

## Some Generalized n-Tuplet Coincidence Point Theorems for Nonlinear Contraction Mappings

Zena Hussein Maibed

Department of Mathematics, College of Education for Pure Science,  
 Ibn Al-Haithem, University of Baghdad, Baghdad, Iraq

**Abstract:** The purpose of this study is to introduce a new concept of generalized n-tupled coincidence point and generalized mixed gT-monotone property. Also, we established the generalized n-tupled coincidence point theorems and we study the existence and uniqueness of generalized n-tupled coincidence point theorems without continuous condition for mappings having generalized mixed gT-monotone property in generalized metric spaces.

**Key words:** Partially ordered metric spaces, mixed monotone property, n-tupled coincidence point, coupled common fixed point, monotone property, generalized mixed

### INTRODUCTION

Bhaskar and Lakshmikantham (2006) introduced mixed monotone and established coupled fixed point theorem for mixed monotone in partially ordered metric spaces. After their research, many researchers studied about coupled fixed point and fixed point in partially ordered metric spaces (Aydi *et al.*, 2011a, b, 2012a-c, Berinde, 2011, 2012; Choudhury and Maity, 2011; Saadati *et al.*, 2010; Samet, 2010; Shatanawi *et al.*, 2011). Mustafa and Sims (2006) introduced the notion of a G-metric spaces as a generalization of the concept of a metric space, many researchers discussed research on the fixed point theory in partially ordered G-metric space (Agarwal and Karapinar, 2013; Alghamdi and Karapinar, 2013; Bilgili and Karapinar, 2013; Ding and Karapinar, 2013; Jleli and Samet, 2012; Karapinar and Agarwal, 2013; Mustafa *et al.*, 2008, 2009, 2011, 2012, 2013; Roldan *et al.*, 2014; Samet *et al.*, 2013; Shatanawi, 2010, 2011; Tahat *et al.*, 2012).

Aydi *et al.* (2011) established coupled coincidence and coupled common fixed point results for a mixed g-monotone mapping in a partially ordered G-metric space. As a continuation of this trend, many researchers have studied coupled coincidence point and coupled common fixed point results for a mixed g-monotone mapping in partially ordered G-metric space, for example (Aydi *et al.*, 2012a-c; Chandok *et al.*, 2013; Cho *et al.*, 2012; Choudhury and Kundu, 2010; Karapinar *et al.*, 2012; Shatanawi, 2011a, b; Chugh and Rani, 2016). In this study, we introduce the concepts of generalized n-tupled coincidence point and generalized mixed gT-monotone

property and we prove the existence and uniqueness of generalized n-tupled coincidence point theorems without continuous condition for mappings having generalized mixed gT-monotone property in generalized metric spaces.

Now, we recall some definitions and properties introduced by Mustafa and Sims (2009) which are useful for the main results in this study.

### MATERIALS AND METHODS

**Definition (1.1):** Let  $X$  be a non empty set,  $G: X \times X \times X \rightarrow \mathbb{R}_+$  be a function satisfying:

- G1.  $G(x, y, z) = 0$  if  $x = y = z$
- G2.  $0 < G(x, x, y)$  for all  $x, y \in X$  with  $x \neq y$
- G3.  $G(x, x, y) \leq G(x, y, z)$   
for all  $x, y, z \in X$  with  $y \neq z$
- G4.  $G(x, y, z) = G(x, z, y) = G(y, z, x) = \dots$ ,  
(Symmetry in all three variable)
- G5.  $G(x, y, z) \leq G(x, a, a) + G(a, y, x, z)$  for all  $x, y, z, a \in X$

Then the function  $G$  is called generalized metric and the pair  $(X, G)$  is called a generalized metric space or more specially G-metric space.

