

## Optimization of Heating, Ventilating and Air Conditioning (HVAC) Systems: A Review

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**Abstract:** Heating, Ventilating and Air Conditioning (HVAC) systems have become a very essential part in our daily lives in providing comfortable and satisfactory indoor conditions. As a consequence, large amount of energy are consumed every year in HVAC application to produce a desirable thermal environment. In light of the increasing energy consumption, energy efficiency improvement has become the ultimate goal of many authorities. Recent advances and revolutionary improvements in computing systems make optimization approaches one of the promising techniques in achieving greatest energy efficiency in HVAC application. This study gives an overview of research conducted on optimization techniques used in HVAC systems for energy and comfort purposes. Current literature are summarized with highlights given to the optimization algorithms used, control scheme, objective function parameters, HVAC simulation tools, optimization programs and HVAC simulation model approaches. Trends in HVAC optimization, performance and efficiency analysis of mostly used algorithms as well as factors affecting selection of algorithms will be mainly discussed in this study.

**Key words:** Energy, thermal comfort, optimization, multi-objective, algorithm

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### INTRODUCTION

**Energy:** Malaysia being a tropical country with hot and humid weather throughout the year, Heating, ventilating and Airconditioning (HVAC) systems are indispensable in providing comfortable living condition in factories, offices and residential areas. According to audit study of Malaysia Green Technology Corporation's, energy consumption of air conditioners accounted for more than 60% of total building energy in Malaysia. Therefore, reducing energy consumption of HVAC systems is very crucial for conservation of energy. This can also lower the energy costs for consumers and businesses, thus, allows energy sustainability. For this reason, efficient control of HVAC systems plays a vital role in achieving the energy saving goals.

**Thermal comfort:** According to a survey carried out by the National Human Activity Pattern Survey (NHAPS), human spend 87% of their time indoors. Thus, maintaining comfortable indoor conditions are important for general well-beings and health of individuals. Thermal comfort is the key factor to indoor comfort condition.

Thermal comfort is the condition of mind that expresses satisfaction with the thermal environment and is assessed by subjective evaluation. It can be measured using Predicted Mean Vote (PMV) index which is calculated using Fanger's equation (ANSI/ASHRAE,

2013). PMV depends on environmental parameters such as air temperature, humidity, mean radiant temperature, air velocity and personal variables such as metabolic rate and clothing insulation. It is associated with thermal sensation scale that runs from cold (-3) to hot (+3) and is adopted as an ISO standard. The recommended acceptable PMV range for thermal comfort from ASHRAE Standard 55 is between -0.5 to 0.5 for indoor spaces (ANSI/ASHRAE, 2013). Often, energy conservation strategies and thermal comfort requirements can conflict with one and another. More energy saving usually results in less comfort while higher comfort level typically results in lower energy saving. Thus, optimal solutions that find the best comprise solution between energy saving and thermal comfort have become the interest of many researchers and engineers.

**Optimization:** In recent years, application of mathematical optimization have seen a lot of advancement in solving various complex engineering problems, including Heating, Ventilating and Air Conditioning (HVAC) systems. Researchers and engineers continuously seek for the best possible solutions in HVAC operation with the most cost-effective energy resources allocation, using different optimization methods. Research on HVAC systems optimization can be traced as early as 1996 by Zheng and Zaheer-Uddin (1996) and 1997 by Huang and Lam (1997) using Sequential Quadratic Programming (SQP) and

Genetic Algorithm (GA), respectively for HVAC Proportional Integral (PI) control, though occupant thermal comfort was not the criteria in their optimization. Since then, there have been increasing trend of studies focusing on optimization of HVAC systems, most with the aims of maintaining the indoor thermal comfort of occupants and indoor air quality but with least energy expenditure possible.

In mathematical aspect, optimization is the selection of a best element from some set of available alternatives with regard to certain objective or objectives (Chiandussi *et al.*, 2012). In HVAC optimization, optimal control search for the best possible operating points of HVAC systems under several dynamic conditions such as outdoor weather and indoor loads with the objectives of maximizing the occupants thermal comfort level and minimizing the energy costs of the system. Generally, HVAC system is the integration of many complicated parts such as heat exchanger, blower, condenser coil, evaporator coil, thermostat, etc. Some researchers examined and optimized single part of a HVAC systems (Lee *et al.*, 2011). While many focused on supervisory control, whereby each individual part of the HVAC

system is being monitored and optimized so that the overall HVAC system is under optimal control conditions (Wright *et al.*, 2002; Nassif *et al.*, 2004; Liang and Du, 2005; Sun and Reddy, 2005; Mossolly *et al.*, 2009).

To date, many studies regarding optimization control of HVAC systems have been carried out and published. This study aims at making a review on the state-of-art of HVAC performance and efficiency analysis using mostly-used optimization techniques.

## CURRENT LITERATURE

A preliminary search was carried out using search engine, Google Scholar and Elsevier's Scopus database with keywords such as "HVAC", "air-conditioning", "optimization", "multi-objective optimization" to examine the trend of optimization algorithms used for HVAC application. The result was derived from more than forty HVAC optimization studies from year 1997-2017 and the articles are summarized in Table 1. List of literature are presented in chronological order with highlights given to optimization algorithms used, control schemes, objective function parameters and simulation tools used.

Table 1: Summary literature of optimization of HVAC systems

Objective functions										
Researchers, Years	Optimization algorithms	Control schemes	Energy usage	Thermal comfort	IAQ	Humidity	Energy cost	Others	HVAC Simulation	Optimization programs
Zheng and Zaheer-Uddin (1996)	Sequential Quadratic Programming (SQP)	Proportional-Integral (PI) control	✓	-	-	-	-	-	*	*
Huang and Lam (1997)	Genetic Algorithm (GA)	Proportional-Integral (PI) control	-	-	-	-	-	-	HVACSIM+	*
Sosa <i>et al.</i> (1997)	Branch-and bound algorithm	Fuzzy predictive control	-	-	-	-	-	Sum of squared error of zone temperature and reference temperature	*	*
Wang and Jin (2000)	Genetic Algorithm (GA)	Proportional-Integral-Derivative (PID) control	✓	✓	✓	✓	-	-	TRNSYS	*
Wright <i>et al.</i> (2002)	Multi-Objective Genetic Algorithm (MOGA)	*	-	✓	-	-	✓	-	*	*
Alcala <i>et al.</i> (2003)	Weighted Multi-Criteria Steady-State Genetic Algorithm (WMC-SSGA)	Fuzzy control	✓	✓	-	-	-	-	*	*
Nassif <i>et al.</i> (2004)	Non-dominated Sorting Genetic Algorithm II (NSGA-II)	Supervisory control	✓	✓	-	-	-	-	*	*
Liang and Du (2005)	Gradient method	Direct Neural Network (Direct NN)	✓	✓	-	-	-	-	*	*
Sun and Reddy (2005)	Sequential Quadratic Programming (SQP)	*	✓	-	-	-	-	-	*	*
Lu <i>et al.</i> (2005a, b)	Modified genetic algorithm	Adaptive neuro-fuzzy inference system (ANFIS)	✓	-	-	-	-	-	*	*
Fong <i>et al.</i>	Evolutionary	*	✓	-	-	-	-	-	TRNSYS	*

