Journal of Engineering and Applied Sciences 13 (24): 10203-10212, 2018
ISSN: 1816-949X
© Medwell Journals, 2018

On the Lindel of Spaces by Using coc-r-Open Sets

Raad Aziz Hussain Al-Abdulla and Fadhel Attala Shneef
Department of Mathematics, College of Computer Sciences and Technology,
University of AL-Qadisiyah, Al Diwaniyah, Iraq

Abstract: The purpose of this research is to extend and study types of topological spaces as Lindelof spaces by using coc-r-open sets. In this study, we study and introduce the concepts of coc-r-open sets. A special intrest types of spaces called coc-r-Lindelof and I-coc-r-Lindelof are studied and obtain some of their basic properties.We introduce new types of functions using coc-r-open sets. Moreover, the relation between Lindelof, I-Lindelof, coc-r-Lindelof and I-coc-r-Lindelof spaces are studied and investigated the topological properties of them using coc-r-open sets.

Key words: coc-r-open, coc-r-β-open and coc-r-regular, open sets, ʂ-coc-r-Lindelof, coc-r-extremally disconnected, coc-r-Lindelof and I-coc-r-Lindelof spaces

INTRODUCTION

We recall the concept of a I-Lindelof space by using coc-r-open sets and give some important generalizations on this concept and also we prove of some results on this concept. By Al-Ghour and Samarah (2012) researchers provided coc-open sets in the topological spaces where they studied continuity by using these sets. Later, some researchers have studied these sets and expanded by Stone (1937), regular open sets were introduced and used to define the semiregularization space of a topological space (Willard, 1970; Bourbaki, 1989) studieded the concept of compact space by Jankovic and Konstadilaki (1996) introduced the concept of rc-compact, rc-Lindelof, countably rc-compact, perfectly k-normal, Luzin space, generalized ordered space by Al-Zoubi and Al-Nashef (2004) introduced the concept of I-Lindelof spaces.

Definition 1.1 (Al Ghour and Samarah, 2012): A subset B of topological space (X, τ) is said cocompact open set (coc-open set) if for each x in B there is open set G in X and compact subset L∈C (X, τ) such that x∈G-L⊆B. The complement of coc-open set they call it coc-closed.

Remarks 1.2 (Jasim, 2014):

- Each open set is a coc-open set
- Each closed set is a coc-closed set
- The reverse of (1, 2) is not always happen

Definition 1.3 (Stone, 1937): A subset A of a topological space (X, τ) they call it regular open set (r-open set) if $A = \bar{A}^{\circ}$. The complement of regular open set they call it regular closed (r-closed) set and it easy to see that A is regular closed if $A = \bar{A^{\circ}}$.

Remarks 1.4 (Willard, 1970):

- Each r-open set is open
- Each r-closed set is a closed
- The converse of (1, 2) not always happen

Remarks 1.5 (Willard, 1970):

- The family of all r-open sets in X is symbolized RO (X, τ)
- The family of all r-closed sets in X is symbolizedd RC (X, τ)

Definition 1.6: A subset B of topological space (X, τ) they call it co-compact regular open set (coc-r-open set) if for each x in B there is r-open set G⊆X and compact set L in X such that x∈G-L⊆B, the complement of coc-r-open set they call it coc-r-close set.

Remark 1.7: Each coc-r-open set is not necessarily to be open set, every coc-r-closed set is not always to be closed set. Also, each open set is not necessarily to be coc-r-open set and every closed set is not necessarily to be coc-r-closed set.

As the next examples

Examples 1.8: Let, X = {a, b, c}, τ = {∅, X, {a}, {b}, {a, b}} be a space on X, the coc-r-open sets are {X, ϕ, {a},

Corresponding Author: Raad Aziz Hussain Al-Abdulla, Department of Mathematics, College of Computer Sciences and Technology, University of AL-Qadisiyah, Al Diwaniyah, Iraq

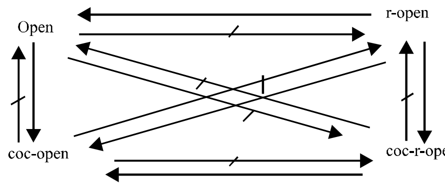

Fig. 1: The following diagram shows the relation between types of coc-r-open sets

{b}, {c}, {a, b}, {a, c}, {b, c}} then {c} is a coc-r-open but it is not open and {b} is coc-r-closed but it is not closed.

Let X = {1, 2, 3, ...,}, τ = {G⊆X: 1∈G}∪{∅} be topology on X, the coc-r-open sets are {G⊆X: G^C}. Notice. that {1} is an open but is not coc-r-open and {2, 3, ...,} is a closed but it is not coc-r-closed.

Remarks 1.9:

- Each r-open set is coc-open
- Each r-closed is coc-closed
- Each r-open set is coc-r-open set
- Each r-closed set is coc-r-closed set
- Each coc-r-open set is coc-open
- Each coc-r-closed set is coc-closed

Proof: It is clear.

Remark 1.10: The converse of remarks 9 is not always happen as the next examples:

Examples 1.11: Let, X = {a, b, c}, τ = {∅, X, {a}, {b}, {a, b}} be a topology on X. It is clear {a, b} is a coc-open: coc-r-open but it is not r-open and {C} is coc-closed; coc-r-closed but it is not r-closed.

Let X = {1, 2, 3, ...,}, τ = {G⊆X: 1∈G}∪{∅} a topology on X, the coc-r-open sets are {G⊆X: G^C is finite}∪{∅}, thus, {1} is a coc-open but it is not coc-r-open and {2, 3, ...,} is a coc-closed but it is not coc-r- closed (Fig. 1). The relation between types of coc-r-open sets.

Remarks 1.12:

- The intersection of two r-open set is r-open (Willard, 1970)
- The intersection of r-open sets and open set is open
- The intersection of two coc-r-open set is coc-r-open
- The union of coc- r-open sets is coc-r-open set
- The intersection of coc-r-open sets and coc-open set is coc-open
- The intersection of two coc-open set is coc-open (Al Ghour and Samarah, 2012)

Proof: Clear. Let C, Dbe coc-r-open, to prove C∩ coc-r-open set. Suppose x∈C∩D, then x∈C and x∈D, since, C, D are coc-r-open, thus, there exist two r-open sets G, W⊆X and two compact subset K, L such that x∈G-K⊆C, x∈W-L⊆D, therefore, x∈(G-K)∩(W-L)⊆C∩D imply that x∈(G∩K^C)∩(W∩L^C)⊆C∩D then, x∈(G∩W)∩(K^C∩L^C)⊆C∩D thus, we get x∈(G∩W)-(K∪L)⊆C∩D by using (1) G∪W is r-open, since, K∪L⊆X is compact set in X. Hence, C∩D is coc-r-open.