**Definition (1.2):** Let  $(X, G)$  be a G-metric space and let  $(x_n)$  be a sequence of points of  $X$ . We say that  $(x_n)$  is G-convergent to  $x$  if  $\lim_{n, m \rightarrow \infty} G(x, x, x_n, x_m) = 0$  that is for any

$\epsilon > 0$  there exist  $N \in \mathbb{N}$  such that  $G(x, x_n, x_m) < \epsilon$  for all  $n, m \geq N$ . We call  $x$  the limit of sequence and write  $x_n \rightarrow x$  or  $\lim_{n \rightarrow \infty} x_n = x$ .

**Definition (1.3):** Let  $(X, G)$  be a G-metric space, a sequence  $(x_n)$  is called G-cauchy sequence if for any  $\epsilon > 0$  there exist  $N \in \mathbb{N}$  such that  $G(x_n, x_m, x_l) < \epsilon$  for all  $n, m, l \geq N$  that is  $G(x_n, x_m, x_l) \rightarrow 0$  as  $n, m, l \rightarrow \infty^+$ .

**Proposition (1.4):** Let  $(X, G)$  be a G-metric space. A mapping is called G-continuous at  $x \in X$  if and if it is G-sequentially continuous at  $x$  that is whenever  $(x_n)$  is G-convergent to  $x$  then  $(f(x_n))$  is G-convergent to  $f(x)$ .

**Proposition (1.5):** A G-metric space  $(X, G)$  is called G-complete if every G-cauchy sequence is G-convergent in  $X, G$ .

## RESULTS AND DISCUSSION

Now, we introduce the concept of generalized n-tupled coincidence point and mixed gT-monotone property as follows:

**Definition (2.1):** Let  $(X, \leq)$  be a partially ordered set. If  $f: X^n \rightarrow X, g: X \rightarrow X$  are there mappings. An element  $(x_1, x_2, \dots, x_n) \in X^n$  is called generalized n-tupled coincidence point of  $f, g$  and  $T$  if:

$$\begin{aligned} f(x_1, x_2, \dots, x_n) &= gT(x_1) \\ f(x_2, x_3, \dots, x_n, x_1) &= gT(x_2) \\ &\vdots \\ f(x_n, x_1, \dots, x_{n-1}) &= gT(x_n) \end{aligned}$$

**Remark (2.2):** If  $g$  is the identity mapping then  $(x_1, x_2, \dots, x_n)$  is called n-tupled coincidence point of  $f$  and  $T$  (Imdad *et al.*, 2013). If  $g$  and  $T$  are the identity mappings then  $(x_1, x_2, \dots, x_n)$  is called n-tupled fixed point of  $f$  (Imdad *et al.*, 2013).

**Definition (2.3):** Let  $(X, \leq)$  be a partially ordered set. If  $f: X^n \rightarrow X$  and  $T, g: X \rightarrow X$  are three mappings, we say that  $f$  have mixed gT-monotone property if:

- $f(x_1, x_2, \dots, x_n)$  is monotone gT-increasing if  $n$  is odd
- $f(x_1, x_2, \dots, x_n)$  is monotone gT-decreasing if  $n$  is even

That is for each  $x_1, x_2, \dots, x_n \in X$ :

$$\begin{aligned} y_1, z_1 &\in X, \quad gT(y_1) \leq gT(z_1) \Rightarrow \\ f(y_1, x_2, x_3, \dots, x_n) &\leq f(z_1, x_2, x_3, \dots, x_n) \\ y_2, z_2 &\in X, \quad gT(y_2) \leq gT(z_2) \Rightarrow \\ f(x_1, y_2, x_3, \dots, x_n) &\geq f(x_1, z_2, x_3, \dots, x_n) \\ &\vdots \\ y_n, z_n &\in X, \quad gT(y_n) \leq gT(z_n) \Rightarrow \\ f(x_1, x_2, \dots, y_n) &\leq f(x_1, x_2, \dots, z_n) \text{ (if } n \text{ is odd)} \\ y_n, z_n &\in X, \quad gT(y_n) \leq gT(z_n) \Rightarrow \\ f(x_1, x_2, \dots, y_n) &\geq f(x_1, x_2, \dots, z_n) \text{ (if } n \text{ is even)} \end{aligned}$$