Table 1: Continue

Researchers, Years			Objective functions							HVAC Simulation	Optimization programs
			Energy usage	Thermal comfort	IAQ	Humidity	Energy cost	Others			
(2006)	Programming (EP)										
Freire <i>et al.</i> (2008)	Sequential Quadratic Programming (SQP)	Model Predictive Control (MPC)	✓	✓	-	-	-	-	PowerDomus	*	
Huh and Brandemuehl (2008)	Complex search	*	✓	✓	-	✓	-	-	*	*	
Nassif <i>et al.</i> (2008)	Genetic Algorithm (GA)	Proportional-Integral (PI) control	-	-	-	-	-	Least square error between estimated data and real data	HVACSIM+	*	
Xu and Wang, (2009)	Genetic Algorithm (GA)	PID for individual component controller; supervisory for integrated system	✓	✓	✓	-	-	-	TRNSYS	*	
Mossolli <i>et al.</i> (2009)	Genetic Algorithm (GA)	*	✓	✓	✓	-	-	-	Visual DOE	MATLAB	
Fong <i>et al.</i> (2009)	Evolution Strategy (ES)	*	✓	-	-	-	-	-	TRNSYS	*	
Congradac and Kulic (2009)	Genetic algorithm (GA)	*	✓	-	-	-	-	-	Energy Plus	MATLAB	
Andrew <i>et al.</i> (2010)	Particle Swarm Optimization (PSO)	*	✓	-	-	-	-	-	*	*	
Magnier and Haghighat (2010)	Non-dominated Sorting Genetic Algorithm II (NSGA-II)	Artificial Neural Network (ANN)	✓	✓	-	-	-	-	TRNSYS	GenOpt	
Kusiak and Li (2010)	Evolution Strategy (ES)	*	-	-	-	-	-	Cooling output power	*	*	
Lee <i>et al.</i> (2011)	Differential Evolution (DE)	*	✓	-	-	-	-	-	*	Visual Basic (VB)	
Kusiak <i>et al.</i> (2011)	Strength-Multi Objective Particle Swarm Optimization (S-MOPSO)	*	✓	-	-	-	-	-	*	*	
Kusiak <i>et al.</i> (2011)	Strength Pareto Evolutionary Algorithm with Local Search (SPEA-LS)	*	✓	-	-	-	-	-	*	*	
Beghi <i>et al.</i> (2011)	Multi-Phase Genetic Algorithm (MPGA)	Proportional-Integral-Derivative (PID) control	✓	-	-	-	-	Partial Loading Ratio (PLR)	MATLAB	MATLAB Simulink	
Kelman and Borrelli (2011)	Sequential Quadratic Programming (SQP)	Model Predictive Control (MPC)	✓	✓	-	-	✓	-	*	BPMPD Solver by C-programming	
Hamdy <i>et al.</i> (2011)	Genetic Algorithm (GA)	*	✓	-	-	-	-	-	IDA-ICE 4.0	MATLAB	
Gacto <i>et al.</i> (2012)	Exploration-Exploitation Strength Pareto Evolutionary Algorithm (LS-SPEA2 E/E)	Fuzzy control	✓	✓	✓	-	-	System stability, number of rules of fuzzy logic control	*	*	
Yang and Wang (2012)	Multi Objective Particle Swarm Optimization (MOPSO)	*	✓	-	-	-	-	Difference of indoor temperature with set temperature	*	*	
Beghi <i>et al.</i> (2012)	Particle Swarm Optimization (PSO)	Supervisory control	✓	-	-	-	-	Partial loading	Design Builder Ratio (PLR)	MATLAB	
Ferreira <i>et al.</i> (2012)	Branch-and bound algorithm	Model Predictive Control (MPC)	✓	✓	-	-	-	-	*	*	
Coelho and	Improved Firefly	*	✓	-	-	-	-	Partial	*	MATLAB	

Table 1: Continue

Researchers, Years	Optimization algorithms	Control schemes	Objective functions						HVAC Simulation	Optimization programs
			Energy usage	Thermal comfort	IAQ	Humidity	Energy cost	Others		
Mariani (2013)	Algorithm (IFA)							Loading Ratio (PLR)		
Alvarez <i>et al.</i> (2013)	Lagrangian dual method	Model Predictive Control (MPC)	✓	-	-	-	-	-	*	*
He <i>et al.</i> (2014)	SPEA, PSO, Harmony search	*	✓	-	-	-	-	Room temperature ramp rate	*	*
Hussain <i>et al.</i> (2014)	Genetic Algorithm (GA)	Fuzzy logic	✓	✓	-	-	-	-	EnergyPlus	MATLAB Simulink
Seo <i>et al.</i> (2014)	Multi-Island Genetic Algorithm (MIGA)	*	✓	-	-	-	-	-	TRNSYS	*
Rackes and Warning (2014)	Hooke and Jeeve's algorithm	*	-	-	✓	-	✓	-	EnergyPlus	GenOpt
West <i>et al.</i> (2014)	Sequential Quadratic Programming (SQP)	Supervisory model predictive control	-	✓	-	-	✓	Greenhouse gas emission	*	*
Zeng <i>et al.</i> (2015)	Firefly Algorithm (FA)	Neural Network (NN)	✓	-	-	-	-	-	*	*
Garnier <i>et al.</i> (2015)	Genetic Algorithm (GA)	Artificial Neural Network (ANN)	✓	✓	-	-	-	-	EnergyPlus	MATLAB
Coelho and Askarzadeh (2016)	Differential Bat Algorithm (DBA)	*	✓	-	-	-	-	Partial Loading Ratio (PLR)	*	MATLAB
Shaikh <i>et al.</i> (2016)	Multi-Objective Genetic Algorithm (MOGA)	Multi-agent control	-	✓	✓	✓	✓	-	MATLAB	MATLAB
Ascione <i>et al.</i> (2016)	Genetic Algorithm (GA)	Model Predictive Control (MPC)	-	✓	-	-	✓	-	EnergyPlus	MATLAB
Chien and Li (2016)	Mixed Integer Linear Programming (MILP)	*	✓	✓	-	-	-	-	*	CPLEX
Risbeck <i>et al.</i> (2017)	Mixed Integer Linear Programming (MILP)	*	-	-	-	-	✓	-	*	Gurobi6

