are many regions are covered by more than one circle (sensor). Such regions are called overlapping regions. In other words such region will be sense and monitor by more than one sensor. In such case same (redundant) data will be delivered to the base station. But in other application it is required and important to increase the system reliability in preventing intruders. Overlapping has many significant effects on the network behavior metrics. It is so important to estimate and optimize the overlapping regions in the process of planning and deploying any new WSN. These overlapping represent the intersection areas (lens) surrounded by arcs resulted from circles intersections. In this study, a new developed Monte Carlo approach is utilized to estimate all the intersection areas among many circles. A computer simulation technique in Net Logo Software is developed to perform this task. The developed approach is found to be useful in estimating all the not uniform regions in any WSN coverage area in a simple manner. Such calculations represent a challenge in mathematics it shows very near exact calculations.

**Key words:** WSNs, overlapping, coverage, Monte Carlo simulation and set theory, preventing intruders, intersection areas

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### INTRODUCTION

“Wireless Sensor Networking (WSN)” represents a developed “wireless communication system”. It composed of many deployed sensors in specific space to achieve certain task. Each sensor node must have at least one link with one of the other sensors. Each sensor has an ability to sense and cover certain circle area called coverage area (Jadhav, 2017).

Depending on the deployment approach, two or more sensors may cover same part (region) of the covered area. Such region is called overlapped region. It represents the intersection of two or more circles. In set theory, the intersection of the set A and the set B is the set that consist of all the elements that belong to mutually A and B, written  $A \cap B$  (Harrison, 2010). For the crop circle problem, we need to compute the total area encompassed by circles that may intersect. A restriction given in the problem was that no point ever belongs to 3 or more circles. This limits us to having to compute, at most, the area of two intersecting circles (Bhavya, 2014). The mathematical methods are too difficult in calculating the area of these regions. These overlapping regions are

significantly affecting the WSN performance, communication interference and energy dissipation (Iqbal and Alimgeer, 2013). Monte Carlo simulation (a “probability simulation”) represents one of the used scientific tools to analyze and solve the analytically intractable problems and for others if their experimentation is costly, time-consuming or impractical.

### MATERIALS AND METHODS

**Wireless Sensor Networks (WSNs):** Wireless Sensor Networks (WSNs) are made up of individual sensors that interact with the surroundings by sensing and controlling physical parameters such as pressure, temperature and volume. The sensors also have to interact between themselves through wireless communication to reach the sensing task and are independent, although, some user driven data collection is also possible. These nodes consist of computation, sensing, actuation and wireless communication functions. Therefore, WSNs are continuously becoming important specifically with the arrival of “Internet of Things (IoT)” which is necessary for monitoring several objects in applications such as

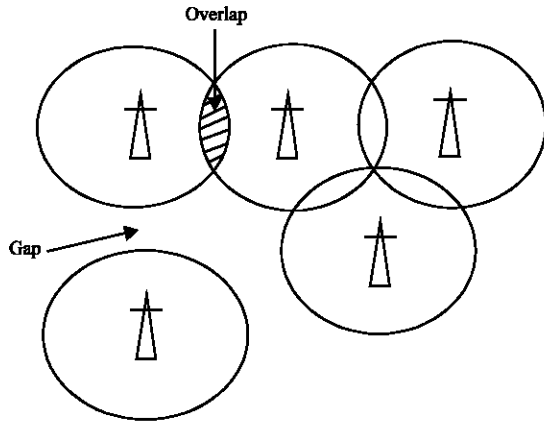


Fig. 1: The overlaps and gaps

smart health care, smart cities, smart water networks, smart power grids, smart farming and intelligent transport systems (Ndiaye *et al.*, 2017). It might so happen that either there may be an overlap in the middle of any two such adjacent circles or there might be a gap between the coverage areas of two adjacent circles (Fig. 1 and 2). Such a circular geometry, therefore, cannot attend as a regular shape (Melorose *et al.*, 2015).

**Overlapping:** Calculating the sensing overlapped areas is a challenging problem since many sensors are overlapped in their coverage areas. Despite of the wide range of its applications in wireless communications so far there is no systematic approach to solve it and for the sake of efficiency in the most proposals of clustering algorithms, each node belongs only to one cluster (Suharjono *et al.*, 2012). However, in several purposes and applications, the clusters are allowed to overlap and ordinary nodes have the ability to join multiple clusters with different membership level for each cluster (Selvakummar *et al.*, 2014; Aydin *et al.*, 2010).