**Remark (2.4):**

- If  $T$  is the identity mapping then  $f$  has mixed g-monotone property (Chugh and Rani, 2016)
- If  $T$  and  $g$  are the identity mappings then  $f$  is said to have the mixed monotone property

Now, we considered the following is the set of all mappings  $\phi: [0, \infty) \rightarrow [0, \infty)$  increasing mapping such that  $\phi(t) \leq t \forall t > 0$ :

- $\phi(0) = 0$  and  $\lim_{n \rightarrow 0} \phi^n(t) = 0$  where  $\phi^n$  denotes the  $n$  the iterate of  $\phi$

$K$  is that set of all mappings  $f: X^n \rightarrow X$  and  $g, T: X \rightarrow X$  such that:

- $gT(X)$  is complete subspace of  $X$  containing  $f(X^n)$
- $f, T$  and  $g$  are commute and the only  $g, T$  are continuous mappings
- $f$  has mixed n-tupled gT-monotone property

**Theorem (2.5):** Let  $(X, G, \leq)$  be a partially ordered generalized metric space,  $f: X^n \rightarrow X$  and  $g, T: X \rightarrow X$  are three mappings lies in  $K$  and satisfy the equations conditions:

$$\begin{aligned} \forall x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n &\in X \text{ and } t > 0 \\ G(f(x_1, x_2, \dots, x_n), f(y_1, y_2, \dots, y_n), t) \\ &\leq \phi \left\{ \max \left\{ \phi_1 G(gT(x_1), gT(y_1), t), \phi_2 G(gT(x_2), gT(y_2), t), \dots, \phi_n G(gT(x_n), gT(y_n), t) \right\} \right\} \end{aligned} \quad (1)$$

$$\begin{aligned} gT(x_0^1) &\leq f(x_0^1, x_0^2, \dots, x_0^n) \\ gT(x_0^2) &\geq f(x_0^2, x_0^3, \dots, x_0^n, x_0^1) \\ &\vdots \\ gT(x_0^n) &\leq f(x_0^n, x_0^1, x_0^2, \dots, x_0^{n-1}) \text{ if } n \text{ is odd} \\ gT(x_0^n) &\geq f(x_0^n, x_0^1, x_0^2, \dots, x_0^{n-1}) \text{ if } n \text{ is even} \end{aligned} \quad (2)$$

If the following conditions hold:

- Every increasing sequence  $\langle x_n \rangle$  converge to  $x$  implies  $x_n \leq x \forall n \in \mathbb{N}$
- Every decreasing sequence  $\langle y_n \rangle$  converge to  $y$  implies  $y_n \geq y \forall n \in \mathbb{N}$

Then  $f$ ,  $g$  and  $T$  have an generalized n-tuplet coincidence point.

**Proof:** We can construct a sequence  $\{x_n\}$  lies in  $gT(X)$

$$\langle gT(x_k^1) \rangle, \langle gT(x_k^2) \rangle, \dots, \langle gT(x_k^n) \rangle \in gT(X)$$

Such that:

$$\begin{aligned} gT(x_k^1) &\rightarrow r^1 = gT(x^1) \in gT(X) \\ gT(x_k^2) &\rightarrow r^2 = gT(x^2) \in gT(X) \\ &\vdots \\ gT(x_k^n) &\rightarrow r^n = gT(x^n) \in gT(X) \end{aligned}$$

Considering the hypothesis 1 and 2 give in the theorem, we get:

$$\begin{aligned} gT(x_k^1) &\leq gT(x^1) = r^1 \\ gT(x_k^2) &\geq gT(x^2) = r^2 \\ &\vdots \\ gT(x_k^n) &\leq gT(x^n) = r^n \text{ (if } n \text{ is odd)} \\ gT(x_k^n) &\geq gT(x^n) = r^n \text{ (if } n \text{ is even)} \end{aligned}$$

