

## Siesmic Response Analysis of Typical Iraqi Buildings Using Ritz Vectors

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**Abstract:** Most of the buildings in Iraq were designed using outdated building codes that did not account for seismic loads. Since, earthquakes are increasing in the area, it is necessary to account for such necessary additional loading in the designs of buildings. It is not clear if typical RC buildings can undergo the effect of real earthquakes taking place in Iraq now. Three buildings with different details and boundary conditions were modeled using 3D finite elements using SAP2000 Software in order to represent scaled versions of typical multistory reinforced concrete commercial buildings generally constructed in Iraq. The main purpose is to extract the response of these buildings to find out to what extent they can sustain earthquakes. Halabja earthquake happened in November 2017 with a magnitude as recorded from Baghdad observatory, of  $PGA = 1.1 \text{ m/sec}^2$   $M_w = 4.9$  is used in this research as the seismic load to examine the response of the buildings. It was concluded that these buildings can sustain actual earthquakes taking place in Iraq safely but increasing the magnitude up to 200%,  $PGA = 2.2 \text{ m/sec}^2$   $M_w = 6.0$ , leads to complete failure of such buildings. The use of load-dependent ritz vectors gave more accurate results than the use of Eigen vectors. It was shown that LDR vectors were faster in the solution than Eigen vectors.

**Key words:** Reinforced concrete building, earthquake, Iraq, ritz vectors, Eigen vectors, boundary

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### INTRODUCTION

Earthquakes generate seismic waves which propagate to the surface of the earth and then propagates in the building above it. Engineers usually analyze the building by free vibration modal analysis but here a wave propagation method will be used and compared to the vibration method. The buildings will be analyzed by load-dependent ritz vectors and eigen vectors using SAP2000 Software.

For modelling wave propagation in the layers of the sub-strata and the structure, eight-node quadrilateral solid elements were used with the algorithm for solution of the dynamic equilibrium equations by superposition of LDR vectors (Wilson *et al.*, 1982).

**Literature review:** Reveals that a few number of studies has been performed till now on wave propagation methods and load-dependent ritz vectors. By Bayo and Wilson (1984 ) presented a numerical method for dynamic analysis of large complex finite element systems in which spatial distribution of the loading was constant. The method was based on the use of special class of ritz vectors which can be generated with minimum numerical effort. The purpose of their research was to extend the use of ritz vectors to the solution of wave propagation and foundation response problems.

The method was applied to one, two and three-dimensional problems in order to illustrate the efficiency and accuracy of the technique. Unless it is necessary to evaluate the very-high frequency behavior of a structural system, it was shown that small number of ritz vectors will produce excellent results. Therefore, LDR vectors can be very effective in the solution of three-dimensional systems including soil-structure interaction subjected to earthquake loading.

The results indicated that it is practical to solve a complex three-dimensional soil-structure finite element system in the time-domain if the system is transformed to a small dynamic response problem by using ritz vectors which can be generated with a minimum of numerical effort. In most practical interaction problems the effect of the interaction has a secondary effect on the displacements and stresses within the structure due to earthquake loading. Also, the most significant response is associated with the lowest frequencies of the combined system. In addition, the soil damping is normally large, compared to the damping in the structure. Therefore, as few as 15 three-dimensional ritz vectors may yield accurate results for complex soil-structure systems.

Safak (1999) presented a discrete-time wave-propagation method to calculate the seismic response of multistory buildings, founded on layered soil media and subjected to vertically propagating waves. Buildings were

modeled as an extension of the layered soil media by considering each story as another layer in the wave-propagation path. The seismic response is expressed in terms of wave travel times between the layers and wave reflection and transmission coefficients at layer interfaces. The method accounts for the filtering effects of the concentrated foundation and floor masses. Compared with commonly used vibration formulation, the wave propagation formulation provides several advantages, including simplicity, improved accuracy, better representation of damping, the ability to incorporate the soil layers under the foundation and providing better tools for identification and damage detection from seismic records. Examples were presented to show the versatility and the superiority of the method. The following formulations were concluded:

$$u_j(t) = A_j(f) \cdot [R_{d,j-1} \cdot d_j(t-\tau_j) + T_{u,j-1} \cdot u_{j-1}(t-\tau_j)] \quad (1)$$

$$d_j(t) = A_j(f) \cdot [R_{u,j-1} \cdot u_j(t-\tau_j) + T_{d,j-1} \cdot d_{j+1}(t-\tau_j)] \quad (2)$$

Where:

- U = Up propagating wave
- D = Down propagating wave
- A = The attenuation function
- R = The Reflection coefficient
- T = The Transmission coefficient
- $\tau$  = The one-way travel time of the wave

## MATERIALS AND METHODS

**Case models:** The first model (B1) is an 18-story building that will only be used to compare the speed of solution. Building B1 is shown in Fig. 1.