Table 2: Classification of optimization algorithm

Category	Examples of algorithms	Characteristics
Direct search	Hooke-and-Jeeves, simplex search	Derivatives free Heuristics For discontinuous function
Gradient based	Sequential Quadratic Programming (SQP) Newton method Gradient-descent method Lagrange method Levenberg-Marquardt algorithm	Derivatives are required For continuous function Rapid convergence in finding local optima Performance depends on the initial values supplied Easily stuck at a single local optima
Integer programming	Branch and bound, Simulated annealing Tabu search	For discrete search space
Single solution-based metaheuristic	Simulated annealing, Tabu search	Search for single solution at a time Stochastics For discrete search space
Population-based metaheuristic	Evolution algorithms family: Genetic Algorithm (GA), Evolution Strategies (ES), Evolution Programming (EP), Differential Evolution (DE) Swarm intelligence family: Particle Swarm intelligence (PSO) ant colony algorithm, bat algorithm	Search for multiple solution at a time Stochastic Large search space thus capable to escape from local optima Global optimal solution is not guaranteed Good for complex non-linear problem

**Classification of optimization algorithms:** There are a lot of mathematical optimization algorithms developed to solve complex engineering problems. Knowing the characteristics and properties of each is important so that a proper selection of algorithms can be achieved. Optimization algorithms can be classified in many ways such as linear or non-

linear, deterministic or stochastics, global or local, single-solution based or population-based, single-objective or multi-objective, etc. Since HVAC systems is highly associated with nonlinearity, only mostly-used nonlinear optimization algorithms in HVAC studies are mainly addressed in this study (Table 2).

## OPTIMIZATION FORMULATION

Optimization of HVAC system can be very well formulated into optimization problems. The steps involved in formulation of HVAC optimization are important to ensure a successful optimal search. These phases include setting of HVAC and building model, objective functions formation, design variables and design constraints determination, coupling between simulation tools and optimization programs and most importantly nature of various optimization algorithms and their efficiencies. These will be discussed in the subsequent sections.

**HVAC and building model:** HVAC system model involves modelling of various HVAC equipment and building with its indoor thermal conditions. Generally, HVAC system is a complex and non-linear system that involves numerous parameters. Up to date, there are many HVAC system modelling approaches that specifically used for energy saving analysis with the latest approaches even incorporated thermal comfort attainment.

The most popular HVAC simulations use physical model or mathematical model (Nassif *et al* 2004; Lu *et al.* 2005; Sun and Reddy, 2005; Fong *et al.*, 2006). It is constructed based on physical and chemical laws of conservation such as component, mass, momentum and energy balance (Raad, 2013). These laws link the input and output with large number of mathematical equations described by fundamental engineering principles. Numerous HVAC parameters, building thermal behaviours, weather data and internal load information are needed to perform simulation and this results in a complicated overall system model. To reduce the complexity of the system, some researchers break down the HVAC Model to several sub-models/sub-systems based on first principles. Besides that, level of details of both building and HVAC equipment models can vary from simple to complex (Trcka and Hensen, 2010). For analysis concerning energy savings, detailed physical model and simulation are usually not necessary as energy consumption can be estimated using simpler modelling approaches. To date, quite a number of mature white box software tools have been widely utilized for energy consumption analysis which include EnergyPlus and TRNSYS.

Another HVAC Model approach is the black-box model or empirical model which is sometimes preferred as it is easier to be constructed (Kusiak *et al.*, 2010, 2011; He *et al.*, 2014). Black box model does not use any physical or mathematical structure of HVAC Model. A system is viewed in terms of its input data and output

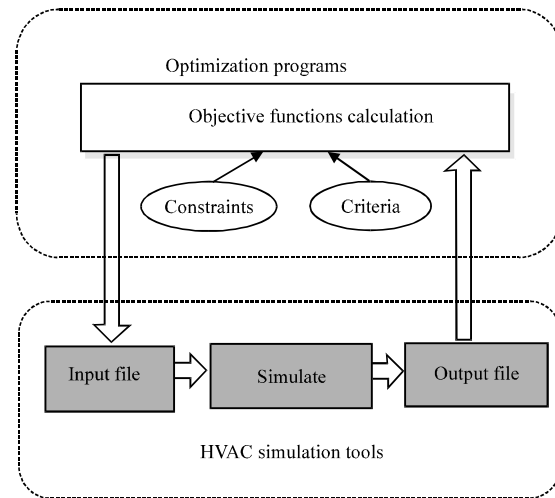


Fig. 1: Coupling between HVAC simulation tools and optimization programs

data without the knowledge of internal structure. The HVAC Model or sub-models are estimated and evaluated using validated input and output data of the system. This needs on-site measurement of energy consumption and HVAC operation data under different conditions for certain period of time. One of the popular black box modelling methods is Neural Network (NN), whereby it is used to predict the energy model of HVAC system. It is found that NN outperforms other black box modelling methods such as autoregressive exogenous (ARX), Autoregressive Moving Average exogenous (ARMAX), Transfer Function (TF) and Box-Jenkins (BJ) (Afram and Janabi-Sharifi, 2015) (Fig. 1).

In the case where mathematical models are combined with black-box models, gray-box models or hybrid models are resulted. In gray-box approach, the model structures are derived using simplified physical models while their parameters are obtained from catalogs or from operation data using some parameters identification methods such as non-linear regression method. Overall by using gray-box approach, the complexity of the model structures as well as computational time to achieve optimality can be significantly reduced. However, the accuracies of these models still greatly rely on precision and richness of data used to train the models (Wang and Ma, 2008).