Since,  $g$  and  $T$  are continuous mapping then, we have:

$$\begin{aligned} gT(gT(x_k^1)) &\rightarrow gT(r^1) \\ gT(gT(x_k^2)) &\rightarrow gT(r^2) \\ &\vdots \\ gT(gT(x_k^n)) &\rightarrow gT(r^n) \end{aligned}$$

And hence:

$$\begin{aligned} gT(gT(x_k^1)) &\leq gT(r^1) \\ gT(gT(x_k^2)) &\geq gT(r^2) \\ &\vdots \\ &\vdots \\ gT(gT(x_k^n)) &\leq gT(r^n) \text{ if } n \text{ is odd} \\ gT(gT(x_k^n)) &\geq gT(r^n) \text{ if } n \text{ is even} \end{aligned}$$

Choose  $t$  satisfy:

$$\begin{aligned} &G(gT(r^1), f(r^1, r^2, \dots, r^n), t) \leq \\ &G(f(r^1, r^2, \dots, r^n), t, gT(gT(x_{k+1}^1))) \\ &= G(f(r^1, r^2, \dots, r^n), gT(gT(x_{k+1}^1)), t) \\ &= G\left(f(r^1, r^2, \dots, r^n), \right. \\ &\quad \left. f(gT(x_k^1), gT(x_k^2), \dots, gT(x_k^n)), t\right) \\ &\quad \left\{ \begin{aligned} &\varnothing_1 G(gT(r^1), gT(gT(x_k^1)), t), \varnothing_2 G\left(gT(r^2), \right. \\ &\quad \left. gT(gT(x_k^2)), t\right) \\ &\quad \dots, \varnothing_n G(gT(r^n), gT(gT(x_k^n)), t) \end{aligned} \right\} \\ &\quad \left\{ \begin{aligned} &G(gT(r^1), gT(gT(x_k^1)), t), G(gT(r^2), gT(gT(x_k^2)), t) \\ &\quad \dots, G(gT(r^n), gT(gT(x_k^n)), t) \end{aligned} \right\} \end{aligned}$$

But:

$$gT(gT(x_k^1)) \rightarrow gT(r^1), gT(gT(x_k^2)) \rightarrow gT(r^2)$$

And:

$$gT(gT(x_k^n)) \rightarrow gT(r^n)$$

Which implies by definition G-convergent in G-metric space:

$$G(gT(r^1), f(r^1, r^2, \dots, r^n), t) = 0 \Rightarrow f(r^1, r^2, \dots, r^n) = gT(r^1)$$

Also, choose  $t^0$  satisfy:

$$\begin{aligned} &G(gT(r^2), f(r^2, r^3, \dots, r^1), t^0) \leq G(f(r^2, r^3, \dots, r^1), t^0, gT(gT(x_{k+1}^2))) \\ &= G(f(r^2, r^3, \dots, r^1), gT(gT(x_{k+1}^2)), t^0) \\ &= G(f(r^2, r^3, \dots, r^1), f(gT(x_k^2), gT(x_k^3), \dots, gT(x_k^1)), t^0) \\ &\quad \left\{ \begin{aligned} &\varnothing_1 G(gT(r^2), gT(gT(x_k^2)), t^0), \varnothing_2 G\left(gT(r^3), \right. \\ &\quad \left. gT(gT(x_k^3)), t^0\right) \\ &\quad \dots, \varnothing_n G(gT(r^1), gT(gT(x_k^1)), t^0) \end{aligned} \right\} \\ &\quad \left\{ \begin{aligned} &G(gT(r^2), gT(gT(x_k^2)), t^0), G(gT(r^3), \\ &\quad gT(gT(x_k^3)), t^0) \\ &\quad \dots, G(gT(r^1), gT(gT(x_k^1)), t^0) \end{aligned} \right\} \end{aligned}$$

But:

$$gT(gT(x_k^1)) \rightarrow gT(r^1), gT(gT(x_k^2)) \rightarrow gT(r^2)$$

And:

$$gT(gT(x_k^n)) \rightarrow gT(r^n)$$

Which is implies by definition G-convergent in G-metric space:

$$G(gT(r^2), f(r^1, r^2, \dots, r^l), t) = 0 \Rightarrow f(r^2, r^3, \dots, r^l) = gT(r^2)$$