The second Building (B2) is a typical commercial RC building in Baghdad which consists of 3-floors in addition to the ground floor. It has a rectangular plan of (20.0×25.0 m). It consists of 5 spans in the x direction and 6 spans in the y direction. The building has a typical story height of 3 m excluding the first story which is 4.5 m height, so that, the total height of the building is 13.5 m.

There are 42 rectangular reinforced concrete columns with dimensions of 0.3×0.5 m for each. All beams of the building are 0.3×0.5 m and slab thickness is 0.2 m. The raft foundation thickness is 0.5 m (Fig. 2).

The first soil layer is made of clayey soil of 14.5 m depth and beneath it a 3 m layer of sand. The water table is at level of -1.5 m from the ground level.

The third and last Building (B3) used in the analysis here is also a RC structure but consists of a basement, a ground floor and four floors above it. It also has a rectangular plan of (20.0×25.0 m). It consists of 5 spans in

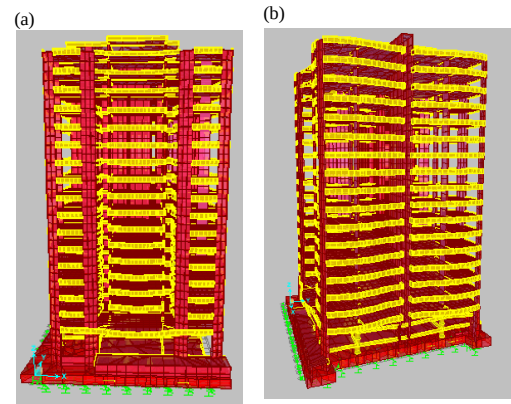


Fig. 1a, b): Building B1

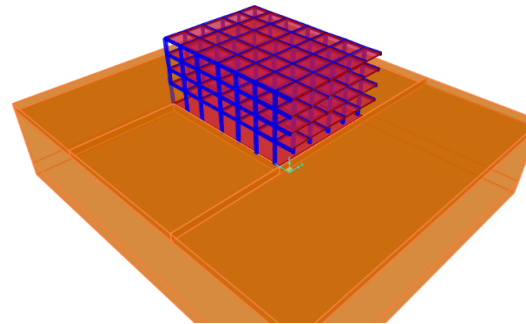


Fig. 2: Building B2

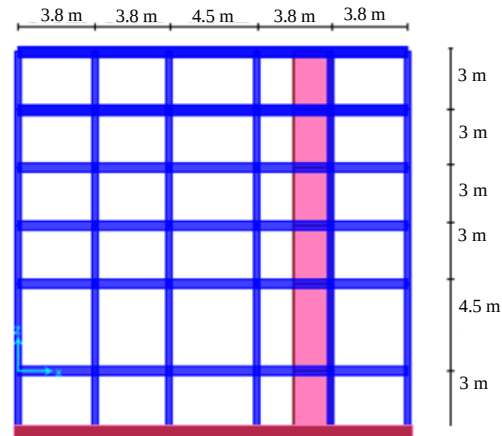


Fig. 3: Building B3-XZ plane (Front elevation)

the x direction and 6 spans in the y direction as shown in Fig. 3. The building has a typical story height of 3 m excluding the ground floor which is 4.5 m height, so that, the total height of the building is 19.5 m.

There are 42 rectangular reinforced concrete columns with dimensions of 0.3×0.5 m each. All beams of the building are 0.3×0.5 m in dimension and the slab thickness

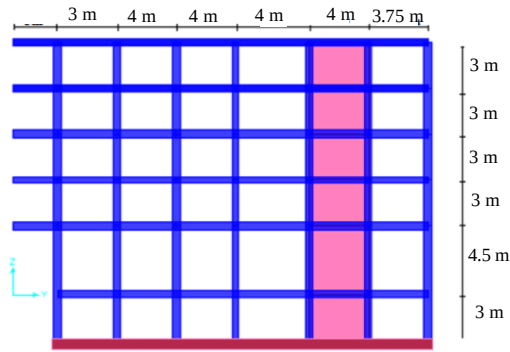


Fig. 4: Building B3-YZ plane (Side elevation)

is 0.2 m. Shear wall thickness is 0.2 m and the basement wall thickness is 0.25 m. Foundation thickness is 0.8 m. Soil specifications are the same as that of Building B2 (Fig. 4).