### HVAC simulation tools and optimization programs:

Table 1 shows the HVAC system simulation tools and optimization programs used by some researchers in their literature. To date, there are more than hundreds of HVAC simulation tools available in the market. However,

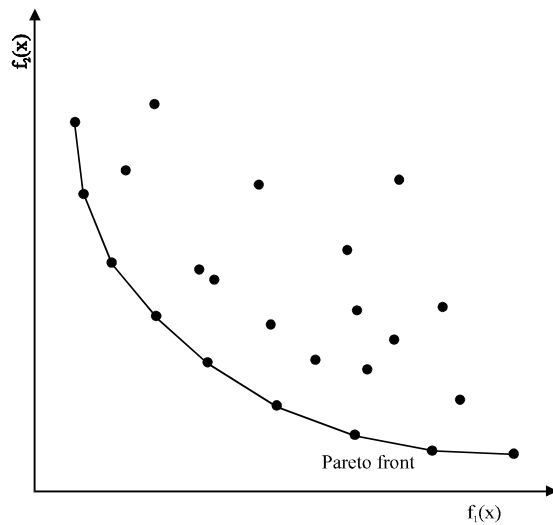


Fig. 2: Pareto front (red points), the set of pareto optimal solutions

programs such as EnergyPlus, TRNSYS, HVACSIM+ and DOE-2 are noticed to be more frequently used for energy performance analysis. These tools are often used to model the energy consumption of a HVAC system in a building by interacting the building thermal model, HVAC component models and the control strategy (Fig. 2).

For optimization processes, programs such as MATLAB/ Simulink and GenOpt are commonly used. This could be due to the reason that these programs are able to couple to various HVAC simulation tools that read input files and write output files in text format. Integration of some popular optimization algorithms such as Genetic Algorithm (GP)/Multi-Objective Genetic Algorithm (MOGA) in GenOpt into HVAC simulation tools such as TRNSYS has eased the efforts of researchers and engineers for the coupling process. Besides that these optimization programs are usually good in solving optimization problems where the objective functions are computationally expensive. For expert users, a software called Building Controls Virtual Test Bed (BCVTB) can also be used to couple different simulation programs for co-simulation or to couple simulation programs with actual hardware for experiment purposes.

## OPTIMIZATION ALGORITHMS

Optimization has become critically important in various engineering applications today. Its ultimate goal is to minimize the resources such as costs and time in a system, yet still yield the greatest efficiency. In recent years, optimization techniques have evolved and improved tremendously, thanks to the rapid growth of

computational power and high computing storage capacity. As a result, almost all engineering problems can be suitably formulated into optimization problems. An optimization problem consists of maximizing or minimizing a real function by analytically selecting some input values from an allowed set and then calculating the value of the function (Chiandussi *et al.*, 2012). Generally an optimization problem can be represented by:

$$\begin{aligned} &\text{Minimize } f_i(x) \quad (i = 1, 2, \dots, M) \\ &\text{s.t. } x \in X \end{aligned} \quad (1)$$

$f_i(x)$  is the objective function or cost function of a problem. If  $M = 1$ , this is a single objective optimization problem. If  $M > 1$ , the optimization problem will be Multi-Objective or multi-criteria Optimization (MOOP) problem.  $X$  is the feasible set of decision vectors. The feasible set is usually defined by some constraint functions.

HVAC systems often deal with two major conflicting design objectives or design issues: maximize the thermal comfort of the occupants at the same time minimize the power consumption or energy cost. Thus, MOOP is more applicable in HVAC control systems than single objective optimization. MOOP search for set of best trade-off solutions in an optimization problem which is called Pareto optimal. Pareto optimal is the solutions that cannot be improved in any of the objectives without degrading at least one of the objectives (Awad and Khana, 2015). A solution  $x_1 \in X$  is said to dominate another solution  $x_2 \in X$  if:

$$\begin{aligned} &f_i(x_1) \leq f_i(x_2) \text{ for all indices } i \in \{1, 2, \dots, k\} \\ &f_i(x_1) < f_i(x_2) \text{ for at least one index } j \in \{1, 2, \dots, k\} \end{aligned} \quad (2)$$

Many researchers regularly performed scalarization to multi-objective optimization (Huang and Lam, 1997; Wang and Jin, 2000; Mossolli *et al.*, 2009) which transform the multi-objective problem into single objective problem by assigning different weight factors to the objective functions:

$$\begin{aligned} &\text{Minimize } \sum_{i=1}^n w_i f_i(x) \\ &x \in X \end{aligned} \quad (3)$$

where,  $w_i$  is the weight factors of the objective functions. The weight factors can be set according to occupant's preferences. This is particularly useful when the preference factor of the objectives is known in advance. However, if one would like to explore and evaluate every

combination of weight factors, large number of reiteration on optimization problem is required. In most cases, the computational requirement are too large to be practical.

**Optimization techniques in HVAC systems:** There are many existing optimization approaches and techniques developed so far and many researchers have reviewed the efficiency and practicality of them in solving HVAC optimization system. Some of the optimization techniques are heuristic or meta-heuristic, some are deterministic or stochastic. Since HVAC is a complex and nonlinear system that involves hundreds of variables, meta-heuristic and stochastics based optimization seem to be more preferred by most researchers.

Out of all the optimization algorithms, Evolutionary Algorithms (EA) and bio-inspired optimization techniques appear to be very popular among the researchers. EA is based on the process of biological evolution where a population adapts to the environment, generating new and better individuals while eliminating the weaker ones. Generally, EA goes through the process of selection, crossover and mutation to get the best solutions. In HVAC system, various set points (population) are continuously searched and objective functions regarding energy cost and thermal comfort are constantly evaluated. Through many iterations, the set points should converge at the optimum value of the objective function. The search process will be terminated according to the criteria set or when the global minima or the optimal solution is found. One of the well-known EA is Genetic Algorithm (GA) which seems to get the most attraction from scholars regarding optimization of HVAC system. Others EA include Evolution Strategies (ES) and Evolution Programming (EP).