**Continue these processes:** Choose  $t^*$  satisfy:

$$\begin{aligned} & G(gT(r^n), f(r^n, r^1, \dots, r^{n-1}), t^*) \\ & \leq G(f(r^n, r^1, \dots, r^{n-1}), t^*, gT(x_{k+1}^n)) \\ & = G(f(r^n, r^1, \dots, r^{n-1}), gT(gT(x_{k+1}^n)), t^*) \\ & = G(f(r^n, r^1, \dots, r^{n-1}), f(gT(x_k^n), gT(x_k^1), \dots, \\ & \quad gT(x_k^{n-1})), t^*) \\ & \leq \max \left\{ \begin{aligned} & \varphi G(gT(r^n), gT(gT(x_k^n)), t^*), \varphi_2 G(gT(r^1), \\ & \quad gT(gT(x_k^1)), t^*) \\ & \quad \dots, \varphi_n G(gT(r^{n-1}), gT(gT(x_k^{n-1})), t^*) \end{aligned} \right\} \\ & \leq \max \left\{ \begin{aligned} & G(gT(r^n), gT(gT(x_k^n)), t^*), G(gT(r^1), \\ & \quad gT(gT(x_k^1)), t^*) \\ & \quad \dots, G(gT(r^{n-1}), gT(gT(x_k^{n-1})), t^*) \end{aligned} \right\} \end{aligned}$$

But:

$$\begin{aligned} & gT(gT(x_k^1)) \text{ is G-convergent to } gT(r^1) \\ & gT(gT(x_k^2)) \text{ is G-convergent to } gT(r^2) \\ & \quad \vdots \\ & gT(gT(x_k^{n-1})) \text{ is G-convergent to } gT(r^{n-1}) \\ & gT(gT(x_k^n)) \text{ is G-convergent to } gT(r^n) \end{aligned}$$

Which is implies by definition of G-convergent in G-metric space:

$$G(gT(r^n), f(r^n, r^1, \dots, r^{n-1}), t^*) = 0$$

hence,  $f(r^n, r^1, \dots, r^{n-1}) = gT(r^n)$ . So  $(r^n, r^1, \dots, r^{n-1})$  is a generalized n-tupled coincidence point of  $f, G$  and  $T$ .

**Corollary (2.6):** Let  $(X, G, \leq)$  be a partially ordered generalized metric space,  $f: X^n \rightarrow X$  and  $g, T: X \rightarrow X$  are three mappings lies in  $K$ . Under the same assumptions of theorem (1) but:

$$G(f^r(x_1, x_2, \dots, x_n), f^r(y_1, y_2, \dots, y_n), t)$$

$$\leq \varphi \left[ \frac{1}{n} \left[ \varphi_1 G(gT^r(x_1), gT^r(y_1), t) + \varphi_2 G(gT^r(x_2), gT^r(y_2), t) + \dots \right] + \varphi_n G(gT^r(x_n), gT^r(y_n), t) \right]$$

Then  $f, G$  and  $T$  have an generalized n-tupled coincidence point.

**Corollary (2.7):** Let  $(X, G, \leq)$  be a partially ordered generalized metric space,  $f: X^n \rightarrow X$  and  $g, T: X \rightarrow X$  are three mappings lies in  $K$ . Under the same assumptions of theorem (1) but:

$$\begin{aligned} & G(f^r(x_1, x_2, \dots, x_n), f^r(y_1, y_2, \dots, y_n), t) \leq \\ & \varphi \left[ \frac{1}{n} \left[ k_1 G(gT^r(x_1), gT^r(y_1), t) + k_2 G(gT^r(x_2), gT^r(y_2), t) + \dots \right] + k_n G(gT^r(x_n), gT^r(y_n), t) \right] \end{aligned}$$

Such that,  $k_i \in (0, 1]$  for all  $i = 1, 2, \dots, n$ . Then  $f, g$  and  $T$  have a generalized n-tupled coincidence point.