**Generation of load-dependent ritz vectors:** The numerical effort required to calculate the exact Eigen solution can be enormous for a structural system if a large number of modes are required. It can be demonstrated that a dynamic analysis based on a unique set of load dependent vectors yields a more accurate result than the use of the same number of exact mode shapes. The efficiency of this technique has been illustrated by solving many problems in structural response and in wave propagation types of problems (Bayo and Wilson, 1984). Several different algorithms for the generation of load dependent ritz vectors have been published since, the method was first introduced in Wilson *et al.* (1982). Therefore, it is necessary to present in Table 1 and 2 the latest version of the method for multiple load conditions.

#### Algorithm for generation of load dependent ritz vectors (Wilson *et al.*, 1982):

##### I. Initial calculations

- Triangularize Stiffness Matrix  $K = L^T DL$
- Solve for block of "b" static displacement vectors  $U_s$  resulting from special load patterns  $F_i$  or  $KU_s = F$
- Make block of vectors us stiffness and mass orthogonal, VI

##### II. Generate blocks of ritz vectors $I=2, \dots, N$

- Solve for block of vectors,  $X_i$ ,  $KX_i = MV_{i-1}$
- Make block of vector,  $X_i$ , Stiffness and mass orthogonal,  $\bar{V}_i$
- Use modified gram-schmidt method (two times) to make  $\bar{V}_i$  orthogonal to all previously calculated vector and normalized, so that,  $V_i^T MV_i = 1$

##### III. Make vectors stiffness orthogonal

- Solve Nb By eigenvalue problem  $[\bar{K} - \Omega^2] Z = 0$  where  $\bar{K} = V^T KV$
- Calculate stiffness orthogonal Ritz vector,  $\Phi = Vz$

**Cases investigated:** The time history analysis in this study is based on modal analysis (mode super-position analysis) to evaluate the dynamic response of the

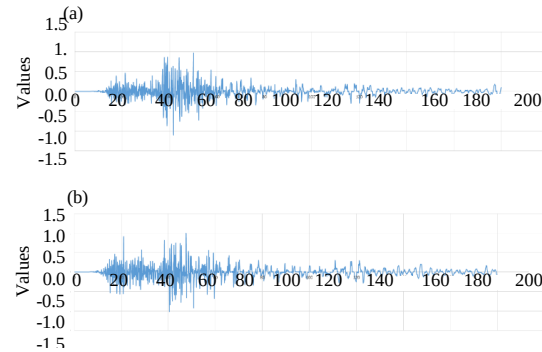


Fig. 5: a) Acceleration Time history in Baghdad, East-West (x) direction and b) Acceleration time history in Baghdad, North-South (y) direction

Table 1: Material properties of building B2

| Property                 | Values |
|--------------------------|--------|
| $f_c$ (MPa)              | 30     |
| $f_y$ (MPa)              | 420    |
| E concrete (MPa)         | 25740  |
| E steel (MPa)            | 200000 |
| Concrete poisson's ratio | 0.17   |
| Steel poisson's ratio    | 0.3    |

Table 2: Soil specifications under building B2

| Property                              | Clay  | Sand  |
|---------------------------------------|-------|-------|
| Density (kg/m <sup>3</sup> )          | 1700  | 1750  |
| Bulk Unit Weight (kN/m <sup>3</sup> ) | 20    | 19.4  |
| Modulus of Elasticity (MPa)           | 20000 | 22000 |
| Angle of Friction                     | -     | 40    |

structure. A constant modal damping of 0.05 is used. The earthquake ground motion subjected to the buildings in this study is Halabja Earthquake as recorded from the station of Baghdad city which happened on 12 Nov. as shown in Fig. 5a shows the East-West direction of the earthquake and Fig. 5b shows the North-South direction of the earthquake. This earthquake ground motion is chosen for the following reasons:

- It is up-to-date recorded Iraqi earthquake which occurred really in Baghdad and affected all buildings of the city
- To check the typical Iraqi building structures to real earthquakes happening in Iraq

Earthquake loads have been applied to the structure models in two perpendicular directions at 17.5 m below the ground level. The first is in the x-direction which is parallel to the structure plane and the other is in y-direction which is perpendicular to the structure plane (Bathe and Wilson, 1972; Wilson and Gauri, 2000; Wilson and Itoh, 1983).