Wang and Jin (2000), Xu and Wang (2009) used GA to perform optimal control of Variable Air Volume (VAV) air conditioning system by simultaneously searching for three HVAC parameters: supply air temperature, chilled water temperature and outdoor ventilation rate. Mossolli *et al.* (2009) used GA to search for the optimal supply air temperature and supply air flow rate by maintaining the PMV and IAQ of multi-zone air conditioning system. Huang and Lam (1997) employed GA to search for the best proportional and integral gain in the Proportional Integral (PI) controller of HVAC systems by constantly evaluating overshoot, settling time and mean squared error of the system. Hussain *et al.* (2014) incorporated GA in the Fuzzy Logic Controller (FLC) dedicated to control of HVAC systems that concerning energy efficiency and thermal comfort requirement. Alcalá *et al.* (2003) used another variant of GA which is Weighted Multi-criteria Steady State Genetic

Algorithm (WMC-SSGA) in searching for optimal setting in its fuzzy logic control of HVAC system. Lu *et al.* (2005) incorporated modified GA in the Adaptive Neuro-Fuzzy Inference System (ANFIS) Model of duct and pipe network in HVAC system to search for the optimal setting of differential pressure. Generally, most writers claimed that optimizations by GA or variants of GA exhibit significant improvement in energy performance of HVAC system.

GA is also extended to solve multi objective problems. Wright *et al.* (2002) employed Multi-Objective Genetic Algorithm (MOGA) to search for the optimum sizing of HVAC system, simultaneously with the optimization of its supervisory control strategy to solve two conflicting objectives regarding system energy use and occupant comfort. Nassif *et al.* (2004) evaluated both Non-dominated Sorting Genetic Algorithm (NSGA) and NSGA-II for two-objectives optimization by tuning four HVAC parameters concurrently. NSGA-II is very well-known as a fast and elitist multi-objective GA. Kusiak *et al.* (2011) applied another siblings of GA, Strength Pareto Evolutionary Algorithm with Local Search (SPEA-LS) in optimizing a scalarized three-objectives functions to search for the best supply air temperature and static pressure set points. Gacto *et al.* (2012) used SPEA2-LS for effective tuning of FLC of a HVAC system. Other variants of GA include Multi-phase Genetic Algorithm (MPGA) by Beghi *et al.* (2011) for optimization of multiple chiller systems, and Multi-Island Genetic Algorithm (MIGA) by Seo *et al.* (2014) for optimal operation of HVAC system in an apartment house.

Another popular metaheuristic optimization technique is the bio-inspired swarm intelligence algorithms such as Swarm Particle Optimization (PSO), Firefly Algorithm (FA), Bat Algorithm (BA), etc. PSO, like EA is a population based optimization algorithms. It works based on the social behaviour of elements in nature such as birds and fish. The population learn from the member at global best position to reach the group objective. Each individual will also moves towards the objective according to its personal best and local best. In HVAC optimization, various set points (population) learn from the neighbour set point with the best fitness function. All the population are supposed to converge at the global minima at last. Kusiak *et al.* (2010), Yang and Wang (2012), He *et al.* (2014) used PSO to find different optimal control set points that applied on different parts of the HVAC system. Kusiak *et al.* (2011) later extended his studies to applying Strength Multi-Objective Particle Swarm Optimization (S-MOPSO) to a scalarized three-objectives optimization of a predictive Air Handling Unit (AHU) system developed using feedforward neural network.

Beghi *et al.* (2012) employed PSO to minimize the overall energy consumption of multiple chiller systems by determining the load fraction that each chiller has to satisfy and on-line or off-line status of multiple chillers.

Other swarm intelligence algorithms include Firefly Algorithm (FA) by Zeng *et al.* (2015) to search for the optimal supply temperature set point and supply air static pressure set point of a multi-zone HVAC system. Coelho and Mariani (2013) used Improved Firefly Algorithm (IFA) to minimize energy consumption of multi-chiller system by determining the part load ratio of each chiller. The same researcher later investigated the same multiple chillers optimization problem using Differential Bat Algorithm (DBA) (Coelho and Askarzadeh, 2016).

Other literature for optimization of HVAC systems include Evolutionary Programming (EP) and Evolution Strategies (ES) by Fong *et al.* (2006, 2009), Kusiak and Li (2010). EP and ES are similar to GA but differ in the way of selection process of the population. Fong *et al.* (2006, 2009) used EP and ES to optimize the energy consumption objective by adjusting the supply air temperature of AHU and chilled water supply temperature. Whereas, Kusiak (2010) used ES to minimize the cooling output while maintaining the corresponding thermal properties of the supply air. Another evolutionary computation algorithm applied to HVAC is Differential Evolution (DE) by Lee *et al.* (2011). They used DE to calculate the optimum Part Load Ratio (PLR) of multiple chiller systems for the minimum energy consumption.

Besides that, there are also some literature regarding HVAC optimization using branch and bound method (Sosa *et al.*, 1997; Ferreira *et al.*, 2012) gradient descent method (Liang and Du, 2005) Sequential Quadratic Programming (SQP) algorithm (Zheng and Zaheer-Uddin 1996; Sun and Reddy, 2005; Roberto *et al.*, 2008) Hooke and Jeeve's algorithm (Rackes and Waring, 2014) Lagrangian method (Alvarez *et al.*, 2013) and Harmony Search (HS) algorithm, although, the application of these algorithms in HVAC systems are considerably lower than that of metaheuristic one.

**Efficiency of optimization algorithms in improving HVAC performance:** It is important to know the efficiency of optimization algorithms in achieving the objective functions of HVAC control which are minimizing of energy usage or costs and maintaining occupant's thermal comfort so that the performance of HVAC system can be continuously improved.

For optimization performed using GA by Mossolli *et al.* (2009) authors claimed that their control strategy resulted in 30.4% savings in energy costs over

summer season of four months compared to conventional control scheme for a building floor in Beirut, Lebanon. By Wang and Jin (2000) writers compared GA with conventional fixed point control of Variable Air Volume (VAV) settings and found GA optimization exhibits significant improvement in cold season compared to hot summer season for online-control application. Alcalá *et al.* (2003) performed fuzzy control of a HVAC system optimized by multi-weighted GA at a large hall situated in France and found about 12% and 15% saving of energy in mid-season and summer season, respectively. Hussain *et al.* (2014) found 16% and 18% reduction in cooling and heating energy for a nine-story hotel building model in Toronto, Canada using fuzzy logic controller with GA optimization algorithm.