Any two overlapping circles must have a certain share part of their areas. Mathematically it means that the intersection of any two sets (B and A) is the set that comprises elements that belong to A and to B at the same time. In sensor forms, it means that one sensor cover part of the other sensor coverage (sensing) space. A Venn diagram or set diagram is a diagram that shows all options of overlap and non-overlap of two or more circles (Chen, 2016). In many WSN applications there are many sensors can belong to two or more clusters (Youssef *et al.*, 2006). The intersection of two sets M and N is the group of all objects that are in both sets. It is written as (Levy, 1979):

$$M \cap N = \{x: (x \in M) \text{ and } (x \in N)\}$$

The other proper description of intersection is:

$$M \cap N = \{x: (x \in M) \wedge (x \in N)\}$$

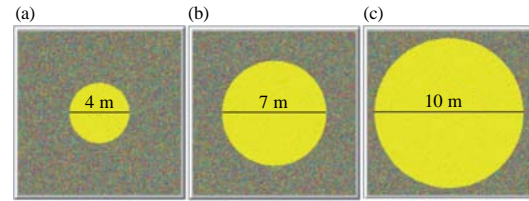


Fig. 2: Three single circles with different diameters: a) Circle of 4 m diameter, b) Circle of 7 m diameter and c) Circle of 10 m diameter

**Monte Carlo methods:** “Monte Carlo (MC)” experiment is one of the wide calculation procedures that depend on the fundamentals of probability and repeated uniformly random numbers to estimate numerical results. MC is utilizing randomness to find a solution for the most complex problems when it is impossible or so difficult to use other methods. It is commonly used in many optimization problems (Hammersley *et al.*, 1965).

In this study, MC is well utilized in the process of estimating the overlap area between any two circles. It also applied in the process of estimating the overlapped and not overlapped areas among different circles.

## RESULTS AND DISCUSSION

**Problem implementation and validation:** Sensor sensing area (coverage area) represents the crucial parameter in all the WSN applications. Sensing area is always a circle with diameter (d) equal to the double of sensor sensing Range ( $R_s \times 2$ ) and its center is on the sensor location (coordinates). Deploying many sensors in a specific area can be represented by locating many circles. Each circle represents one sensor. Managing and controlling such circles is a challenging problem. These circles are either overlapping or mutually exclusive depending on the deployment approach. Specific designed applications may prefer or aim to increase the coverage overlapping area or decreasing it. In this study, a developed Monte Carlo approach is applied to help in estimating the various overlapping (intersecting) areas.

Finding the intersecting areas will help in improving the designing process of any planned WSN by controlling its energy, coverage, connectivity and reliability. The following cases are adopted to achieve this study aim and validate its algorithms.

**Case 1 single circle:** This case is implemented, applied and simulated to estimate many different areas by this developed approach and compare the results with the mathematical calculations. The accurate results will ensure the validity of this developed approach. In the first

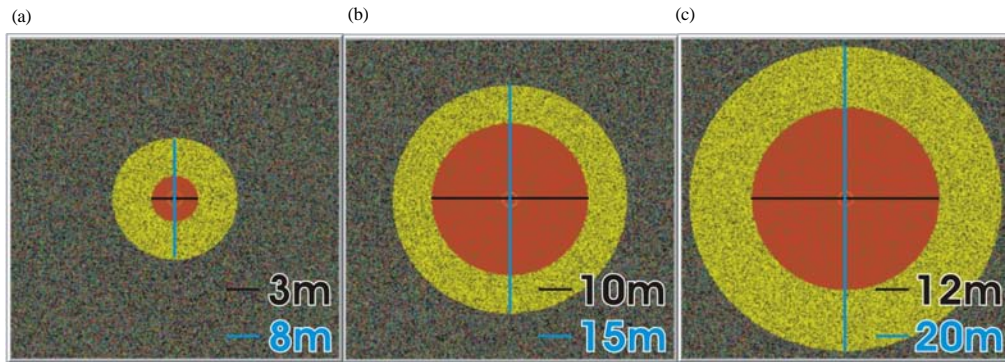


Fig. 3: Three pairs of fully overlapped circles with different diameters: a) 3 and 8 m diameters; b) 10 and 15 m diameters and c) 12 and 20 m diameters

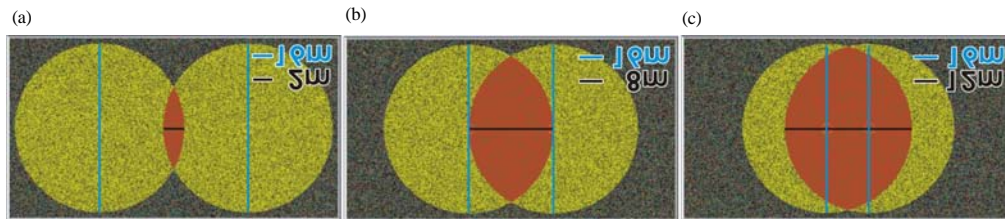


Fig. 4: Three pairs of similar diameters intersected circles: a) Small intersection; b) Medium intersection and c) Large intersection