## RESULTS AND DISCUSSION

**Analysis of Building B1:** Here, the building will not be fully analyzed but only the maximum displacement will be extracted to show the difference between eigen vectors and LDR vectors. Then the results will be compared to find out the advantages of each method.

The 100% mass participation displacement occurs in the building by using LDR vectors and eigen vectors is 94.5 mm under the 100% seismic loading. The results using LDR vectors are shown in Table 3 below. The results using eigen vectors are shown in Table 4. The analysis by LDR vectors needed only 14 modes of vibration to get the exact solution while the eigen modes used 46 modes to get the exact solution. Moreover, the software needed less time to analyze the building using LDR vectors which means the computational effort for LDR vectors mode shapes is very much less than the time needed for eigen analysis. Table 5 shows the percentage of time needed for eigen analysis as a ratio of time needed for ritz analysis.

As shown, ritz analysis is always faster than eigen analysis when using the same number of modes and when comparing time to get the exact response, eigen analysis needed 443.8% of the time needed to get the exact response by ritz analysis.

**Analysis of Building B2:** The building will be analyzed in the x- and y-directions using 100% and then 200% of earthquake intensity. Using 100% earthquake intensity, the absolute maximum displacement for this building is

Table 3: Analysis results using LDR vectors

| Number of LDR modes used | Mass participation factor (%) |
|--------------------------|-------------------------------|
| 1                        | 44.12                         |
| 5                        | 94.07                         |
| 10                       | 99.57                         |
| 14                       | 100.0                         |

Table 4: Analysis results using eigen vectors

| Number of eigen modes used | Mass participation factor (%) |
|----------------------------|-------------------------------|
| 1                          | 2.400                         |
| 5                          | 18.30                         |
| 10                         | 97.03                         |
| 20                         | 97.03                         |
| 30                         | 97.46                         |
| 40                         | 97.24                         |
| 46                         | 100.0                         |

Table 5: Comparison in solution speed as a percentage between eigen vectors and LDR vectors

| Number of modes used | Percentage of time needed for Eigen analysis as a ratio of the time needed for LDR analysis (%) |
|----------------------|-------------------------------------------------------------------------------------------------|
| 1                    | 187.5                                                                                           |
| 5                    | 191.6                                                                                           |
| 10                   | 178.5                                                                                           |

52.4324 mm in the x-direction while it is just 28 mm in the y-direction, also the moment and shear forces are greater in the x-direction. Accordingly, the results of only the x-direction will be presented.

The building was analyzed using LDR vectors where it needed 34 mode shapes to get the full response. The first 6 mode shapes gave %90 mass participation.

All results will be demonstrated using the 100 % mass participation. The first four mode shapes in the xz plane are shown in (Fig. 6).

Using 100% earthquake intensity, the maximum beam moment was 270.5 kN.m while the capacity of the beam is 340 kN.m (using singly reinforced capacity for the beams with ratio of 0.02) (Fig. 7).

The maximum beam shear is 212 kN while the maximum shear capacity is 345 kN (using 10 mm stirrups @ 100 mm c/c) (Fig. 8).

The maximum column moment is 247.03 kN.m while the maximum capacity is 466 kN.m (using 12 bars of 25 mm longitudinal reinforcement) (Fig. 9).

The maximum column shear force is 107.1 kN while the maximum shear capacity is 250 kN (using 10 mm stirrups @ 150 mm c/c) (Fig. 10).

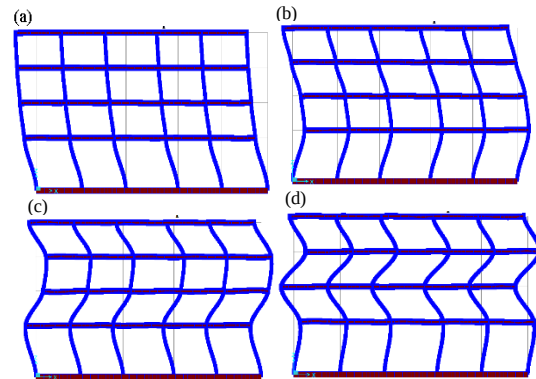


Fig. 6: First four mode shapes in xz plane of building B1  
a) First mode shape; b) Second mode shape; c) Third mode shape and d) Fourth mode shape

