Journal of Engineering and Applied Sciences 13 (21): 9106-9110, 2018
ISSN: 1816-949X

Electromagnetic Problems Solving by MoM Method

[1]A. Faize, [2]G. Alsharahi, [2]A. Driouach and [1]M. Atounti
[1]Department of Physic, Faculty of Multidisciplinary, University Mohammed I, BP. 300, 62700 Nador, Morocco
[2]Department of Physic, Faculty of Sciences, Abdelmalek Essaadi University, 93000 Tetouan, Morocco

Abstract: This research is devoted to the presentation of the method of moments: brief overview of other numerical methods, theoretical foundation formulation of integral equations, development of the matrix method and examples. In this study, we will discuss the detailed study of the Method of Moments (MoM) while making a brief description of the theoretical basis of some other numerical methods used in solving electromagnetic problems. For this, we have written a code in Fortran 90 based on the MoM numerical method that we have applied to some electromagnetic problems and also, a model which is interesting in electronic, the charge density of conductive wire in this study.

Key words: MoM method, Fortran code, charge density, straight yam, numerical methods, conductive wire

INTRODUCTION

The method of moments, developed for the first time in electromagnetic problems by Newman (1988), Gibson (2008, 2014) and De Doncker (2003). It solves the integral equation by transforming it into a linear equation system (Miano *et al*., 1996). To use this method, we must break down the structure studied in several parts or cells. The numerical resolution of Maxwell's equations of the study structure, allows to write the electric fields or magnetic a function of a sum of induced currents. Calculating the current distribution (Zhou *et al*., 2009) allows to obtain the parameters $[l_{mn}]$. In the method of moments, we have the linear equation to solve: $[l_{mn}][\alpha_n] = [g_m]$ (Harrington, 1993; El-Misilmani *et al*., 2015). The parameters $[l_{mn}]$ are calculated from integral equations. The structure is excited with the source $[g_m]$. And consequently the vector $[\alpha_n]$ will be calculated.

Application of MoM to calculate the cross section by cylinder: Either g(x) a well-defined function on its domain, one seeks f(x) in the range $0 \le x \le 1$ which check:

$$\frac{d^2f}{dx^2} = g(x) \quad (1)$$

$$f(0) = f(1) = 0 \quad (2)$$

For solving $l(f) = g$ or $g(x) = 1\text{-}4x^3$ and considered a source function and the operator $l = d^2/dx^2$ and the function f to be determined. By simply calculates we find f(x) using the initial conditions:

$$f(x) = \frac{-3}{10}x + \frac{1}{2}x^2 - \frac{1}{5}x^5 \quad (3)$$

To illustrate the procedure, the solution will be resolved by the method of moments. We choose the series of power f:

$$f_n = 1\text{-}x^{n+1} \quad (4)$$

n = 1, 2, 3, ..., N the series of power f will be:

$$f = \sum_{n=1}^{N} \alpha_n (1\text{-}x^{n+1}) \quad (5)$$

For a better solution, the probationary function w_n must be in the area of deputy operator. The evaluation of the coefficients of the matrices given by:

$$l_{mn} = \prec w_n, lf_n \succ = \prec 1\text{-}x^{n+1}, \frac{d^2}{dx}(1\text{-}x^{n+1}) \succ \quad (6)$$

$$\frac{d^2}{dx^2}(1\text{-}x^{n+1}) = -n(n+1)x^{n-1} \quad (7)$$

Corresponding Author: A. Faize, Department of Physic, Faculty of Multidisciplinary, University Mohammed I, BP. 300, 62700 Nador, Morocco

$$l_{mn} = -\int_0^1 (1-x^{m+1})n(n+1)x^{n-1}dx \quad (8)$$

Or:

$$C = \int_0^1 (1-x^{m+1})x^{n-1}dx$$
$$C = \left[\frac{x^n}{n} - \frac{x^{m+n+1}}{m+n+1}\right]_0^1$$
$$C = \left[\frac{1}{n} - \frac{1}{(m+n+1)}\right] \quad (9)$$
$$l_{mn} = -n(n+1)\left[\frac{1}{n} - \frac{1}{(m+n+1)}\right] = \frac{(m+)(n+1)}{m+n+1}$$

We find, after some calculation steps:

$$l_{mn} = \prec w_n, lf_n \succ = \frac{-(m+1)(n+1)}{m+n+1} \quad (10)$$

By involving the weight function w_n, we calculate the elements g_n:

$$g_m = \prec w_n, g \succ = -\int_0^1 (1-x^{m+1})(1-4x^3)dx \quad (11)$$

$$g_m = \int_0^1 (1-4x^3)dx - \int_0^1 (x^{m+1}-4x^{m+3}dx) \quad (12)$$
$$g_m = A+B$$

$$A = -\int_0^1 (1-4x^3)dx = [x-x^4]_0^1 = 0 \quad (13)$$

$$B = \int_0^1 (x^{m+1}-4x^{m+3}dx = \frac{-(m+4)+4(m+2)}{(m+2)(m+4)} = \frac{3m+12}{(m+2)(m+4)} \quad (14)$$

