

Numerical Technique for Autonomous Robot Path Planning Based on QSAOR Iterative Method Using Indoor Environment

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Abstract: In recent years, a significant amount of research on robot path planning problems has been devoted. The main goal of this problem is to construct a collision-free path from arbitrary start location to a specified end position in their environment. In this study, numerical technique, specifically on family of Accelerated Over-Relaxation (AOR) iterative methods will be used in attempt to solve mobile robot problem iteratively. It's lean on the use of Laplace's equation to constrain the generation of a potential values. By applying a finite-difference technique, the experiment shows that it is able to generate smooth path between the starting positions to specified destination. The simulation results shows the proposed methods performs faster solution and smoother path compared to the previous research.

Key words: Autonomous navigation, elliptic partial differential equation, finite difference scheme, iterative method, collision-free and optimal path, Malaysia

INTRODUCTION

Path planning or navigation problem plays an important role in the autonomous mobile robots. In order to achieve a successful autonomous mobile robot it must have the capability to efficiently and reliably plan a route from initial to the final configurations without colliding with obstacles in between. Efficient algorithm for solving problems of this type have important applications in areas such as constructions, manufacturing, space exploration and military transportation. It is therefore, not surprising that the research activities in this field becomes one of the most key topics of this generation of research.

Based on the heat transfer theory, modeled with Laplace's equation, this study presents the implementation simulation of a point-robot path planning using numerical potential function. This heat transfer model created a free local minima environment which also, beneficial for robot navigation control. The temperature values in the environment will be used for paths simulation derived from harmonic functions which is the solutions of Laplace's equation. The numerical techniques is been used in order to find the harmonic

functions. This is because of having the availability of fast processing machine and the competency in solving the problem. In this study, the performance of Quarter-Sweep Accelerated Over-Relaxation (QSAOR) iterative method were tested and creating autonomous path planning in different environments.

Literature review: By Khatib (1985)'s Model where he demonstrated the use of potential functions for robot path planning, every obstacles produce repelling force. At the same time, the goals exert an attractive force. Conversely, Koditschek (1987) concluded that the potential functions can be used to guide the robot from almost any point to a given point in certain types of domains. The shortcoming from these potential fields is that they suffer from the generation of local minima. Meanwhile, a global method to generate a smooth collision-free path planning were independently developed by Connolly *et al.* (1990) and Akishita *et al.* (1993) using harmonic functions which also, known as the solutions to Laplace's equations. This functions offer a fast method of finding the path lines over the entire region and avoids the spontaneous creation of local minima. Later, the use of

numerical technique in solving path planning problem is introduced by Sasaki (1998). He shows that by using complicated maze problems simulation, the new computational approach to motion planning is successfully worked. Whereas, Hachour (2008) proposed an autonomous mobile path planning strategy designed in a grid-map form by using hybrid intelligent. He discussed the use of the best path of biological genetic principle in deciding the best avoidance direction for getting a big safety of obstacle danger. The simulation is demonstrated using two programming languages: in visual basic language the robot reaches the target by avoiding all of the obstacles whilst in Delphi language, the robot takes the shortest path to reach the target.

In the previous researches, the standard Gauss-Seidel (GS) and Successive Over-Relaxation (SOR) iterative methods were applied to compute the harmonic potentials (Connolly *et al.*, 1990; Sasaki, 1998; Connolly and Gruppen, 1993; Saudi and Sulaiman, 2013). Apart from autonomous mobile robot path planning, harmonic potentials were also, implemented to many other applications such as UAV motion planning (Liang *et al.*, 2014; Motonaka *et al.*, 2014) marine vessel path planning (Pedersen and Fossen, 2012) ship navigation (Shi *et al.*, 2007) trajectory control (Masoud and Al-Shaikhi, 2015) space exploration (Vallve and Andrade-Cetto 2015) etc.

MATERIALS AND METHODS

The simulation concept of robot vehicle movement were built by using a point moving concept in known environment. The path planning problem of the robot can be formulated as a steady-state heat transfer problem. In the configuration space, the end point is treated as a sink pulling heat in. The heat sources with fixed constant temperature values are set in the environment boundaries, including outer boundaries and inner walls and obstacles. A temperature distribution develops from the heat conduction process and thus the heat flux lines flowing to the sink fill the configuration space. This field shows a transmission medium between goal, obstacles and robots. By following the heat flux, the temperature distribution in the field can then be used as a design for mobile robot to move from the start point to the goal point which will flow from high temperature sources to the lowest temperature point in the environment. The temperature distribution is computed by applying harmonic function to model the environment setup. A harmonic function on a domain $\Omega \subset \mathbb{R}^n$ is a function that satisfies Laplace's Eq. 1, shown as:

$$\nabla^2 \phi = \sum_{i=1}^n \frac{\partial^2 \phi}{\partial x_i^2} = 0 \quad (1)$$

where, x_i is the i th Cartesian coordinates and n is the dimension. For robot path development, the domain Ω includes the outer boundary walls, all obstacles in the environment, start points and goal point. Laplace's equation can be well solved through numerical method. Standard methods that been used to solve the problem are Jacobi and Gauss-Seidel whilst in this study, Eq. 1 was solved via, two stage iterative method for faster computation.