Let Aα, α∈Λ be coc-r-open to prove $\cup_{\alpha\in\Lambda}$ Aα is coc-r-open. Suppose x∈$\cup_{\alpha\in\Lambda}$, then x∈Aα for some α∈Λ, since, Aα is coc-r-open, thus, there is r-open sets Uα⊆X, compact subset Kα such that x∈Uα-Kα⊆Aα for some α∈Λ, since, Aα⊆$\cup_{\alpha\in\Lambda}$Aα. Hence, ⊆$\cup_{\alpha\in\Lambda}$ is coc-r-open. Clear.

Definition 1.13 (Radhy, 2010): Let X be a topological space and C be a subset of X. A point x∈C they call it r-interior point in C if there is r-open set G in X containing x such that x∈G⊆C.

The set of all r-interior points of B they call it r-interior subset of B, it is symbolized and C^{or} r-open set in X such that {B⊆C}.

Definition 1.14 (Radhy, 2010): Let X be a topological space and B be a subset of X. The intersection of all r-closed sets of X which contains B are called r-closure of B and is denoted $\overline{B}^r$.

Remarks 1.15 (Radhy, 2010): Let X be a topological space and B be a of subset, then:

- B^{or}⊆B^o
- B⊆$\overline{B}^r$
- If x∈$\overline{B}^r$, then for any r-open set G in X contains x we have G∩B≠ϕ
- If B a closed subset of X, then, B^o is a r-open subset
- If A an open subset of X, then, $\overline{A}$ is a r-closed set
- If A a r-closed set, then A is closed set

Definition 1.16 (Radhy, 2010): A space X is said to be r-compact if every r-open covering of X has a finite sub covering.

Proposition 1.17 (Radhy, 2010):

- Every compact topological space is r-compact
- Eech r-compact subset of T_2 topological space is r-closed set set

Theorem 1.18: Let X be T_2-space and A a subset of X:

- If A is a coc-r-open in X, then, $A = A^{or}$
- If A is a coc-r-closed in X, then, $A = \overline{A}^{r}$

Proof: Let A is coc-r-open in X, since, $A^{or} \subseteq A^{o} \subseteq A$, we need to prove that $A \subseteq A^{or}$. Let $x \in A$, since, A is coc-r-open, thus, there exist r-open G, compact set K such that $x \in G-K \subseteq A$. Since, every compact is r-compact and X be T_2-space, thus, K is r-closed set (by using Proposition (1.17), (1), (2)), so, K^C r-open subset in X and $x \in G \cap K^C \subseteq A$ and G, K^C are r-open sets in X, therefore, $G \cap K^C$ is r-open in X, hence, $x \in A^{or}$.

Let $x \in \overline{A}^{r}$ and $x \notin A$, then $x \in A^C$, since, A is coc-r-closed in X, thus, A^C is coc-r-open in X and $x \in A^C$, there exist r-open U, compact subset K such that $x \in U-K \subseteq A^C$. Since, K is compact subset in X, therefore, K is r-compact, so, K is r-closed (by using proposition (1.17), (1), (2)), then K^C r-open, since, $U \cap K^C$ is r-open, $x \in U \cap K^C \subseteq A^C$, $x \in \overline{A}^{r}$ and using by remarks (1.15), (3) then $(U \cap K^C) \cap A \neq \phi$ this is contradiction with $U \cap K^C \subseteq A^C$, thus, $x \in A$, since, $A \subseteq \overline{A}^{r}$ hence, $A = \overline{A}^{r}$.

Remarks 1.19:

- The coc-r-open sets is a topology on X symbolized by τ^{rk}
- If X is a finite set then τ^{rk} is a discrete topology
- A closed subset of compact space X is compact relative to X (Lipschutz, 1995)
- In each topological space, the interssection of compact set and a closed set is compact (Lipschutz, 1995)
- Each compact subset of T_2topological space is closed set (Lipschutz, 1995)
- A topological space (X, τ) is regular space if for each $x \in$ in x and open set U in X such that x there is an ope n set W such that $x \in W \subseteq \overline{W}^{r}$ (Dugundji, 1978)
- A space (X, T) is called T_3-space if X is regular space and T_1-space (Dugundji, 1978)
- Every T_3-space is T_2-space (Dugundji, 1978)

Proposition 1.20 (Radhy, 2010): Let X be regular space, if $A \subseteq X$ is an open then $A \in RO$ (X, τ).

Corollary 1.21: Let X be regular space, if $F \subseteq X$ is a closed then $F \in RC$ (X, τ).

Proof : It is clear.

Theorem 1.22: Let (x, τ) be T_2 topological space, thus, $\tau^{rk} \subseteq \tau$.

Proof: Used $B \in \tau^{rk}$ to prove $B \in \tau$. Let $x \in B$, thus, there is r-open set $G \subseteq X$ and compact subset L such that $x \in G-L \subseteq B$, thus, $x \in G \cap L^C \subseteq B$. Since, L is compact and X is T_2-space, therefore, L is closed, so, L^C is open. By using remarks (1.19), (5), so, $G \cap L^C$ is open set in X. Hence, $B \in \tau$.

Remarks 1.23: Let (X, τ) be a T_2 topological space, thus:

- Each coc-r-open set is open
- Each coc-r-closed set is closed

Proof: It clear.

Theorem 1.24: Let (X, τ) be a regular-space, then $\tau \subseteq \tau^{rk}$

Proof: Clear by using proposition (1.20) and remarks (1.9), (3).

Theorem 1.25: Let (X, τ) be a T_3-space, then $\tau = \tau^{rk}$.

Proof: It is clear.

Definition 1.26: Let (X, τ) be a topological space and B be subset of X. The intersection of all coc-r- closed subsets of X which contains B called coc-r-closure of B and is denoted $\overline{B}^{rK}$ such that $\overline{B}^{rK} = \cap$ {F: F coc-r-closed set in X and $B \subseteq F$}

Remark 1.27: $\overline{B}^{rK}$ smallest coc -r-closed set contains B

Proposition 1.28: Let (X, τ) be a topological space and $A \subseteq B \subseteq X$. Thus:

- $\overline{A}^{rK}$ is an coc-r-closed set
- A is coc-r-closed set if $A = \overline{A}^{rK}$
- $\overline{A \cup B}^{rk} = \overline{A}^{rk} \cup \overline{B}^{rk} = \overline{\overline{A}^{rK}}^{r}$
- $\overline{A}^{rK} \subseteq \overline{A}^{rK}$

Proof: It is clear.

Proposition 1.29: Let X be a topological space and $A \subseteq X$. Then, $x \in \overline{A}^{rK}$ iff for every coc-r-open setg in x contained point X we get $G \cap A \neq \phi$.

Proof: It is clear.

Proposition 1.30: Let X be space, A and B be subsets of X:

- $\overline{\phi}^{rk} = \phi$, $\overline{X}^{rk} = X$
- $\overline{A \cup B}^{rk} = \overline{A}^{rk} \cup \overline{B}^{rk}$
- $\overline{A \cup B}^{rk} \subseteq \overline{A}^{rk} \cup \overline{B}^{rk}$

Proof: It clear.