Nassif *et al.* (2004) used controlled elitist NSGA-II to optimize the set points of an existing HVAC system with a detailed VAV Model and found a 19.5% of energy reduction for one week simulation in summer season. Magnier and Haghighat (2010) too used NSGA-II to optimize the HVAC settings along with building behaviour and found significant reduction in total energy consumption when thermal comfort is slightly compromised compared to base case configuration. With an existing HVAC system model optimized using evolutionary computation algorithm. Kusiak *et al.* (2011) showed an energy saving of 21.4% without violating the indoor air quality constraints. The saving can go up to 22.6% if occasional violations of IAQ is allowed.

By Kusiak *et al.* (2010), PSO optimization is used to search for the optimal control settings of AHU simulated with different internal loads. The results demonstrated a 7.7% savings of the total energy for simulation carried out in cooling season mainly due to the energy reduction in chiller. Beghi *et al.* (2012) simulated a multiple-chillers system by means of PSO algorithm in a directional building in Milan, Italy on a typical cooling season ranging from May to September. The simulation results were compared with the standard sequencing strategies namely sequential strategy (MS) and Symmetric Strategy (SS) and they found that PSO exhibits 13.8% and 7.1% seasonal electric energy saving with respect to MS and SS, respectively. Yang and Wang (2012) employed MOPSO for HVAC VAV optimization in an office building for 24 hour period. Simulation was carried out for both hot and cold weather to examine the control ability of cooling unit and heating unit respectively. Compared to Constant Air Volume (CAV) system and non-optimized VAV system, their MOPSO-optimized HVAC system features the highest energy efficiency, especially in high load time. Kusiak *et al.* (2011) tried to improve the optimization



of a predictive HVAC Model using Strength Multi-Objective Particle Swarm Optimization (S-MOPSO). Their results showed that S-MOPSO demonstrated even higher energy saving of AHU (13.4%) compared to MOPSO (3.3%) taking into consideration of humidity quality and temperature quality.

Coelho and Mariani (2013) optimized the power consumption of a six chillers system with six different cooling load cases. They compared improved Firefly Algorithm (IFA) with Average Loading method (AVL), Genetic Algorithm (GA), Simulated Annealing (SA), Binary Genetic Algorithm (BGA), Continuous Genetic Algorithm (CGA), Particle Swarm Intelligence (PSO) and Evolution Strategy (ES). From the simulation results obtained, IFA exhibited the best objective function value and they concluded that it is possible to achieve substantial energy savings while granting good satisfaction of the cooling demand when compared with all the standard algorithms presented in recent literature. Fong *et al.* (2006) also demonstrated a near 7% energy saving potential compared to conventional setting when both chilled water supply temperature and supply air temperature set points are optimized in a monthly basis using Evolutionary Programming (EP). The same researchers Fong *et al.* (2009) also optimized the heat rejection system of HVAC using Evolution Strategy (ES) and found that ES could achieve 6.13% lower daily total energy consumption compared to GA.

Undoubtedly optimization approaches offer great potential in improving the performance of HVAC system, both in cooling season and hot season as proved by the literature. These optimization algorithms can significantly reduce the power consumption of the overall system without sacrificing the comfort of the indoor condition. However, due to dissimilar objective functions from various literature, a clear-cut efficiency evaluation of different optimization algorithms might require further analysis.

**Selection of optimization algorithms:** The choice or selection of optimization algorithms for HVAC specific application depends on many factors such as performance of the algorithms, natures of objective functions (single objective or multi-objectives), design variables involved, complexity of the optimized systems, etc.

Figure 3 shows the frequency of usage of different optimization algorithms in HVAC application which is derived from Table 1. It can be seen that stochastic population-based algorithms such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO) and other Evolution Algorithms (EA) receive most attentions from

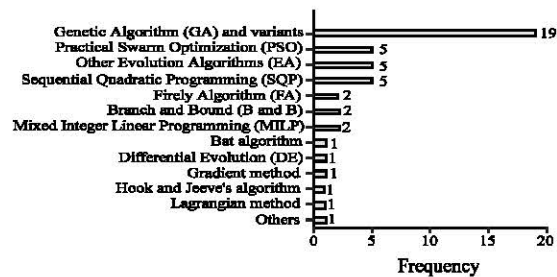


Fig. 3: Frequency of usage of different optimization algorithms

researchers for optimization of HVAC system in the past 20 years. These algorithms are non-deterministic and random in new population selection. They maintain a diverse set of points providing a means of escaping from one local optimum. As a result, global optimum and better-quality solution can be obtained. Besides that, these algorithms provide a means of handling large and discontinuous search spaces which are frequently seen in HVAC problem (Eiben and Smith, 2003).

**Performance of optimization algorithms:** The performance of various optimization algorithms in searching for the best solutions of dynamic HVAC systems is always the interest of many researchers. Performance of the algorithms can be evaluated from the efficiency in converging to a low value of objective function in short period of time. It is one of the main factors to be considered when selecting optimization algorithm to an engineering problem. To date, many literature have been published to compare and validate the effectiveness, efficiency and robustness of different optimization algorithms for HVAC application, either quantitatively or qualitatively.

Popular optimization algorithm GA was first adopted by Huang and Lam (1997) to search for the proportional and integral gain of a Proportional-Integral (PI) controller of a HVAC system, they proved that GA showed better controller performance than conventional Ziegler-Nicholas method.

Wright *et al.* (2002) used Multi-Objective Genetic Algorithm (MOGA) to investigate the pay off between energy cost and occupant discomfort in a HVAC conditioned zone and concluded that MOGA exhibits fast progress towards the Pareto optimal solutions. They found that MOGA is able to find feasible solutions within a very few trail solutions.

Nassif *et al.* (2004) compared the performance of Non-dominated Sorting Genetic Algorithm (NSGA) and NSGA-II in solving two-objective optimization HVAC problem. They evaluated the performance of these two

algorithms using two metrics: closeness to the Pareto-optimal front, diversity among the non-dominated solutions. The results showed that NSGA-II produces better convergence and distribution of optimal solutions located along the Pareto front.