In this model, a point in the configuration space which designed in a grid form, act as the robot. By using numerical technique to satisfy Eq. 1, the function values associated with each node are computed iteratively. The starting point is appointed as the highest temperature while the lowest is assigned as the goal point. Meantime, the outer wall boundaries and obstacles has different initial temperature values. In this research, solution to the Laplace's equation were subjected to Dirichlet boundary conditions, $\Phi|_{\partial\Omega} = c$ where c is constant. Once the potential values of the environment is gained, the smooth path can be easily found by tracing the temperature distributing using steepest descent method. The algorithm will descent through successive points starting with lower potential values leading to lowest temperature goal point.

Formulation of accelerated over-relaxation iterative

method: In solving problem 1 based on the robotics literature, the standard GS (Connolly *et al.*, 1990) and SOR (Saudi and Sulaiman, 2013; Saudi *et al.*, 2014) had been used. The computation for Laplace's Eq. 1 solution for this study is by using faster numerical solver namely Accelerated Over-Relaxation (AOR) iterative method. Consider the two-dimensional Laplace's equation in Eq. 1 be defined as:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0 \quad (2)$$

The approximation of Eq. 2 can be simplify through the 5 point second-order standard finite difference formula as generally stated in the following Eq. 3:

$$U_{i-1,j} + U_{i+1,j} + U_{i,j-1} + U_{i,j+1} - 4U_{i,j} = 0 \quad (3)$$

To boost up the convergence speed, AOR iterative scheme is added to Eq. 3 and the standard five-point formula of AOR can be written as:

$$u_{i,j}^{(k+1)} = \frac{r}{4} \left(u_{i-1,j}^{(k+1)} - u_{i-1,j}^{(k)} + u_{i,j-1}^{(k+1)} - u_{i,j-1}^{(k)} \right) + \frac{\omega}{4} \left(u_{i-1,j}^{(k)} + u_{i+1,j}^{(k)} + u_{i,j-1}^{(k)} + u_{i,j+1}^{(k)} \right) + (1-\omega) u_{i,j}^{(k)} \quad (4)$$

where r and ω are the optimum relaxation parameters. The uncertain optimum values of r and ω gave no restriction in getting the minimum number of iterations. Hadjidimos (1978) stated that, the value is normally chosen to be close to the value of the corresponding SOR where $1 \leq \omega \leq 2$.

RESULTS AND DISCUSSION

This study proposes various shapes of static environment with three different sizes and different number of obstacles. Dirichlet boundary condition was used in the initial setting where the walls and obstacles were fixed with highest temperature values. The lowest temperature values in the configuration space will be set as the goal point whereas there are no specific temperature values for starting and all other points. The computation process was run on AMD A10-7400P Radeon R6, 10 Compute Cores 4C+6G running at 2.50 GHz speed with 8 GB of RAM. Not until the stopping condition is met, the iteration process for temperature values at all points will computed continuously. The loop of the iteration is terminated once there's no changes in temperature values. The difference of calculation values was very small, i.e., 1.0^{-10} in order to avoid the saddle points which will cause the formation of path failed. Table 1 and 2 show the number of iterations and execution time (sec) for all numerical techniques compared in the experiments. Based on the results from Table 1 and 2, QSAOR iterative method evidently proved to be very fast compared to other proposed methods.

By executing steepest descent search from start points to the specific destination, the required path was generated once the temperature values were attained. Based on the Laplacian potential profile, Fig. 1 shows the paths in a known static environment were successfully generated obtained via., numerical computation. All various obstacles setting in the various environment was successfully avoided, so, too the specified goal point (red dot/circle dot) was successfully reached from all start points (green dot/square dot).

Table 1: Performance of the considered methods in terms of number of iteration

Cases/Methods	-----N×N-----		
	300×300	600×600	900×900
Case 1			
FSSOR	2168	9951	21755
HSSOR	1076	5070	11125
QSSOR	510	2587	5708
FSAOR	1760	8222	18081
HSAOR	856	4165	9196
QSAOR	359	2107	4698
Case 2			
FSSOR	3572	13671	29331
HSSOR	1794	6977	15010
QSSOR	899	3571	7720
FSAOR	2930	11325	24380
HSAOR	1454	5754	12448
QSAOR	704	2928	6373
Case 3			
FSSOR	6730	25847	564560
HSSOR	3408	13210	28896
QSSOR	1731	6780	14853
FSAOR	5535	21503	47037
HSAOR	2790	14055	24049
QSAOR	1400	7208	12318
Case 4			
FSSOR	3664	14022	30639
HSSOR	1838	7156	15660
QSSOR	917	3664	8052
FSAOR	3005	11624	25482
HSAOR	1489	5896	12989
QSAOR	721	3008	6640

Table 2: Performance of the considered methods in terms of execution time (sec)