Definition 1.31: Let X be topological space and A be a subset of X. The union of all coc-r-open subsets of X containing in A they call it coc-r-interior of A symbolized. A^{ork} such as $A^{ork} = \cup$ {U: U coc-r-open set in X and $U \subseteq A$}.

Proposition 1.32: Let X a topological space and B a subset of X, thus, B^{ork} is the largest coc-r-open set containing in B.

Proof: Clear by definition of B^{ork}.

Proposition 1.33: Let X be a topological space and B be a subset of X, then $x \in B^{ork}$ if and only if there is coc-r-open set U containing x such that $x \in U \subseteq B$.

Proof: It is clear.

Proposition 1.34: Let X be a space and $A \subseteq B \subseteq X$, thus:

- A^{ork} is coc-r-open set
- A is coc-r-open ifff A^{ork}
- $A^{ork} = (A^{ork})^{ork}$
- If $A \subseteq B$ then $A^{ork} \subseteq B^{ork}$
- $A^{ork} \cup B^{ork} \subseteq (A \cup B)^{ork}$
- $A^{ork} \cap B^{ork} \subseteq (A \cap B)^{ork}$

Proof: It is clear.

Proposition 1.35: Let (X, τ)a topological space, A a sub set of X, thus:

- $\left(\overline{A}^{rk}\right)^c = \left(A^C\right)^{ork}$
- $(A^{ork})^c = \overline{\left(A^c\right)}^{ork}$
- $\overline{A}^r = \left(A^{cork}\right)^c$
- $A^{or} = \left(\overline{A^c}^{rk}\right)^c$

Proof: Since, $A \subseteq \overline{A}^{rk}$, then $\left(\overline{A}^{rk}\right)^c \subseteq A^c$ and $\overline{A}^{rk}$ coc-r - closed subset in X, then $\left(\overline{A}^{rk}\right)^c$ coc-r-open set in X but $\left(A^c\right)^{ork}$ coc-r-open subset in X with $\left(A^c\right)^{ork} \subseteq A^c$. By using proposition (1.32), then:

$$\left(\overline{A}^{rk}\right)^c \subseteq \left(A^c\right)^{ork} \quad (1)$$

Now: Let $x \in (A^c)^{ork}$, thus, there is coc-r-open set G in X such that $x \in G \subseteq A^c$, to prove $x \in \left(\overline{A}^{rk}\right)^c$.

Let $x \notin \left(\overline{A}^{rk}\right)^c$, thus, $x \in \overline{A}^{rk}$, since, $x \in G$ and G coc-r-open subset in X.

Hence, $G \cap A \neq \phi$, this is contradiction with $G \subseteq A^c$, so, $x \in \left(\overline{A}^{rk}\right)^C$. Hence:

$$\left(A^c\right)^{ork} \subseteq = \left(\overline{A}^{rk}\right)^c \quad (2)$$

From Eq. 1 and 2, we get $\left(A^c\right)^{o}k = \left(\overline{A}^{rk}\right)^c$
By using Eq. 1, $A^{ork} = \left(\overline{A^c}^{rk}\right)^c$, then, $\overline{A^c}^{rk} = \left(A^{ork}\right)^c$
By using Eq. 1, $\left(\overline{A}^{rk}\right) = \left(A^c\right)^{ork}$, then, $\left(A^{c\,ork}\right)^c = \overline{A}^{rk}$
By using Eq. 1, $\overline{A^c}^{rk} = (A^{ork})^C$, then, $A^{ork} = \left(\overline{A^c}^{rk}\right)^c$

On coc-r-β-open and coc-r-regular open sets

Definition 2.1 (Kumar and Chowdhary, 2011): A subset A is said to be β-open set if $A \subseteq \overline{A^{\circ}}$. The complement of β-open is said β-closed. A subset A is called β-closed set if $\overline{A^{\circ}} \subseteq A$.

Definition 2.2: A subset B is said coc-r-β-open set if $B \subseteq \overline{\overline{B}^{rk\,ork}}^{rk}$. The complement of coc-r-β-open is called coc-r-β-closed. A subset B is called coc-r-β-closed set if $\overline{B^{\circ rk}}^{rk\,a} \subseteq B$.

Remark 2.3: β-open $\not\rightarrow$ coc-r-β-open.
coc-r-β-open $\not\rightarrow$ β-open. As the following examples shows:

Examples 2.4: Let x = {1, 2, 3, ...,}, τ = {G⊆X: 1∈G}∪{ϕ} then {G⊆X: G^c is finite}. ∪{ϕ}, let A = {1}, then $\overline{\overline{A}^{rk\,a}}^{rk} = \phi$, then A is not coc-r-β- open but $\overline{A^{\circ}} = X$, then A is β- open. Let X = {a, b, c}, τ = {∅, X, {a}}, then τ^{rk} = {A: A⊆X}, then {b} is coc-rβ-open but is not β-open because $\overline{\overline{\{b\}}^{\circ}} = \varnothing$.

Remark 2.5: Every coc-r-open is coc-r-β-open set in X but the convers is not true in general as the following example shows.

Example 2.6: Let X = R with usual topology, since, X is T_2 and regular space, then $\tau = \tau^{rk}$, A = (0, 1], thus, A is coc-r-β-open but is not coc-r-open in X.

Remark 2.7: The intersection of two coc-r-β-open sets is not necessary coc-r-β-open set as the following example show.

Example 2.8: Let X = R with usual topology, since, X is T_2 and regular space, then $\tau = \tau^{rk}$, C = (1, 2], D = [2, 3), thus, C, D are coc-r-β-open but C∩D = {2} is not coc-r-open in X.

Remarks 2.9: A subset B is called coc-r-X-open in (X,T) if B is called β-open in (X,τ^{rk}). The family of all coc-r-β-open subsets in X is symbolized βO (X, τ^{rk}). Every r-open is coc-r-β-open set.

Definition 2.10: Let (X, τ) be a topological space and $B \subseteq X$. Then, B is said coc-r-regular open subset in X if $B=\overline{B}^{rk\,ork}$. The complement of coc-r-regular open set is said coc-r-regular closed and it easy to see that B is coc-r- regular closed if $B=\overline{B^{ork}}^{rk}$.

Remarks 2.11: A subset B is called coc-r-regular open in (X,T) if B is called r-open in (X, τ^{rk}). The family of all coc-r-regular open subset in X is symbolized RO (X, τ^{rk}). The family of all coc-r-regular closed subset in X is symbolized RC (X,τ^{rk}).

Remarks 2.12: If $A \in$ RO (X, τ^{rk}) then A is coc-r-open but the convers is not always happen such that the next example.