**Remark (2.8):** If  $T = I$  (identity map) and  $f$  has mixed  $g$ -monotone property then, we get,  $f$  and  $g$  have n-tupled coincidence point. If  $T = g = I$  (identity map) and  $f$  has mixed monotone property then, we get  $f$  has n-tupled fixed point.

## CONCLUSION

In this study, we introduce the concepts of generalized n-tupled coincidence point and generalized mixed  $gT$ -monotone property and we prove the existence and uniqueness of generalized n-tupled coincidence point theorems without continuous condition for mappings having generalized mixed  $gT$ -monotone property in generalized metric spaces.

## REFERENCES

- Agarwal, R.P. and E. Karapinar, 2013. Remarks on some coupled fixed point theorems in G-metric spaces. Fixed Point Theor. Appl., 2013: 2-33.
- Alghamdi, M.A and E. Karapinar, 2013. G- $\beta$ - $\psi$ -contractive type mappings in G-metric spaces. Fixed Point Theor. Appl., 2013: 1-17.
- Aydi, H, E. Karapinar and W. Shatanawi, 2012b. Tripled common fixed point results for generalized contractions in ordered generalized metric spaces. Fixed Point Theor. Appl., 2012: 1-29.
- Aydi, H., B. Damjanovic, B. Samet and W. Shatanawi, 2011a. Coupled fixed point theorems for nonlinear contractions in partially ordered G-metric spaces. Math. Comput. Mod., 54: 2443-2450.