Finally, we find:

$$g_m \prec w_n, g \succ = \frac{3m+12}{(m+2)(m+4)} \quad (15)$$

For any fixed n (the number of expansion function is given), we calculate α_n by the above Eq. 15 and the approximation of the function f by Eq. 5 for finding the convergence, one makes a successive approximation by giving different values for n. For n = 1, we have: $l_{11} = -4/3$, $g_1 = 1$, $\alpha_1 = -3/4$. For n = 2, we have:

$$\begin{bmatrix} \frac{-4}{3} & \frac{-3}{2} \\ \frac{-3}{2} & \frac{-9}{5} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{3}{4} \end{bmatrix} \quad (16)$$

the coefficients α_n:

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} -9075 \\ 15 \end{bmatrix} \quad (17)$$

For n = 3, we have:

$$\begin{bmatrix} \frac{-4}{3} & \frac{-3}{2} & \frac{-8}{5} \\ \frac{-3}{2} & \frac{-9}{5} & -2 \\ \frac{-3}{5} & -2 & \frac{-16}{7} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{3}{4} \\ \frac{3}{5} \end{bmatrix} \quad (18)$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} \frac{-4}{3} & \frac{-3}{2} & \frac{-8}{5} \\ \frac{-3}{2} & \frac{-9}{5} & -2 \\ \frac{-8}{5} & -2 & \frac{-16}{7} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ \frac{3}{4} \\ \frac{3}{5} \end{bmatrix} \quad (19)$$

the coefficients α_n:

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} -9.75 \\ 15 \\ -6.56 \end{bmatrix} \quad (20)$$

$$f = \sum_{n=1}^{N} \alpha_n f_n = \alpha(1-x^2) + \alpha(1-x^3) + \alpha(1-x^4) \quad (21)$$

$$f = -9.75(1-x^2) + 15(1-x^3) - 6.56(1-x^4) \quad (22)$$

These results are calculated by via. a numerical code we realized by using MATLAB:

$$l = \begin{bmatrix} -1.333333 & -1.5 & -1.6 \\ -1.5 & -1.8 & -2 \\ -1.6 & -2 & -2028571 \end{bmatrix} \quad (23)$$

$$g = \begin{bmatrix} 1 \\ 7.5e\text{-}001 \\ 6.0e\text{-}001 \end{bmatrix} \quad (24)$$

$$\alpha = \begin{bmatrix} -9.750000000000014e{+}000 \\ 1.500000000000004e{+}001 \\ -6.562500000000021e{+}000 \end{bmatrix} \quad (25)$$

We notice from Fig. 1 and 2a, b that more n is large, more the results obtained by method of moments approach the results of the analytical solution.

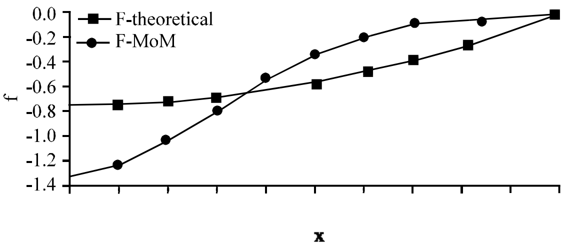

Fig. 1: Curves representative of the results obtained by the method of moments and those analytical for n = 1

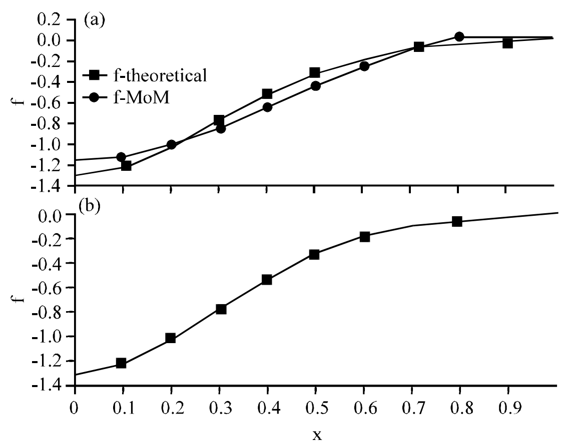

Fig. 2: Curves representative of the results obtained by the method of moments and those analytical for n = 2 and n = 3