Example 2.13: Let, $X = \{1, 2, 3, ...,\}$, $\tau = \{G \subseteq X: 1 \in G\} \cup \{\varnothing\}$, then $\tau^{rk} = \{G \subseteq X: G^c \text{ is finite}\} \cup \{\varnothing\}$, let $A = \{1, 3, 4, 5, ...,\}$ is coc-r-open in X but $\overline{A}^{rk\,ork} = X$, hence, $A \notin$ RO (X,τ^{rk}).

Proposition 2.14:

- If A is coc-r-open, then $\overline{A}^{rk} \in RO(X, \tau^{rk})$
- If A is coc-r-closed, then $A^{ork} \in RO(X, \tau^{rk})$
- If A, $B \in$ RO (X, τ^{rk}), then $A \cap B \in$ RO (X, τ^{rk})
- If $A \in \beta O$ (X, τ^{rk}), then $\overline{A}^{rk} \in RO(X, \tau^{rk})$

Proof: It is clear.

Remarks 2.15:

- If $A \in$ RO (X, τ^{rk}), then $A \in \beta O$ (X, τ^{rk})
- If $A \in$ RC (X, τ^{rk}), then $A \in \beta O$ (X, τ^{rk})
- If $A \in$ RC (X, τ^{rk}), then A is coc-r-closed

Proof: It is clear.

On I-coc-r-Lindelof spaces

Definition 3.1 (Bourbaki, 1989): A topological space (X, τ) is said Lindelof if each open cover of X has a countableb sub cover.

Definition 3.2: A space X is said to be a coc-r-Lindelof if each coc-r-open covering of X has a countable sub covering.

Definition 3.3 (Al-Zoubi and Al-Nashef, 2009): A topological space (X, T) is said I-Lindelof if each covering F of X by r-closed subsets of the topological space (X, T) containing a countable subcover L such that $X = \cup \{F^{\circ}: F \in L\}$.

Definition 3.4: A topological space (X, T) is said to be a I-coc-r-Lindelof if each covering F of X by coc-r-regular closed subsets of the topological space (X, T) containing countable subcover L such that $X = \cup\{F^{ork}: F \in L\}$.

Examples 3.5: The following are straight forward examples of I-coc-r-Lindelof spaces.

Let $X = \{1, 2, 3, ...,\}$, $\tau = \{G \subseteq X: 1 \in G\} \cup \{\phi\}$ then, $\tau^{rk} = \{G \subseteq X: G^c \text{ is finite}\} \cup \{\varnothing\}$, since, all coc-r-regular closed sets of the topological space (X,T) are $\varnothing$, X, thus, X is I-coc-r-Lindelof.

Let $X = \{1, 2, 3, 4\}$, $\tau = \{\varnothing, X, \{4\}, \{2, 3\}, \{2, 3, 4\}\}$ then $\tau^{rk} = \{G: G \subseteq X\}$. Thus, X is I-coc-r-Lindelof.

Theorem 3.6: The next are equivalent for a topological space (X, T):

- X is a I-coc-r-Lindelof
- Each cover $\{U_\alpha: \alpha \in \Lambda\}$ of X by coc-r-β-open subsets containing a countable subcover such that $X = \cup_{\alpha \in N} \overline{U_\alpha}^{rk\,ork}$
- Every family $\{U_\alpha: \alpha \in \Lambda\}$ of X by coc-r-regular open subsets with empty intersection containing a countable subfamily such that

$$\cap_{n \in N} \overline{U_{\alpha n}}^{rk} = \varnothing$$

Proof: (I)→(ii) Let $\{U_\alpha: \alpha \in \Lambda\}$ cover of X by coc-r-β-open subsets, then $\overline{U_\alpha}^{rk} \in RO(X, \tau^{rk})$, (by using proposition (2.14), (4)) for all $\alpha \in \Lambda$. Thus, $\{\overline{U_\alpha}^{rk}: \alpha \in \Lambda\}$ forms covering of X, since, X is I-coc-r-Lindelof, therefore, $\{\overline{U_\alpha}^{rk}: \alpha \in \Lambda\}$ has a countable subcover such that $X = \cup_{n \in N} \overline{U_{\alpha n}}^{rk\,ork}$

(ii)→iii)

Used $\{U_\alpha: \alpha \in \Lambda\}$ be a family of coc-r-regular open sets of X with empty intersection. Since, $U_\alpha^c \in$ RC (X, τ^{rk}) for all $\alpha \in \Lambda$. by using (remarks (2.15), (2)) we get $U_\alpha^c \in \beta O$ (X, τ^{rk}) for all $\alpha \in \Lambda$. Since, $\cap_{\alpha \in \Lambda} U_\alpha = \varnothing$, then $X = \cup_{\alpha \in \Lambda} U_\alpha^c$, thus, $\{U_\alpha^c: \alpha \in \Lambda\}$ is cover of X. By assumption, $\{U_\alpha: \alpha \in \Lambda\}$ has a countable subcover such that Therefore, $X = \cup_{n \in N} \overline{U_{\alpha n}^{c}}^{rk\,ork} = \cup_{n \in N} \left(U_{\alpha n}^{ork^c}\right)^{ork}$ $\varnothing = \cap_{n \in N} \overline{U_{\alpha n}^{ork}}^{rk} = \bigcap_{n \in N} \overline{U_{\alpha n}}^{rk}$

(iii)→(i)

Let $\{F_\alpha: \alpha \in \Lambda\}$ be covering of X by coc-r-regular closed sets in X, thus, $X = \cup_{\alpha \in \Lambda}, F_\alpha$, thus, $\varnothing = \cap_{\alpha \in \Lambda} F_\alpha^c$. Since, $F_\alpha \in$ RC (X, τ^{rk}), therefore, F_α^c RO (X, τ^{rk}) for all $\alpha \in \Lambda$ by assumption The family $\{F_\alpha^c: \alpha \in \Lambda\}$ has a countable subfamily such that $\varnothing = \bigcap_{n \in N} \overline{F_{\alpha n}^{c}}^{rk}$. Then $X = \bigcup_{n \in N} \left(\overline{F_{\alpha n}^{c}}^{rk}\right)^c = \bigcup_{n \in N} F_{\alpha n}^{ork}$, hence, X is I-coc-r-Lindelof.

Definition 3.7: A topological space (X, T) is said to be I-coc-r-compact if each covering u of X by coc-r-regular closed subsets of the topological space (X, T) containing finite subcover v such that:

$$X = \cup\left\{U^{\circ rk} : U \in V\right\}$$

Remark 3.8: Every I-coc-r-Compact space is I-coc-r-Lindelof and the converse is not necessary as the next example.

Example 3.9: Let, X = {1, 2, 3, ...,}, τ = {A: A⊆X} then τ^{rk} = {A: A⊆X}, since, (X, τ^{rk}) is desecrate topology, then A∈RC (X, τ^{rk}) for each A subset of X, X is a countable set, thus, X is I-coc-r-Lindelof but {{x}: x∈X} is a cover of X such that {x}∈RO (X, τ^{rk}) but it has not a finite subcover, hence, X is not I-coc-r-compact.