By Magnier and Haghighat (2010) the researchers first used Artificial Neural Network (ANN) to characterize the building behaviour and then NSGA-II optimization to search for the optimal settings of building design, including HVAC system. They claimed that the optimization was very efficient in terms of convergence and spreading of Pareto solutions with significant reduction in terms of energy consumption as well as improvement in thermal comfort even though the exact comparison values were not presented.

He *et al.* (2014) compared two popular evolutionary algorithms: MOGA and NSGA-II with PSO and Harmony Search (HS) algorithm. They found that the average computational time of both EAs are about 900 sec which is too high for online implementation of HVAC settings. Both PSO and HS outperform EAs by showing a high convergence frequency and a much shorter computing time than EAs (0.26 and 0.04 sec, respectively). They concluded that PSO and HS have very good potential in online optimization of HVAC systems.

Zeng *et al.* (2015) investigated energy savings of a multi-zone HVAC system using Firefly Algorithm (FA) and compared it with PSO and Evolution Strategies (ES). They showed that FA demonstrates the lowest energy consumption under almost 300 combinations of optimization parameters settings and 10 randomly selected data points. Moreover, the average CPU time used by FA per parameter setting per data point is the shortest (12.3 sec) compared to PSO (13.0 sec) and ES (19.5 sec).

Coelho and Mariani (2013) simulated a three chillers systems with Improved Firefly Algorithm (IFA) and showed that IFA is able to search for the optimal solution as Differential Evolution (DE) and PSO can. It outperforms GA by showing quality optimization solutions (minimal energy consumption) for all six different loading cases from high cooling load to low cooling load. Besides that, IFA overcomes the divergence problem caused by Gradient Method (GM) occurring at low cooling demands.

Beghi *et al.* (2012) implemented both Multi-Phase Genetic Algorithm (MPGA) and PSO for optimal multi-chiller operation. From the simulation results, they showed that both algorithms are able to converge to their optimum values but PSO can reach a fixed average fitness in fewer numbers of generations with low memory requirements and smaller numbers of parameter settings.

Lee *et al.* (2011) compared Differential Evolution (DE) with GA, PSO and Lagrange method in searching for the optimum Part Load Ratio (PLR) of multiple chiller system for minimal energy consumption and they proved that DE outperforms GA in finding optimal solution, overcomes the divergence problem caused by Lagrange method and could show better average solutions compared to PSO.

Performance evaluation of different optimization algorithms is still very subjective as most of the literature do not present the convergence speed and cost function value of their optimization. Even if they do, the literature usually cover different objective functions and optimize different parts of the HVAC systems. Thus, it is not easy to achieve an equal comparison.

**Optimization design variables:** Selection of optimization algorithm also affected by types and number of design variables in an optimization problem. Table 3 tabulated the design variables of eight random literature and its associated optimization algorithm, according to chronological order.

Generally, HVAC optimization often deal with both continuous variables and discrete variables. Continuous variables are the thermo-fluid properties in primary and secondary components of HVAC system such as temperature, mass flow rate, humidity ratio and pressure of supply/return water at primary side and supply/return air at secondary side. Commonly, design constraints are imposed to define the low limits and high limits of the variables in optimization search. Discrete variables include parameters such as number of coils, number of water circuits, number of chillers in operation, sequence of operation, etc. Sometimes, HVAC optimization also deal with design variables with binary state such as on or off operation of a HVAC component that takes up a value of 0 and 1. Thus, discrete variables are usually constrained to be integers that take up a finite value. Solving optimization problems that include both continuous variables and discrete variables are denoted as Mixed-Integer Programming (MIP) problems. These problems usually make the optimization problem non-convex and discontinuous, hence, much more challenging in searching for optimality. Even though conventional methods such as branch and bound can be used to solve mixed integer problems, metaheuristic algorithms such as evolutionary algorithms (GA, ES, EP) and Swarm Optimization (PSO) provide a potential alternative in looking for optimality in high dimensional problems. These metaheuristic algorithms can handle both

Table 3: HVAC set-point variables and design variables in some arbitrary studies

researchers, Year	Optimization algorithm	Set points to be optimized		Variable types		Problem	variables	
				Continuous	Discrete		Continuous	Discrete
Wang and Jin (2000)	Genetic Algorithm (GA)	Outdoor ventilation airflow rate	Supply air temperature	Chilled water supply temperature	3	0	Outdoor ventilation airflow rate Supply air temperature Chilled water supply temperature	-
Wright <i>et al.</i> (2002)	Multi-Objective Genetic Algorithm (MOGA)	Supply air flow rate	Supply air temperature		2	7	Supply air temperature Supply air flow rate	On/ off status Coil width Coil height No. of coil rows No. of water circuits Maximum water to flow rate each coil Supply fan sizes
Nassif <i>et al.</i> (2004)	Non-dominated temperature Sorting Genetic Algorithm 2 (NSGA-2)	Zone temperature	Supply air temperature	Chilled water supply temperature supply duct static pressure	4	0	Zone temperature Supply air temperature Chilled water supply Supply duct static pressure	-
Sun and Reddy (2005)	Sequential Quadratic Programming (SQP)	Chilled water supply	Relative speed of condenser water pump	Relative speed of cooling tower fan	8	0	Chiller design cooling load Chilled water mass flow rate Chilled water supply temperature Chilled water return temperature Condenser water mass flow rate Condenser water supply temperature Condenser water return temperature Cooling tower air flow rate	-
Lu <i>et al.</i> (2005)	Modified genetic algorithm	Chilled water supply temperature	supply air flow rate		6	3	Chilled water supply temperature Supply air flow rate Room temperature Head pressure provided by chilled water pump Air pressure of cooling coil Supply air temperture	No. of chillers No. of chilled water pump No. of cooling coils

Table 3: Continue

Researchers, Year	Optimization algorithm	Set points to be optimized	Variable types	Problem	variables	
					Continuous	Discrete
Fong <i>et al.</i> (2009)	Evolution Strategy (ES)	Supply air temperature	Chilled water supply temperature	4	3	Flow rate of condenser water pump Air flow rate of cooling tower fan Condenser water supply temperature Condenser water return temperature No. of chillers in operation No. of condenser water pump in operation No. of cooling tower fans in operation
Kusiak <i>et al.</i> (2010)	Particle Swarm Optimization (PSO)	Supply air temperature	Supply air static pressure	7	2	Supply air temperature Supply air static pressure Chilled water supply temperature (mean) Chilled water supply temperature (standard dev) Outdoor air temperature Solar normal flux (mean) Solar normal flux (standard dev) Internal load Internal load at previous time
Magnier and Haghighat (2010)	Non-dominated Sorting Genetic Algorithm 2 (NSGA-2)	Zone temperature	Supply air flow rate Humidity	7	5	Heating temperature Cooling temperature Relative humidity Supply air flow rate Thermostat on delays time Thermostat off delays time Thermal mass of house Sizes of five South and North Windows (building related)

continuous and discrete variables which are commonly found in HVAC application. They integrate one or more properties of good populations when generating new populations and do not keep trying the same solutions. Thus, they are capable to escape from local minima, although, these algorithms do not guarantee that the global minima can be found. They tend to move towards good solutions in a faster manner, hence, they can provide a more efficient way to deal with large complicated problems that involve a sizable number of variables.