- Aydi, H., E. Karapinar and W. Shatanawi, 2011b. Coupled fixed point results for  $(\psi, \phi)$ -weakly contractive condition in ordered partial metric spaces. *Comput. Math. Applic.*, 62: 4449-4460.
- Aydi, H., E. Karapinar and W. Shatanawi, 2012a. Tripled fixed point results in generalized metric spaces. *J. Appl. Math.*, 2012: 1-10.
- Aydi, H., M. Postolache and W. Shatanawi, 2012b. Coupled fixed point results for  $(\psi, \phi)$ -weakly contractive mappings in ordered G-metric spaces. *Comput. Math. Applic.*, 63: 298-309.
- Berinde, V., 2011. Generalized coupled fixed point theorems for mixed monotone mappings in partially ordered metric spaces. *Nonlinear Anal. Theor. Methods Appl.*, 74: 7347-7355.
- Berinde, V., 2012. Coupled fixed point theorems for  $\phi$ -contractive mixed monotone mappings in partially ordered metric spaces. *Nonlinear Anal. Theor. Methods Appl.*, 75: 3218-3228.
- Bhaskar, T.G. and V. Lakshmikantham, 2006. Fixed point theorems in partially ordered metric spaces and applications. *Nonlinear Anal. Theor. Methods Appl.*, 65: 1379-1393.
- Bilgili, N. and E. Karapinar, 2013. Cyclic contractions via auxiliary functions on G-metric spaces. *Fixed Point Theor. Appl.*, 2013: 1-16.
- Chandok, S., W. Sintunavarat and P. Kumam, 2013. Some coupled common fixed points for a pair of mappings in partially ordered G-metric spaces. *Math. Sci.*, 2013: 1-7.
- Cho, Y.J., B.E. Rhoades, R. Saadati, B. Samet and W. Shatanawi, 2012. Nonlinear coupled fixed point theorems in ordered generalized metric spaces with integral type. *Fixed Point Theory Appl.*, 2012: 1-14.
- Choudhury, B.S. and A. Kundu, 2010. A coupled coincidence point result in partially ordered metric spaces for compatible mappings. *Nonlinear Anal. Theor. Meth. Appl.*, 73: 2524-2531.
- Choudhury, B.S. and P. Maity, 2011. Coupled fixed point results in generalized metric spaces. *Math. Comput. Mod.*, 54: 73-79.
- Chugh, R. and R. Rani, 2016. A weak convergence theorem for variational inequalities and fixed point problems in a real Hilbert space. *J. Eng. Appl. Sci.*, 11: 2962-2970.
- Ding, H.S. and E. Karapinar, 2013. Meir-Keeler type contractions in partially ordered G-metric spaces. *Fixed Point Theor. Appl.*, 2013: 1-10.
- Imdad, M., A.H. Soliman, B.S. Choudhary and P. Das, 2013. Onn-tupled coincidence point results in metric spaces. *J. Operators*, 2013: 1-8.
- Jleli, M. and B. Samet, 2012. Remarks on G-metric spaces and fixed point theorems. *Fixed Point Theor. Appl.*, 2012: 1-7.
- Karapinar, E., B. Kaymakçalan and K. Tas, 2012. On coupled fixed point theorems on partially ordered G-metric spaces. *J. Inequal. Appl.*, 2012: 1-13.
- Karapinar, E. and R.P. Agarwal, 2013. Further fixed point results on G-metric spaces. *Fixed Point Theor. Appl.*, 2013: 11-14.
- Mustafa, Z. and B. Sims, 2009. Fixed point theorems for contractive mappings in complete-metric spaces. *Fixed Point Theor. Appl.*, 2009: 1-10.
- Mustafa, Z., H. Aydi and E. Karapinar, 2013. Generalized Meir-Keeler type contractions on G-metric spaces. *Applied Math. Comput.*, 219: 10441-10447.
- Mustafa, Z., H. Aydi and E. Karapinar, 2012. On common fixed points in G-metric spaces using (E.A) property. *Comput. Math. Appl.*, 64: 1944-1956.
- Mustafa, Z., H. Obiedat and F. Awawdeh, 2008. Some fixed point theorem for mapping on complete G-metric spaces. *Fixed Point Theor. Appl.*, 2008: 1-12.
- Mustafa, Z., M. Khandagji and W. Shatanawi, 2011. Fixed point results on complete G-metric spaces. *Stud. Sci. Math. Hungarica*, 48: 304-319.
- Mustafa, Z., W. Shatanawi and M. Bataineh, 2009. Existence of fixed point results in G-metric spaces. *Intl. J. Math. Sci.*, 2009: 1-10.
- Roldan, A., E. Karapinar and P. Kumam, 2014. G-Metric spaces in any number of arguments and related fixed-point theorems. *Fixed Point Theor. Appl.*, 2014: 1-18.
- Saadati, R., S.M. Vaezpour, P. Vetro and B.E. Rhoades, 2010. Fixed point theorems in generalized partially ordered G-metric spaces. *Math. Comput. Mod.*, 52: 797-801.
- Samet, B., 2010. Coupled fixed point theorems for a generalized Meir-Keeler contraction in partially ordered metric spaces. *Nonlinear Anal. Theor. Methods Appl.*, 72: 4508-4517.
- Samet, B., C. Vetro and F. Vetro, 2013. Remarks on G-metric spaces. *Intl. J. Anal.*, 2013: 1-6.
- Shatanawi, W., 2010. Fixed point theory for contractive mappings satisfying  $\phi$ -maps in G-metric spaces. *Fixed Point Theor. Appl.*, 2010: 1-9.
- Shatanawi, W., 2011b. Coupled fixed point theorems in generalized metric spaces. *Haceteppe J. Math. Statist.*, 40: 441-447.
- Shatanawi, W., 2011a. Some fixed point theorems in ordered G-metric spaces and applications. *Abstr. Appl. Anal.*, 2011: 1-11.
- Shatanawi, W., M. Abbas and T. Nazir, 2011. Common coupled coincidence and coupled fixed point results in two generalized metric spaces. *Fixed Point Theor. Appl.*, 2011: 1-13.
- Tahat, N., H. Aydi, E. Karapinar and W. Shatanawi, 2012. Common fixed points for single-valued and multi-valued maps satisfying a generalized contraction in G-metric spaces. *Fixed Point Theor. Appl.*, 2012: 1-9.