Remark 3.10: Every coc-r-Lindelof space is not necessary to be I-coc-r-Lindelof as the next example.

Example 3.11: Let X = {1,2 , 3, ...,}, τ = {G⊆X: 1∉G}∪{X} then τ^{rk} = {G⊆X: 1∉G}∪{G⊆X: 1∈G, G^c is finite}, thus, X is coc-r-Lindelof space but the cover {A = {1, 2, 4, 6, ...,}}∪{{X}, x∉A} of X by coc-r-regular closed sets is a countable cover but X≠$A^{ork}\cup_{x\notin A}$ = {2, 4, 6, ...,} $\cup_{x\notin A}$ = {x}.

Definition 3.12: A topological space (X, T) is said coc-r-extremally disconnected (coc-r-e.d) if $\overline{G}^{rk}$ is coc-r-open for every coc-r- open G in X.

Proposition 3.13: If C∩D = ∅, C is coc-r-open, then $C \cap \overline{D}^{rk} = \varnothing$.

Proof : Let C∩D = ∅, then there is x∈C∩D^{rk}, since, C∩D = ∅ and C coc-r-open, thus, x∉D, so, x∈D^{rk} and C∩D-{x}≠∅. Therefore, C∩D = ∅, this is contradiction, hence, $C \cap \overline{D}^{rk} = \varnothing$.

Proposition 3.1: A space X is coc-r-e.d if for all G, W∈RO (X, τ^{rk}) with G∩W = ∅, then $\overline{G}^{rk} \cap \overline{W}^{rk} = \varnothing$.

Proof: Let X be coc-r-e.d and G, W∈RO (X, τ^{rk}) with G∩W = ∅. Since, G∈RO (X, τ^{rk}), then G is coc-r-open, thus, $G^{rk} \cap \overline{W}^{rk} = \varnothing$, (By using proposition (3.13)). Since, X is coc-r-e.d and W∈RO (X, τ^{rk}), therefore, $\overline{W}^{rk}$ is coc-r-open in X, so, $\overline{W}^{rk} \cap \overline{G}^{rk} = \varnothing$ by using proposition (3.13). Conversely : Let G be coc-r-open, then $\overline{G}^{rk} \in RC(X, \tau^{rk})$ (By using proposition (2.14), (1)), thus, $\overline{G}^{rk} \in RC(X, \tau^{rk})$, since, $\overline{G}^{rk}$ is coc-r-closed , therefore, $\overline{G}^{rkork} \in RO(X, \tau^{rk})$, (proposition (2.14), (2)), since, $\overline{G}^{rk^c} \cap \overline{G}^{rk^{ork}} = \varnothing$ by using assumption we get $\overline{\overline{G}^{rk^c}}^{rk} \cap \overline{\overline{G}^{rk^{ork}}}^{rk} = \varnothing$, then $\overline{G}^{rk} \cap \overline{\overline{G}^{rk^c}}^{rk} = \varnothing$, so, $\overline{G}^{rk} \subseteq \overline{G}^{rk^{ork}}$, then $\overline{G}^{rk}$ is coc-r-open. Hence, X is coc-r-e.d.

Proposition 3.15: If X is coc-r-e.d, L is subset of X such that L∈RC (X, τ^{rk}), then L coc-r-open.

Proof: Let L∈RC (X, τ^{rk}), then $L = \overline{F^{ork}}^{rk}$, since, X is coc-r-e.d, thus, $\overline{L^{ork}}^{rk}$ is coc-r-open, hence, L coc-r-open.

Proposition 3.16: Every I-coc-r-Lindelof space is coc-r-e.d.

Proof: Let X be I-coc-r-Lindelof space. Suppose X is not coc-r-e.d, then there is G, W∈RO (X, τ^{rk}) such that G∩W = ∅ but $\overline{G}^{rk} \cap \overline{W}^{rk} \neq \varnothing$. Then there is $x \in \overline{G}^{rk} \cap \overline{W}^{rk}$, since, G, W∈RO (X, τ^{rk}), thus, G^c, W^c∈RC (X, τ^{rk}), therefore, {G^c, W^c} forms a covering of X, since, X is I-coc-r-Lindelof space, so, X = G^{cork}∪W^{cork}. Let x∈G^{cork} but $x \in \overline{G}^{rk}$, then G^{cork}∩G ≠ ∅. Since, G^{cork}∩G⊆G^c∩G, this is contradiction, thus, $\overline{G}^{rk} \cap \overline{W}^{rk} = \varnothing$. Hence, X is coc-r-e.d (By using proposition (3.14)).

Remark 3.17: The convers of proposition 3.13) is not necessary true as the next example.

Example 3.18: Let, X = R with indiscrete topology, then τ^{rk} = {A: A⊆X}, then X is coc-r-e.d. Since, {x}∈RC (X, τ^{rk}) for every x in x, since, the cover {{x} x∈X} of X has not a countable subcover such that X = $\cup_{x\in X}\{x\}^{ork}$.

Theorem 3.19: Every coc-r-Lindelof, coc-r-e.d space is I-coc-r-Lindelof space.

Proof: Let {F_α: α∈Λ} be covering of X, F_α∈ RC (X, τ^{rk}) for all α∈Λ, then, F_α is coc-r-open for all α∈Λ (By using Proposition (3.15)). Thus, {F_α: α∈Λ} is covering of X by coc-r-open subsets, since, X is coc-r-Lindelof space, therefore, {F_α: α∈Λ} has countable subcover such that $X = \cup_{n\in N} F_{\alpha n} = \cup_{n\in N} F_{\alpha n}^{ork}$. Hence, X is I-coc-r-Lindelof space.

Remark 3.20:

- I-coc-r-Lindelof ↛ I-Lindelof
- I-Lindelof ↛ I-coc-r-Lindelof

As the following examples show.

Examples 3.21: 1) Let X = {1-4}, τ = {∅, X, {4}, {2, 3}, {2, 3, 4}}, then, τ^{rk} = {G: G ⊆ X}.

Thus, X is I-coc-r-Lindelof, since, {1, 2, 3}, {1, 4} are r-closed cover of X but X ≠ $\{1, 2, 3\}^{ork} \cup \{1, 4\}^{ork}$ = {2, 3} ∪ {4} = {2, 3, 4}, thus, X is not I-Lindelof.

Let X = R with indiscrete topology, then τ^{rk} = {A: A⊆X}, thus, X is I-Lindelof but X is not I-coc-r-Lindelof.

Definition 3.22: A topological space (X, T) is said S-coc-r-Lindelof if each covering F of X by coc-r-regular closed subsets of the topological space (X, T) containing countable subcover L such that X = ∪ {F:F∈L}.

Remark 3.23: Every I-coc-r-Lindelof space is S-coc-r-Lindelof but the convers is not necessary as the next example.