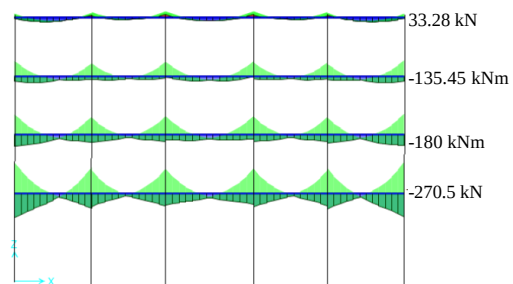


Fig. 7: Maximum beam moments (kNm) in Building B2

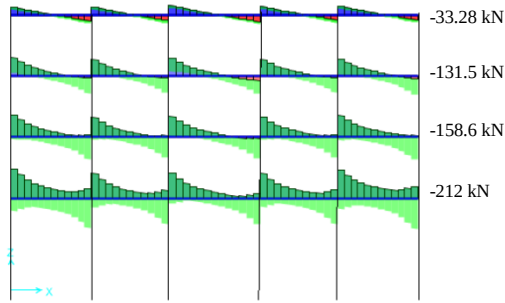


Fig. 8: Maximum beam shear force (kN) in Building B2

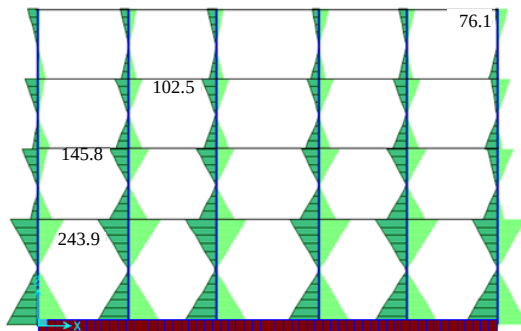


Fig. 9: Maximum moments in columns (kNm) for Building B2

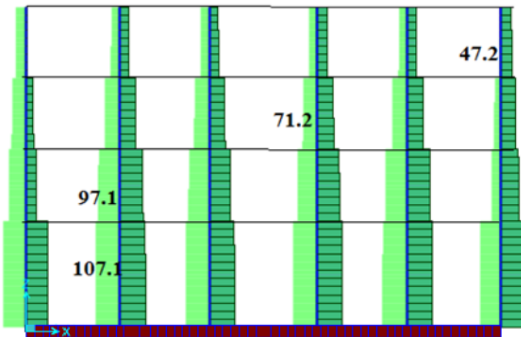


Fig. 10: Maximum shear force in columns (kN) for Building B2

Finally the maximum displacement in the 1st-4th floors are 30.6, 41, 48.4 and 52.4 mm, respectively and the maximum drift occurs in the first story which is 6.66 mm/m.

Increasing the earthquake intensity gradually till 200% of the original earthquake intensity led to the complete failure of the building. The maximum beam moment reaches 477.4 kNm, the maximum beam shear is 354.1 kN, the maximum column moment is 494 kN.m and the maximum column shear force is 216.2 kN.

**Analysis of Building B3:** In comparison with the previous building, this building will have larger frame with shear

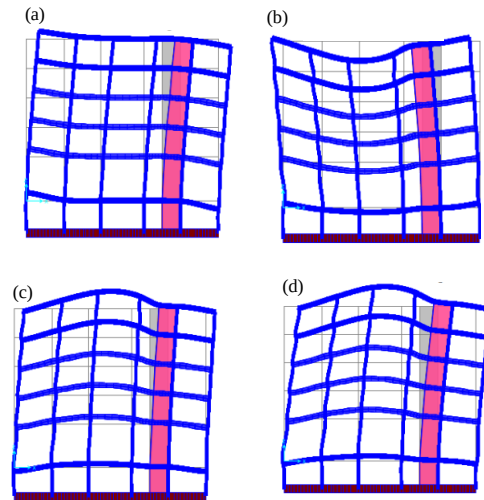


Fig. 11: First four mode shapes in xz-plane of building B3  
a) First mode shape; b) Second mode shape; c) Third mode shape and d) Fourth mode shape