Table 1: Circles areas estimation

Circles	Diameter (m)	Circles areas		
		Mathematical (Ma)	Simulation (Si)	Error (%)
1	4	12.5663	12.5598	0.00052
2	7	38.4845	38.5022	0.00045
3	10	78.5398	78.5169	0.00030

part of this case, estimating a circle area is used as a sample. Different circles with diameter are used and tested. Net Log Version 6.0 is used as a simulation tool in this study. Figure 2 and 3 single cycle circles with different diameters estimated by Monte Carlo approach. Table 1 shows a comparison sample between the mathematical and simulation approach in calculating the circles area. Algorithm 1 shows an algorithm used in calculating such areas.

#### Algorithm 1; An algorithm used in calculating single circle area:

```

Area = Sensing_Range2×Pi
For each dot
    Begin for
        If (Distance to WS 0 ≤ Sensing_Range) then
            Set type inside_dots
            Set color "Yellow"
        End if
    End for
Area = World_Width×World_Length×Count of Inside_dots/No. of dots
    
```

From Table 1 and its difference error, it is clear that this simulation manner is suitable to be applied in other area calculation cases such as the circles intersections. In the second part of this case, we will use two fully overlapped not similar circles. The estimating and comparison between computing the with overlapping area using mathematical method and computing such area using Monte Carlo approach is done to prove the validity of proposed approach. Figure 3 and 4 shows three pairs of fully overlapping samples of different diameters circles estimated by Monte Carlo approach. Table 2 show the calculation of three sample intersection, union, difference and areas of circles.

**Case 2; Two overlapped sensors:** Two overlapped sensors can be easily represented by two intersected circles. These two circles (A and B) are tested in this case to find their possible (overlapping) intersection area.

In this approach, a uniform (L×W) rectangle shape is suggested to represent the total area. Two intersection circles with known diameters (d<sub>i</sub>) are placed inside this square area. Very large number of pseudo random numbers are generated as points (dots) and distributed randomly in this area. Probability or ratio of points numbers are used to estimate the area of each part. For example the area of the intersection part is estimated as a ratio of the number of points in this part to the total

Table 2: The calculation of three sample intersection, union, difference and areas

Diameters		Intersections		Union		A-B		A		B	
A	B	Ma	Si	Ma	Si	Ma	Si	Ma	Si	Ma	Si
8	3	7.07	7.13	50.28	50.14	43.21	43.01	50.28	50.14	7.07	7.13
15	10	78.57	78.40	176.78	175.49	98.21	97.09	176.78	175.49	78.57	78.40
20	12	113.04	112.97	314.00	314.11	200.96	201.13	314.00	314.11	113.04	112.97

Table 3: Two circles intersection area calculation

Intersection	Diameters	Intersection area (A∩B)	Union area (A∪B)	Difference (A-B)	Difference (B-A)	Area of A		Area of B	
						Ma	Si	Ma	Si
Small	16	10.43	391.65	190.69	190.52	201.06	201.12	201.06	200.95
Medium	16	79.78	323.05	121.47	121.78	201.06	201.26	201.06	201.57
Large	16	137.31	264.85	63.71	63.83	201.06	201.02	201.06	201.14

Table 4: Two not similar circles intersection area calculation

Intersection	Diameter (A)	Diameters (B)	Intersection area (A∩B)	Union area (A∪B)	Difference (A-B)	Difference (B-A)	Area of A		Area of B	
							Ma	Si	Ma	Si
Small	10	16	16.54	262.68	62.00	184.13	78.53	78.54	201.06	200.67
Medium	10	16	43.94	235.55	34.76	156.84	78.53	78.71	201.06	200.78
Large	10	16	63.50	215.60	15.03	137.06	78.53	78.54	201.06	200.57

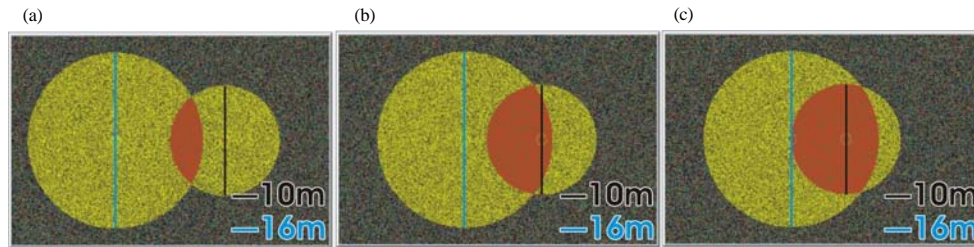


Fig. 5: Two different diameters intersected circles: a) Small intersection; b) Medium intersection and c) Large intersection