Example 3.24: Let X = {1, 2, 3, 4, ...,}, τ = {G⊆X: 1∉G}∪{X}, then, τ^{rk} = {G ⊆ X: 1∉G}∪{G⊆X: 1∈G, G^c is finite}, thus, X is S-coc-r-Lindelof, since, A = {3, 5, 7, ...,} is coc-r-open in X but $\overline{A}^{rk}=\{1, 3, 5, 7,...,\}$ is not coc-r-open in X, therefore, X is not coc-r-e.d, hence, X is not I-coc-r-Lindelof (By using the convers of Proposition (3.13)).

Theorem 3.25: A topological space (X, T) is I-coc-r-Lindelof iff it a coc-r- e.d and S-coc-r-Lindelof space.

Proof: The necessity is clear. We prove the sufficiency. Let {F_α: α∈∧} be covering of X by coc-r-regular closed sets, since, X is coc-r-e.d, then F_α is coc-r-open for each α∈∧ (By using Proposition (3.15)), thus, $F_\alpha = F_\alpha^{ork}$. Since, X is S-coc-r-Lindelof space, therefore, {F_α: α∈∧} has countable subcover such that $X = \cup_{n\in N} F_{\alpha n} = \cup_{n\in N} F_{\alpha n}^{ork}$, hence, X is I-coc-r-Lindelof.

Definition 3.26: A topological space (X, T) is said to be coc-r-regular space iff for every x in x and closed subset C of X such that x∉C there exist coc-r-open sets G, W such that G∩W = ∅, x∈G and C⊆W.

Definition 3.27: A space (X, T)is said to be coc-r-regular space if for each x in X and coc-r-closed subset F of X such that x∉F there exist disjoint coc-r-open sets G, W such that x∈G and C⊆W.

Proposition 3.28: A topological space (x, T) is coc-r-regular space iff for all x in X and all open set U in X such that x∈U there is coc-r-open set G such that $X \in G \subseteq \overline{G}^{rk} \subseteq U$.

Proof: Let X be coc-r-regular space and x∈X be open subset in X such that x∈U. Then, U^c is closed set in X and x∉U^c then, there is coc-r-open sets G, W such that G∩W = ∅, x∈G and U^c⊆W. Since, G∩W = ∅, thus G⊆W^c and W^c⊆U. Hence, $x \in G \subseteq \overline{G}^{rk} \subseteq \overline{W^c}^{rk} \subseteq W^c \subseteq U$.

Conversely: Let x in X and L be closed set in X such that x∉L then L^c open set in X and x∈L^c, thus, there is coc-r-open set G such that $x \in G \subseteq \overline{G}^{rk} \subseteq L^c$. Therefore, $x \in G \subseteq (\overline{G}^{rk})^c$ and $G, (\overline{G}^{rk})^c$ coc-r-open sets which have empty intersection. Hence, X coc-r-regular.

Remark 3.29: A space X is said to be coc'-r-regular space iff (X, τ^{rk}) is regular space.

Proposition 3.30: Let X be coc'-r-regular space, if G is coc-r-open then G ∈ (X, τ^{rk}).

Proof: Let G is coc-r-open in X, since, X is coc'-r-regular space, then for each x∈G there exists an coc-r-open set W_x such that:

$$x \in W_x \subseteq \overline{W_x}^{rk} \subseteq G$$

(By using Proposition (3.28)). Thus:

$$G = \cup \left\{ \overline{W_x}^{rk} : x \in G \right\}$$

So = G^{ork}. Hence, G ∈ RO (X,τ^{rk}):

$$\overline{G}^{rkork} = \overline{\left(\cup_{x\in G} \overline{W_x}^{rk} \right)}^{rk\,ork} = \left(\cup_{x\in G} \overline{W_x}^{rk} \right)^{ork}$$

Remark 3.31: If X is coc'-r-regular space, C is coc-r-closed then C∈RC (X, τ^{rk}).

Proof: It is clear.

Definition 3.32 (Gleason, 1958): A topological space (X, T) is said to besaid to bextremally disconnected (e.d) in case Gis open for each open set G in X.

Remarks 3.33 (Gleason, 1958):

- A space X is e.d iff for all G, W∈RO (X, τ) with G∩W = ∅, then, $\overline{G} \cap \overline{W} = \varnothing$
- If a topological space X is e.d, then every r-open, r-closed in X is open set

Theorem 3.34: If a topological space X is e.d space, then every coc-r-Lindelof is I-Lindelof.

Proof: Let {F_α: α∈∧} be r-closed cover of X, then F_α is closed for each α∈∧, thus, F_α is r-open for each α∈∧ (by using remarks (1.4), (2) and Remarks (1.15), (4)). Since, F_α is r-closed for each α∈∧ and X is e.d space, therefore ,F_α is open set in X for each α∈∧ (by using remarks (3.33), (2)), so, F_α is r-open, then, F_α is coc-r-open set in X for each α∈∧. Since, X is coc-r-Lindelof, thus, the cover {F_α: α∈∧} has a countable sub cover such that X = ∪ {$F_{\alpha i}$: i = 1, 2, ..., n}∪{$F_{\alpha i}^{o}$: i = 1, 2, ..., n}. Hence, X is I-Lindelof.

Theorem 3.35: If X is coc-r-e.d , coc'-r-regular space, then the following are equivalent:

- X is S-coc-r-Lindelof
- X is I-coc-r-Lindelof
- X is coc-r-Lindelof

Proof:

(i) $\longrightarrow$ (ii)

Clear by using Theorem (3.22).

(ii) $\longrightarrow$ (iii)

Let $\{U_\alpha : \alpha \in \Lambda\}$ be cover of X by coc-r-open subsets, since, X is coc'-r-regular space, then by using proposition (3.30) we get $U_\alpha \in RO(X, \tau^{rk})$ for each $\alpha \in \Lambda$. Thus, $\overline{U_\alpha}^{rk} \in RC(X, \tau^{rk})$ for each $\alpha \in \Lambda$, therefore, $\{\overline{U_\alpha}^{rk} \alpha \in \Lambda\}$ forms a cover of X, since, X is I-coc-r-Lindelof, therefore, $\{\overline{U_\alpha}^{rk} \alpha \in \Lambda\}$ has countable subcover such that $X = \cup_{n \in N} \overline{U}^{rk^{\alpha n}}$. Because $U_\alpha \in RO(X, \tau^{rk})$ for each $\alpha \in \Lambda$, so, $X = \cup_{n \in N} U_{\alpha n}$. Hence, X Is coc-r-Lindelof.

(iii) $\longrightarrow$ (i)

It is clear by using Theorem (3.19) and remark (3.23).

Remarks 3.36:

- If X is T_3-space (T_1 regular space), then, every coc-r-closed is r-closed
- If X is T_3-space, then X is e.d if and only if X is coc-r-e.d
- If X is T_3-space, then X is regular space if and only if X is coc'-r-regular space

Proof: Used $A \subseteq X$ be coc- r-closed, since, X is T_3-space, then, X is T_3-space, thus, A is closed subset in X, therefore, A is r-closed (X is regular space). From (ii), (iii) it is clear, since, X is T_3-space, then $\tau = \tau^{rk}$.