**Single objective and multi-objective optimization:** Optimization problems can be classified as single

objective or multi-objective problem. Single objective optimization minimizes or maximizes one objective that are functions of some integer variables. Whereas multi-objective optimization handles simultaneous optimization of two or more conflicting objectives with regards to certain constraints or limits (Chiandussi *et al.*, 2012). The latter one is usually the case in our real-life situations. In multi-objective optimization, enhancement of one objective always leads to degradation of another. In this scenario, a trade-off must be created. Same goes for HVAC system optimization where energy or cost objective function conflicts with thermal comfort objective function and sets of compromise points need to be found. This adds complexity to the optimization of HVAC systems.

Many researchers transform multiple objective to single objective by means of scalarization. This is achieved by allocating different weight factors to each element in the objective function. Wang and Jin (2000) assigned five weight factors to five elements in the objective function including Predicted Mean Vote (PMV), energy use, Indoor Air Quality (IAQ), maximum allowed Relative Humidity (RH) and minimum allowed ventilation rate with each fitness function element represents the quantitative penalty when an index moves away from the expected value. Minimization of the objective function results in optimal control of the whole air conditioning system. Huang and Lam (1997) optimized settling time, overshoot and mean squared error with three weight factors in his tuning of HVAC Proportional, Integral and Differential (PID) controller. Their weight factors selection is based on the rule of keeping the product of the weight factors and its respectively fitness function element at the same value to ensure the importance of every element in the objective function. Mossolli *et al.* (2009) too using the same approach as Wang and Jin (2000) with six weight factors assigned to PMV, IAQ, fan operation energy, cooling energy, heating energy and supply temperature. However, this approach exhibits some disadvantages because it is not easy to assign the weight factors value at priori level as all factors usually have different significance. Moreover, this approach will provide just one set of optimal solution.

Another approach is to use true multi objective optimization where a set of Pareto optimal solutions are produced. In this regards, some of the metaheuristic single-objective optimization algorithms such as GA, NSGA-II, PSO and DE have their multi-objective version developed. Nassif *et al.* (2004) optimized two objective functions simultaneously using NSGA-II to find the optimal settings of supply air temperature, supply duct static pressure, chilled water temperature and zone temperature to give the minimum energy usage and maximum PMV. Magnier and Haghighat (2010) too optimized thermal comfort and energy consumption using NSGA-II by searching for optimal heating set points, cooling set points, RH set points, supply air flow rates and thermostat delays. Wright *et al.* (2002) optimized energy cost and occupant thermal discomfort using MOGA by adjusting supply air temperature and supply air flow rate. For this approach, many Pareto optimal solutions are produced and all solutions are considered equally good in satisfying the objective functions. It then depends on the user's preferences or prioritization in choosing for the best solution from the Pareto set and this system is called Multi-Criteria Decision Making (MCDM).

## CONCLUSION

Optimization undeniably demonstrates great energy and cost saving potentials, along with indoor thermal comfort in HVAC systems. This study gives an overview of the optimization algorithms applied on HVAC system for optimal control. First, the problem is addressed as a whole where energy, thermal comfort and optimization are involved. Next, some preparation of optimization are discussed which include the modelling approaches and simulation tools used. The remaining parts discussed about the mostly-used optimization techniques, efficiency in energy saving, performance analysis of various algorithms and the factors affecting the selection of algorithms.

From the review, it appears that population-based metaheuristic optimization algorithm such as Genetic Algorithm (GA) and Particle Swarm intelligence (PSO) and their multi-objective versions draw the most attention of researchers. It looks like these algorithms will continue to be the trends of optimization in HVAC systems in view of the remarkable advances in optimization to solve complex problems of multitude of variables in reasonable time. However, it is suggested that other new intelligence optimization algorithms (such as Biogeography-Based Optimization (BBO), Cultural Algorithm (CA), etc.) or even hybrid of optimization algorithms (genetic algorithm mixed simulated annealing, backpropagation algorithm mixed genetic algorithm) to be further explored for HVAC application in order to find a new breakthrough in energy efficiency.

For HVAC simulation tools, Energy Plus and TRNSYS are getting more popularity among researchers as these tools provide a very comprehensive energy analysis and performance assessment. Users can model any scale of HVAC systems from simple to complex using these well-developed tools. In terms of optimization simulation programs, MATLAB and GenOpt seem to be preferred by researchers as these high-quality software are very powerful and flexible in programing. These software provide optimization toolboxes of some established algorithms such as quadratic programming and genetic algorithm for the ease of application. Researchers also have the freedom to program or modify their own optimization algorithms using the said software. However, coupling of simulation tools with optimization programs is a very complicated and tedious process. It requires in-depth knowledge in managing communication between the software involved. For ease of optimization job in HVAC application, it is recommended to develop some user-friendly coupling methods for co-simulation of

different programs. From the review, it is found that most of the current research lack quantifiable data of both energy saving level and occupant's thermal comfort level to reflect the effectiveness and performance of various optimization techniques. Thus, it is suggested that upcoming research should comprise quantification evaluation of both conflicting objectives, so that more accurate and thorough performance comparison of different optimization techniques can be obtained.

Although the study of optimization in HVAC systems has been going on for almost two decades, the implementation of the algorithms in real world application is still at a challenging stage. Thus far, application of these algorithms are only on experimental basis carried out at research centers or universities. Therefore, continuous and much efforts are very much needed to transform these optimization techniques to commercial usage.

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