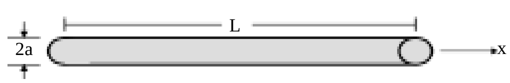

Fig. 3: Thin wire brought to the potential V

MATERIALS AND METHODS

Application of the method of moments to calculate the charge density of conductive wire brought to the potential V: One considers a thin thread of length L and radius a oriented along the axis OX as illustrated in Fig. 3. If the radius of the wire is very small compared to the length, the electric potential on the wire can be expressed by the integral (De Doncker, 2003):

$$\varnothing_e(r) = \int_0^L \frac{q_e(x')}{4\pi \in |r-r'|} dx' \tag{26}$$

$$|r-r'| = \sqrt{(x-x')^2+(y-y')^2} \tag{27}$$

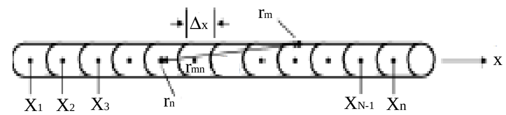

Fig. 4: Segmentation of thin wire

With the intention to convert Eq. 26 in a system of linear equations, we cut out the wire into N sub-segments, each of length Δ_x as shown in Fig. 4.

Within each sub-segment, we assume that the charge density of has a constant value, so that, $q_e(x')$ is constant piecewise along the length of the wire.
Mathematically, we write:

$$q_e(x') = \sum_{n=1}^{N} a_n f_n(x') \tag{28}$$

where, a_n are unknown weighting coefficients and $f_n(x')$ a set of pulse functions which are constant over a segment but no on all other segments, they are defined by:

$$f_n(x') = \begin{cases} 0 & x<(n-1)\Delta_x \\ 1 & (n-1)\Delta_x \le x' \le n\Delta_x \\ 0 & x'>n\Delta_x \end{cases} \tag{29}$$

were fixed to the potential V 1 and replacing Eq. 29 into Eq. 26:

$$1 = \int_0^L \sum_{n=1}^{N} a_n f_n(x') \frac{1}{4\pi \in |r-r'|} dx' \tag{30}$$

Using the above definition, of the pulse function, we can rewrite:

$$1 = \frac{1}{4\pi \in} \sum_{n=1}^{N} a_n \int_{(n-1)\Delta_x}^{n\Delta_x} \frac{1}{|r-r'|} dx' \tag{31}$$

where, we have now a sum of integrals, on each the domain of a single pulse function. Now, let us fix the source points on the wire axis and observation points onto the surface of the canvas. This choice ensures that there is no singularity in the integral.

The denominator of the integral becomes now:

$$|r-r'| = \sqrt{(x-x')^2+a^2} \tag{32}$$

And Eq. 31 can be written:

$$4\pi\in = a_1\int_0^{\Delta_x}\frac{1}{\sqrt{(x-x')^2+a^2}}dx' + a_2\int_{\Delta_x}^{2\Delta_x}\frac{1}{\sqrt{(x-x')^2+a^2}}dx' + ... + a_{N-1}\int_{(N-2)\Delta_x}^{(N-1)\Delta_x}\frac{1}{\sqrt{(x-x')^2+a^2}}dx' + a_N\int_{(N-1)\Delta_x}^{N\Delta_x}\frac{1}{\sqrt{(x-x')^2+a^2}}dx' \quad (33)$$

Which includes an equation in N unknowns. We can solve this equation by ordinary routines of the matrix algebra if we can get N equations with N unknowns. We choose N independent observation points x_m on the wire surface:

$$4\pi\in = a_1\int_0^{\Delta_x}\frac{1}{\sqrt{(x_1-x')^2+a^2}}dx' +, ..., +a_N\int_{(N-1)}^{N\Delta_x}\frac{1}{\sqrt{(x_1-x')^2+a^2}}dx'$$

$$\vdots$$

$$4\pi\in = a_1\int_0^{\Delta_x}\frac{1}{\sqrt{(x_N-x')^2+a^2}}dx' +, ..., +a_N\int_{(N-1)\Delta x}^{\Delta_x}\frac{1}{\sqrt{(x_N-x')^2+a^2}}dx' \quad (34)$$

Which includes the matrix system [Z][a] = [b]

$$\begin{bmatrix} Z_{11}Z_{12}Z_{13}, \cdots, \cdots, Z_{1N} \\ Z_{21}Z_{22}Z_{23}, \cdots, \cdots, Z_{2N} \\ \vdots \\ \vdots \\ Z_{N1}Z_{N2}Z_{N3}\cdots\cdots Z_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ \vdots \\ b_N \end{bmatrix} \quad (35)$$