Theorem 3.37: If X is T_3, e.d space, thus, the next statements are equivalent:

- X is coc-r-Lindelof
- X is I-Lindelof
- X is Lindelof
- X is I-coc-r-Lindelof

Proof:

(i) $\longrightarrow$ (ii)

Since, X is e.d , thus, X is I-Lindelof (theorem (3.34))

(ii) $\longrightarrow$ (iii)

Let $\{U_\alpha : \alpha \in \Lambda\}$ be open covering of X. Because X is T_3-space, then X is regular space, thus, U_α is r-open in X (Proposition (1.20)) for every $\alpha \in \Lambda$.

Since, U_α is open in X, therefore, $\overline{U_\alpha}$ is r-closed (Remarks (1.15), (5)) for each $\alpha \in \Lambda$.

Then, $\{\overline{U_\alpha} : \alpha \in \Lambda\}$ forms a covering of X . Because X is I-Lindelof, therefore, $\{\overline{U_\alpha} : \alpha \in \Lambda\}$ has countable subcover such that $X = \cup_{n \in N} \overline{U_{\alpha n}} = \cup_{n \in N} U_{\alpha n}$, hence, X is Lindelof space.

(iii) $\longrightarrow$ (iv)

Let $\{F_\alpha : \alpha \in \Lambda\}$ be covering of X by coc-r-regular closed subsets, then, F_α is coc-r-closed of X, thus, F_α is r-closed for each $\alpha \in \Lambda$ (By using Remarks (3.36), (1)), thus, F_α is open for each $\alpha \in \Lambda$ (By using Remarks (3.33), (2)), therefore, $\{F_\alpha : \alpha \in \Lambda\}$ forms a cover of X, since, X is Lindelof, so, $\{F_\alpha : \alpha \in \Lambda\}$ has countable subcover such that $X = \cup_{n \in N} F_{\alpha n}$. Because X is T_3-space, then F_α coc-r-open in X for each $\alpha \in \Lambda$, thus, $X = \cup_{n \in N} F_{\alpha n}^{ork}$, hence, X is I-coc-r-Lindelof.

(iv) $\longrightarrow$ (i)

It is clear by using (Remarks (3.36), (2), (3)) and Theorem (3.35).

Theorem 3.38: Let g: X→Y be a co-r-continuous function, onto and (Y, τ_Y) be coc-r-e.d space, incase X is coc-r-Lindelof, thus, Y is I-coc-r-lindelof.

Proof: Let $\{F_\alpha : \alpha \in \Lambda\}$ be covering of Y by coc-r-regular closed subsets, since, Y is a coc-r-e.d, then, F_α is coc-r- open in Y for each $\alpha \in \Lambda$ (By using Proposition (3.15)) because g is coc-r-continuous function, thus, $g^{-1}(F_\alpha)$ is coc-r-open in X. Because $Y \subseteq \cup_{\alpha \in \Lambda} F_\alpha$ therefore, $X = g^{-1}(Y) \subseteq \cup_{\alpha \in \Lambda} g^{-1}(F_\alpha)$, so, $\{g^{-1}(F_\alpha) : \alpha \in \Lambda\}$ forms a covering of X. Since, X is coc-r-Lindelof, then, $\{g^{-1}(F_\alpha) : \alpha \in \Lambda\}$ has countable subcover such that $X \subseteq \cup_{n \in N} g^{-1}(F_{\alpha n})$, since, g onto, thus, $g(X) = Y \subseteq \cup_{n \in N} g(g^{-1} F_{\alpha n}) = \cup_{n \in N} F_{\alpha n} = \cup_{n \in N} F_{\alpha n}^{ork}$, hence, Y is I-coc-r-Lindelof.

Theorem 3.39: Let g: X→ Y be a coc-r-open function, bijective and (X, τ_X) be coc-r-e.d space, if Y is coc-r-Lindelof, thus, X is I-coc-r-Lindelof.

Proof: Let $\{F_\alpha : \alpha \in \Lambda\}$ be covering of X by coc-r-regular closed sets. Because X is coc-r-e.d, thus, F_α is coc-r-open in X for each $\alpha \in \Lambda$ (By using Proposition (3.18)) because g is a coc-r-open function, thus, $g\{F_\alpha\}$ is coc-r-open in Y but $X \subseteq \cup_{\alpha \in \Lambda} F_\alpha$, therefore, $Y = g(X) \subseteq \cup_{\alpha \in \Lambda} g(F_\alpha)$, so, $\{g(F_\alpha) : \alpha \in \Lambda\}$ forms a cover of Y, since, Y is coc-r-Lindelof, then, $\{g(F_\alpha) : \alpha \in \Lambda\}$ has a countable subcover such that $Y \subseteq \cup_{n \in N} g(F_{\alpha n})$, thus, $X = g^{-1}(Y) \subseteq \cup_{n \in N} g^{-1}(g F_{\alpha n}) = \cup_{n \in N} F_{\alpha n} = \cup_{n \in N} F_{\alpha n}^{ork}$, hence, X is I-coc-r-Lindelof.

Theorem 3.40: Let g: X→Y be a coc-r-continuous, coc-r-open function, onto and X coc'-r-regular space, if x is I-coc-r-Lindelof, thus, Y is also.

Proof: Let $\{F_\alpha : \alpha \in \Lambda\}$ be cover of Y by coc-r-regular closed subsets, then, $F_\alpha^c \in RO(X, \tau^{rk})$, thus, F_α^c coc-r-open in Y for each $\alpha \in \Lambda$, since, g is a coc-r-continuous function, therefore, $g^{-1}(F_\alpha^c)$ is coc-r-open in X for each $\alpha \in \Lambda$, so, $(g^{-1}(F_\alpha^c))^c$ is coc-r-closed in X for each $\alpha \in \Lambda$, since, X coc'-r-regular space, then $g^{-1}(F_\alpha^c) \in RC(X, \tau^{rk})$ (By using Remark (3.31)), since, $Y \subseteq \cup_{\alpha \in \Lambda} F_\alpha$, thus, $X = g^{-1}(Y) \subseteq \cup_{\alpha \in \Lambda} g^{-1}(F_\alpha)$, therefore, $\{g^{-1}(F_\alpha) : \alpha \in \Lambda\}$ forms a covering of X, since, X is I-coc-r-Lindelof space, then, $\{g^{-1}(F_\alpha): \alpha \in \wedge\}$ has countable subcover such that $X \subseteq \cup_{n\in N} (g^{-1}(F_{\alpha n}))^{ork}$.

Thus, we get $Y = f(x) \subseteq g((g^{-1}(F_{\alpha n}))^{ork}) \subseteq \cup_{\alpha \in \wedge} ((g^{-1}(F_{\alpha n})))^{ork}$, hence, Y is I-coc-r-Lindelof.