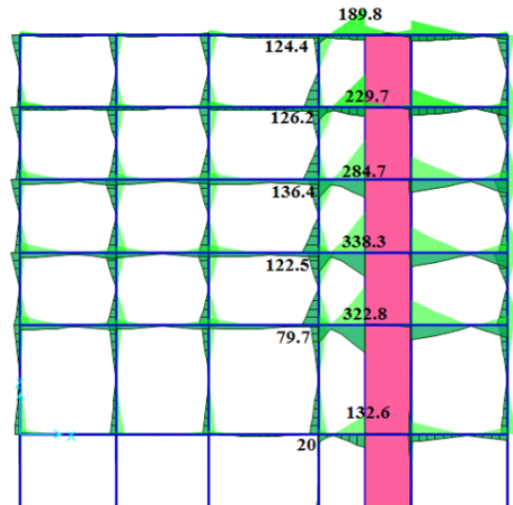


Fig. 12: Maximum beam and column moments (kNm) for Building B3

wall and soil beneath the building with its raft foundation which makes the analysis very much time consuming than the second typical building. So, it will be analyzed using LDR vectors only and in the x-direction since, it is the critical one.

The first four mode shapes of the building are shown in Fig. 11. The first thing to be noted here is that all the critical values occurred at the sections of the shear wall and near it. Using 100% earthquake intensity, the Maximum beam moment is 338.3 kNm while the maximum capacity is 340 kNm (using singly reinforced beam with ratio of 0.02) (Fig. 12). The maximum column moment is 136.4 kNm while the maximum moment capacity is 516 kNm (using 14 longitudinal bars of 25 mm) (Fig. 12).



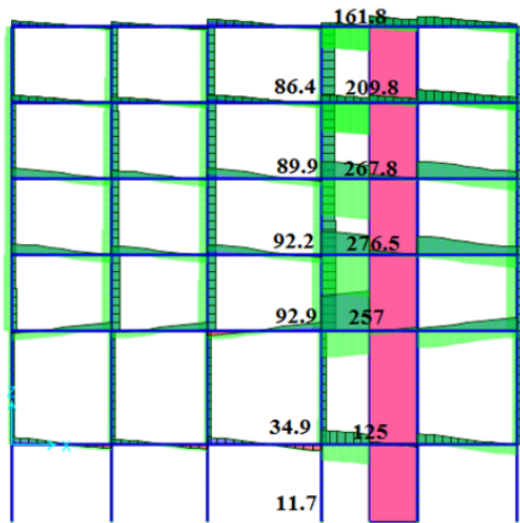


Fig. 13: Maximum beam and column shear forces (kN) in Building B3

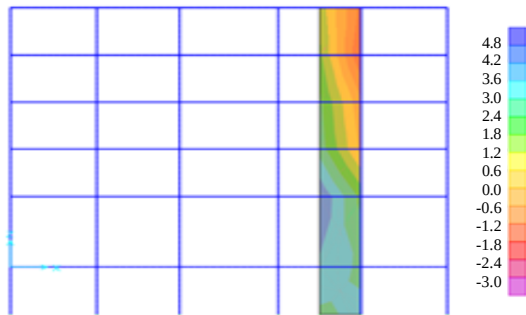


Fig. 14: Stresses in shear wall (MPa) of Building B3

The Maximum beam shear force is 276.5 kN while the shear capacity is 345 kN (using 10 mm-stirrups @ 100 mm c/c) (Fig. 13).

The maximum column shear force is 92.9 kN while the shear capacity is 250 kN (using 10 mm stirrups @ 150 mm c/c) (Fig. 13).

The maximum story displacement in the basement, 1st-5th floors are 0.2, 8.2, 14.2, 20.1, 25.1 and 29.1 mm. The maximum drift is 3.1 mm/m and the allowable drift is 20 mm/m.

The maximum stress in the shear wall is 4.09 MPa as shown in Fig. 14 and in the basement wall is 3.7 MPa as shown in Fig. 15.

Using 200% earthquake intensity, the maximum displacement became 58.9 mm and the maximum drift didn't pass the 6.1 mm/m.