Individual matrix elements z_{mn} are:

$$Z_{mn} = \int_{(n-1)\Delta_x}^{n\Delta_x}\frac{1}{\sqrt{(x-x')^2+a^2}}dx' \quad (36)$$

and the right lateral vector elements b_m are:

$$b_m = 4\pi\in \quad (37)$$

RESULTS AND DISCUSSION

Evaluation of matrix elements: The integral of the elements of the matrix [Z] for this problem can be assessed in closed form. The execution of this integration gives:

$$Z_{mn} = \log\left[\frac{(x_b-x_m)+\sqrt{(x_b-x_m)^2+a^2}}{(x_a-x_m)+\sqrt{(x_a-x_m)^2+a^2}}\right] \quad (38)$$

where, $x_b = n\Delta_x$ and $x_a = (n-1)\Delta_x$. This gives a symmetric matrix called Toeplitz, namely:

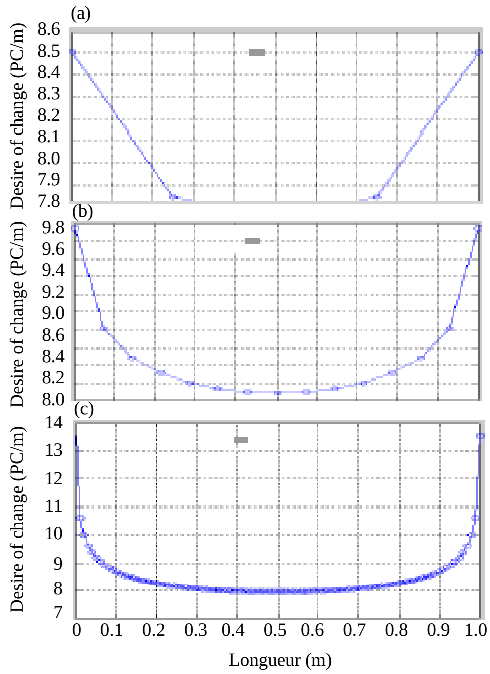

Fig. 5: Load distribution straight yarn: a) Methode of momnet N = 6; b) Methode of momnet N = 15 and c) Methode of momnet N = 100

$$[Z] = \begin{bmatrix} Z_1 & Z_2 & Z_3 & \cdots\cdots & Z_N \\ Z_2 & Z_1 & Z_2 & \cdots\cdots & Z_{N-1} \\ Z_3 & Z_2 & Z_1 & \cdots\cdot & Z_{N-2} \\ \vdots & & & & \vdots \\ & & \ddots & & \\ & & \vdots & & \\ & & \vdots & & \\ Z_N & Z_{N-1} & Z_{N-2} & \cdots\cdots & Z_1 \end{bmatrix} \quad (39)$$

In Fig. 5a-c, we show the charge density calculated on the wire by using progressively 5, 15 and 100 segments. We notice that switching to 100 unknown considerably improves the results.

CONCLUSION

This study has enabled us to learn about the numerical simulation of physical problems who is here, calculating the cross section by cylinder homogeneous and dielectric and undefined, the realization of computer programs based on the numerical method known: Method of Moments (MoM).

REFERENCES

De Doncker, P., 2003. A volume/surface potential formulation of the method of moments applied to electromagnetic scattering. Eng. Anal. Boundary Elem., 27: 325-331.

El-Misilmani, H.M., K.Y. Kabalan, M.Y. Abou-Shahine and M. Al-Husseini, 2015. A method of moment approach in solving boundary value problems. J. Electromagn. Anal. Appl., 7: 61-65.

Gibson, W.C., 2008. The Method of Moments in Electromagnetics. Taylor & Francis, Abingdon, England, UK., ISBN:9781420061451, Pages: 288.

Gibson, W.C., 2014. The Method of Moments in Electromagnetics. 2nd Edn., CRC Press, Boca Raton, Florida, USA., ISBN-13:978-1-4822-3579-1, Pages: 426.

Harrington, R.F., 1993. Field Computation by Moment Methods. Oxford University Press, Oxford, England, UK., ISBN:9780198592174, Pages: 240.

Miano, G., L. Verolino and V.G. Vaccaro, 1996. A new numerical treatment for Pocklington's integral equation. IEEE. Trans. Magn., 32: 918-921.

Newman, E.H., 1988. Generation of wide-band data from the method of moments by interpolating the impedance matrix (EM problems). IEEE. Trans. Antennas Propag., 36: 1820-1824.

Zhou, Y., P. Yan, W.Q. Yuan, J. Wang and M.T. Li, 2009. Current distribution and inductance gradient calculation at different rail geometric parameters. Proceedings of the 2009 IEEE International Conference on Pulsed Power (PPC'09), June 28-July 2, 2009, IEEE, Washington, DC, USA., ISBN:978-1-4244-4064-1, pp: 1290-1293.