Definition 3.41: Let X→Y be a function of space X into space Y thus, gis said S-coc-r-β-closed function if for every y∈Y and for every $G \in \tau^{rk}$ with $g^{-1}(y) \subseteq G$, there exist β-open set V such that y∈V, $g^{-1}(V) \subseteq G$.

Definition 3.42: A topological space (X, T) is said coc-r-P-space if the countable union of coc-r-closed subsets is coc-r-closed.

Definitions 3.43 (Jankovic and Konstadilaki, 1996): A topological space (X, T) is said rc-Lindelof if each covering of X by regular closed subsets of the topological space (X, T) containing countable subcover.

Proposition 3.44 (Jankovic and Konstadilaki, 1996): A topological space (X, T) is said rc-Lindelof iff each covering i u of X by β-open subsets containing countable subcover v such that X = ∪ {U:U∈v}.

Proposition 3.45 (Al-Zoubi and Al-Nashef, 2003): Each I-Lindelof space is rc-Lindelof space.

Definition 3.46: A function g: (X, τ)→(Y, τ) is said super coc-r-open if g (U) is open in Y for each coc-r-open U in X

Theorem 3.47: Let g: X→Y be a function of space X into space Y if g super coc-r-open function then $g^{-1}(\overline{B}) \subseteq \overline{g^{-1}(B)}^{rk}$, for each B subset of V.

Proof: Let g: X→Y is super coc-r-open function, $x \in g^{-1}\overline{(B)}$ and U coc-r-open inX contain x, thus, $g(x) \in \overline{B} \cap g(U)$, since, g is super coc-r-open function, U coc-r-open inX, thus, g (U) is open set in Y, therefore, $g(U) \cap B \neq \varnothing$, so, $g^{-1}(B) \cap U \neq \varnothing$, then $x \in \overline{g^{-1}(B)}^{rk}$, hence, $g^{-1}(\overline{B}) \subseteq \overline{g^{-1}(B)}^{rk}$, for all B⊆Y.

Theorem 3.48: Let g: $(X, \tau_X) \to (Y, \tau_Y)$ be a s-coc-r-β-closed, super coc-r-open function with $g^{-1}(Y)$ s-coc-r-Lindelof for every y in Y and X coc-r-e.d, coc-r-P-space, incase Y is I-Lindelof, thus, X is I-coc-r-Lindelof.

Proof: We have to prove that X is s-coc-r-Lindelof. Used F a covering of X by coc-r-regular closed subsets, thus, for every y in Y, F forms a cover of $g^{-1}(Y)$, since, $g^{-1}(Y)$ s-coc-r-Lindelof, thus, we get a countable subcover F_Y of F such that $g^{-1}(Y) \subseteq \cup \{F: F \in F_Y\}$. Put

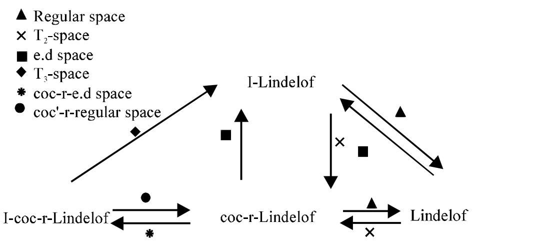

Fig. 1: The relationhip among the types of Lindelof spaces

$G_Y = \cup \{F: F \in F_Y\}$, therefore, G_Y is union of coc-r-regular closed subsets, since, $F \in RC(X, \tau^{rk})$ for every $F \in F_y$ and since, X coc-r-e.d, so, we get G_Y is coc-r-open (By using Proposition (3.15)), $g^{-1}(y) \subseteq G_y$ for every y in y in Y. Since, g is s-coc-r-β-closed, then, there is β-open V_y such that $y \in V_y$, $g^{-1}(V_y) \subseteq G_y$.

Then, the family $\{V_y: y \in Y\}$ is a covering of Y by β-open subsets, since, Y is I-Lindelof and By using Proposition (3.44), (3.45) then the cover $\{V_y: y \in Y\}$ contains a countable subcover such that $Y = \cup \{\overline{V_{yk}} : n \in N\}$. Let $L = \cup \{F_{yn}: n \in N\}$, it is clear that L is a countable family. Let x∈X and y = g(x), thus, $Y \in \overline{V_{yk}}$, k ∈ N, there fore, $x \in g^{-1}(\overline{V_{yk}})$, since, g super coc-r-open function and by using Theorem (3.47) we get $x \in g^{-1}(\overline{V_{yk}}) \subseteq \overline{g^{-1}(\overline{V_{yk}})}^{rk} \subseteq \overline{G_{yk}}^{rk}$ by using remarks ((2.15), (3)) and since, X coc-r-P- space, so, we get $x \in \overline{G_{yk}}^{rk} = G_{yk} = \cup \{F: F \in F_{yk}\} \subseteq \cup \{F: F \in L\}$, then, X is s-coc-r-Lindelof, hence, X is I-coc-r-Lindelof (By using Theorem (3.25)). (Fig. 2).

CONCLUSION

In this study, we study the concept of coc-r-open sets and investigate some of their fundamental properties. New type of spaces, that related to these concepts, such as coc-r-Lindelof and I-coc-r-Lindelof are introduced and studied. Also, new classes of functions using coc-r-open sets are introduced and studied. This study will open a new way for other researchers to study the applications of the concepts of coc-r-open sets.

REFERENCES

Al-Ghour, S. and S. Samarah, 2012. Cocompact open sets and continuity. Abstr. Appl. Anal., 2012: 1-9.

Al-Zoubi, K. and B. Al-Nashef, 2004. I-Lindelof spaces. Intl. J. Math. Sci., 2004: 2299-2305.

Bourbaki, N., 1989. Elements of Mathematics: General Topology. 1st Edn., Springer, Berlin, Germany, ISBN-13:9783540645634,.

Dugundji, J., 1978. Topology. Allyn & Bacon, Boston, Massachusetts, USA.,.

Gleason, A.M., 1958. Projective topological spaces. Illinois J. Math., 2: 482-489.

Jankovic, D. and C. Konstadilaki, 1996. On covering properties by regular closed sets. Math. Pannonica, 7: 97-111.

Jasim, F.H., 2014. On compactness via cocompact open sets. MSc Thesis, University of Al-Qadisiyah, Al Diwaniyah, Iraq.

Kumar, M.C.V. and S. Chowdhary, 2011. On topological sets and spaces. Global J. Sci. Front. Res., 11: 29-34.

Lipschutz, S., 1995. Theory and Problem of General Topology. Schaum Publications, Inc., Mequon, USA.,.

Radhy, F.K., 2010. On regular proper mapping. MSc Thesis, University of Kufa, Kufa, Iraq.

Stone, M.H., 1937. Applications of the theory of Boolean rings to general topology. Trans. Am. Math. Soc., 41: 375-481.

Willard, S., 1970. General Topology. Addison-Wesley, Boston, Massachusetts, USA.,.