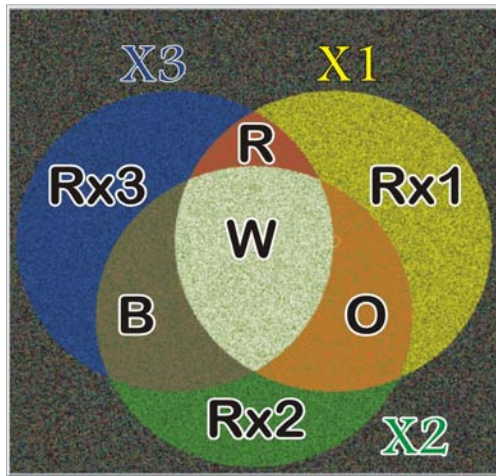


Fig. 6: Intersection regions for three similar diameter circles; X1-X3 are circles; W: White area is  $X1 \cap X2 \cap X3$ ; R: Red area is  $(X1 \cap X3) - (X1 \cap X3 \cap X2)$ ; O: Orange area is  $(X1 \cap X2) - (X1 \cap X3 \cap X2)$ ; B: Brown area is  $(X2 \cap X3) - (X1 \cap X3 \cap X2)$ ; Rx1: Yellow area is  $X1 - (OUWUR)$ ; Rx2: Green area is  $X2 - (OUWUB)$ ; Rx3: Blue area is  $X3 - (RUWUB)$

number of the used points multiplied by the uniform area (LW). Figure 4 and 5 show the three possible intersection situations (large, medium and small) for similar diameters circles. The mathematical methods for calculating such areas are too difficult and require different assumptions and complex calculations see (Bhavya, 2014; Iqbal and Alimgeer, 2013; Fewell, 2006). Table 3 and 4 show the simulation results. Algorithm 2 shows an algorithm used in calculating such intersection areas.

**Algorithm 2; An algorithm used in calculating overlapping area of two intersected circles:**

```

For each inside_dots
    Begin for
        If (Distance ws 0 ≤ Sensing_Range) and
            (Distance ws 1 ≤ Sensing_Range) then
            Set type Ov_dots
            Set color "Red"
        End if
    End for
Area of over lapping = World_Width×World_Length×Count of
Ov_dots/No. of dots

```

their possible intersection area. Figure 6 shows the four intersection regions red, orange, brown and white for

Table 5: Region areas calculations

Not overlapped areas		Overlapped areas	
Region	Area	Region	Area
Circle $X_1$	454.78	R	24.7789
Circle $X_2$	452.59	O	96.7800
Circle $X_3$	450.53	B	98.8400
$X_1 \cup X_2 \cup X_3$	821.81	W	157.8400
$R \times 1$	175.36	RUOUBUW	157.8400
$R \times 2$	99.11	-	378.2500
$R \times 3$	169.07	-	-

Table 6: The intersection regions areas between circles

$\cap$	Circle $X_1$	Circle $X_2$	Circle $X_3$
Circle $X_1$	454.78	254.62	182.61
Circle $X_2$	254.62	452.59	256.68
Circle $X_3$	182.61	256.68	450.53

Table 7: The union regions areas between circles

$\cup$	Circle $X_1$	Circle $X_2$	Circle $X_3$
Circle $X_1$	454.78	652.75	722.69
Circle $X_2$	652.75	452.59	646.44
Circle $X_3$	722.69	646.44	450.53

similar diameters 24 m circles and the area of simulation is  $37 \times 37$ . Table 5 shows region area statistics of three similar circles not intersected and intersected area calculations. Table 6 shows the intersection regions area between circles. Table 7 shows the union regions area between circles.

## CONCLUSION

In this study, new algorithms were developed and implemented to estimate the coverage overlapping areas between or among sensors. The sensors coverage area is always represented by a circle. The circles intersection represents the sensors coverage overlapped area. Monte Carlo experiments are known and applied in many applications. With the vast developed computers, Monte Carlo simulation can be suitably utilized in complex random calculations. Net Logo is applied as a simulation tool. In this study, MC technique was applied to estimate different known areas and compare the results to ensure this simulation approach validity. Results showed an intangible error between the mathematical and the simulation values. This approach is then applied in calculating any coverage overlapping between two and among three sensors. The calculated results were also tested and proved to be very close to the real values. It can be easily applied for any number of sensors.

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