The maximum beam bending moment is 589.5 kNm and all the beams attached to the shear wall fail, except the roof beam. Maximum column bending moment 194 kNm

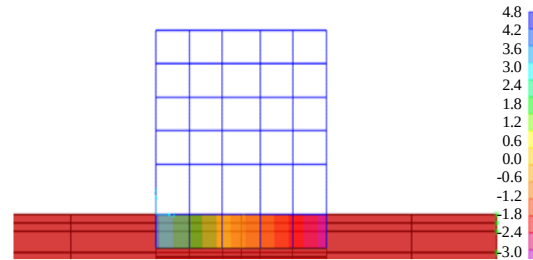


Fig. 15: Stresses in basement wall (MPa) of Building B3

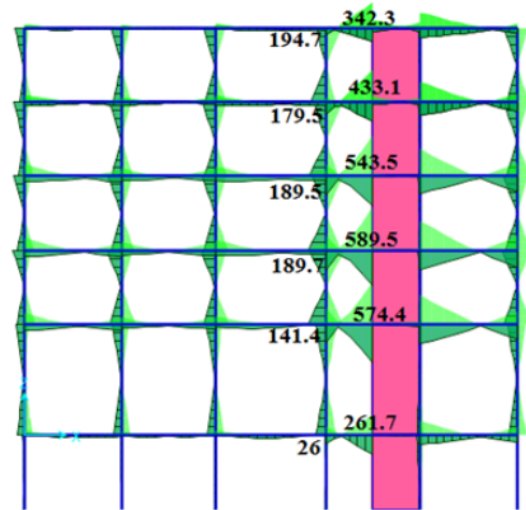


Fig. 16: Beam and column moments (kNm) for Building B3

while the maximum beam shear force is 461 kN and maximum column shear force is 149.1 kN. The maximum stress in the shear wall is 7.8 MPa and in the basement wall is 4.0 MPa. Figure 16 shows the beam and column moments for B3 and Fig. 17 shows shear force in beams and columns for B3.

From Building B1, one can note that LDR vectors gave more accurate and faster results than Eigen vectors. Using the same number of mode shapes, LDR vectors needs less computational effort than eigen vectors and gave more mass participation which makes LDR vectors more useful in analyzing buildings under earthquakes.

For Building B2, using 100% earthquake intensity, the building withstand it without any failure. In fact, the maximum moment is only 71% of the maximum capacity of the beam moment if considered as singly reinforced. For the columns, the first floor moment is 46% of the column full capacity. The moments induced in columns above it are smaller, so, the designer can use less reinforcement gradually. Using 200% earthquake intensity, gives a flexural and shear failure in the exterior beams of the first floor.

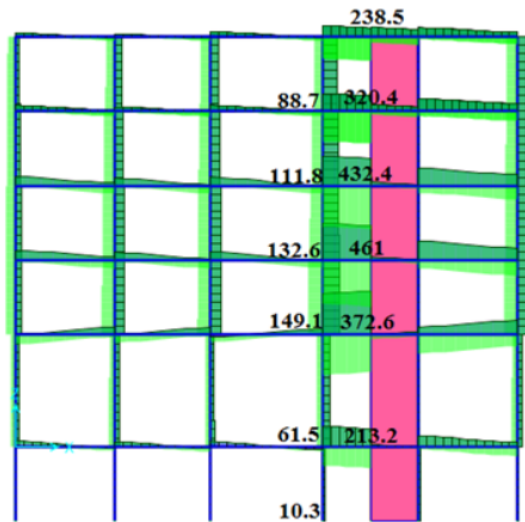


Fig. 17: Shear force in beams and columns (kN) of Building B3

For Building B3, using 100% earthquake intensity, the maximum beam moment was 99% of the capacity and the shear was 80% of its capacity. Columns can easily bear the maximum moment of 136.4 kNm and maximum shear of 92.9 kN. The shear wall gave much strength to the building but the beams attached to it suffer too much load, so, they need special treatment. Using 200% earthquake intensity, most of the beams attached to the shear wall fail in flexure and shear.

One may ask what happens if, we remove the live load, well, the live load direction is downward so, it reduces the positive moment and increases the negative moment and also it increases the shear force.

### CONCLUSION

The typical RC commercial buildings in Iraq can withstand a 4.9 earthquake ( $\text{PGA } 1.1 \text{ m/sec}^2$ ) but cannot withstand 200% of the intensity of this earthquake

which is of Mw of 6.0 ( $2.2 \text{ m/sec}^2$ ) and undergoes a local failure in the beams at the top of the ground floor. The typical RC 5-story with basement and shear walls building can almost withstand the 4.9 earthquake with minimum factor of safety and failed when subjected to an earthquake with doubled intensity.

The shear wall gave much strength to the building but the beams attached to it need special treatment such as increasing the depth and main reinforcement with stirrups. Modal analysis using LDR vectors are more accurate and requires less computational effort than Eigen vectors.

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