Cases/Methods	-----N×N-----		
	300×300	600×600	900×900
Case 1			
FSSOR	10.21	273.57	1477.21
HSSOR	2.70	100.12	502.43
QSSOR	0.47	19.94	103.78
FSAOR	9.63	248.85	1319.96
HSAOR	2.61	82.86	424.50
QSAOR	0.42	17.78	96.10
Case 2			
FSSOR	17.27	376.26	1947.15
HSSOR	5.32	133.49	659.67
QSSOR	0.86	26.08	138.11
FSAOR	15.76	336.73	1744.58
HSAOR	4.79	111.06	553.73
QSAOR	1.00	23.51	127.17
Case 3			
FSSOR	32.62	758.04	4024.04
HSSOR	9.81	273.59	1369.79
QSSOR	2.22	54.02	310.01
FSAOR	32.07	691.97	3617.46
HSAOR	9.22	226.61	1176.52
QSAOR	1.97	47.25	277.58
Case 4			
FSSOR	18.56	410.51	2042.55
HSSOR	6.15	144.12	705.06
QSSOR	0.88	28.28	151.52
FSAOR	17.49	370.12	1849.85
HSAOR	5.26	122.74	596.07
QSAOR	0.98	26.42	139.90

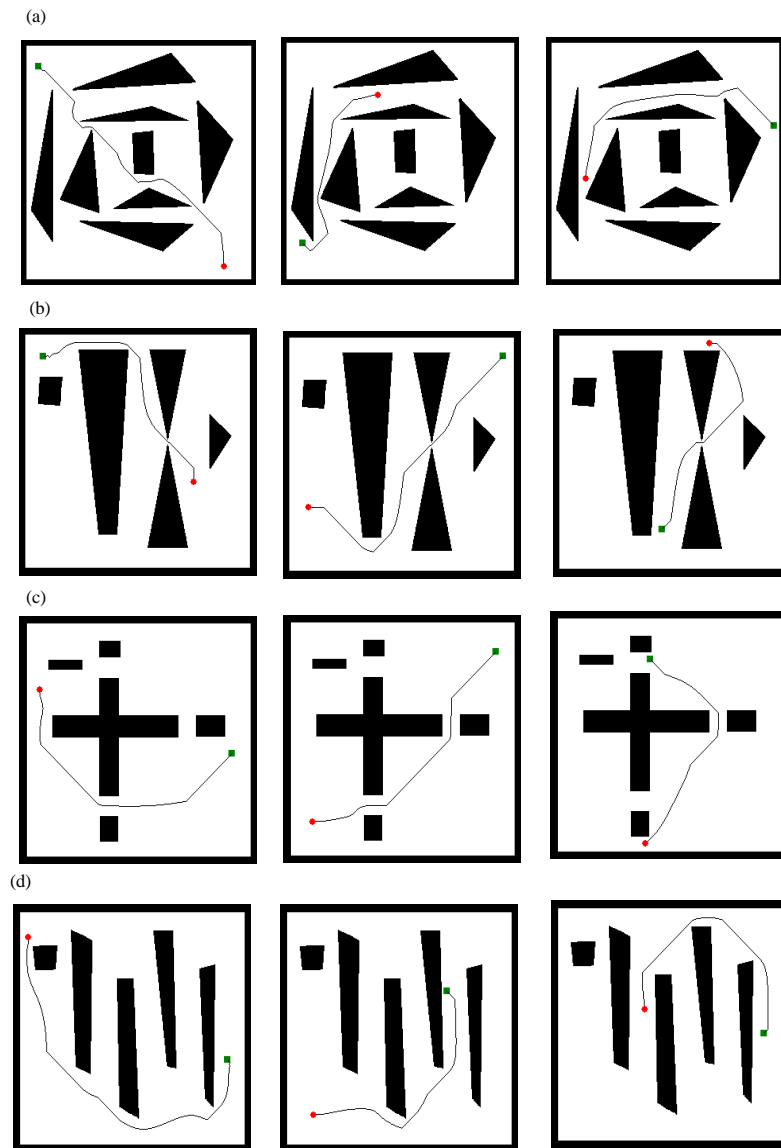


Fig. 1: The generated paths from several different start and goal positions for various environment: a) Case 1; b) Case 2; c) Case 3 and d) Case 4

CONCLUSION

The research in this study shows that solving autonomous navigation problem using numerical techniques are indeed very attractive and feasible due to the recent advanced and new found techniques as well as the availability of fast machine now a days. From the table of results, compared to the other iterative methods, the QSAOR method proved to be rapidly increased. An enlargement and enhancement of obstacles does not affect the performance negatively in fact the computation get faster since the occupied areas by obstacles are ignored during computation. Since, this study can be

classified as a point iterations problem it is hoped that the capability of the proposed method will be helpful for the further investigation such as block iterations problem (Saudi and Sulaiman, 2009; Dahalan *et al.*, 2017) in order to speed up the convergence rate of the standard proposed iterative methods.

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REFERENCES

- Akishita, S., T. Hisanobu and S. Kawamura, 1993. Fast path planning available for moving obstacle avoidance by use of Laplace potential. Proceedings of the 1993 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS'93) Vol. 1, July 26-30, 1993, IEEE, Yokohama, Japan, pp: 673-678.
- Connolly, C.I. and R.A. Grupen, 1993. The applications of harmonic functions to robotics. *J. Robotic Syst.*, 10: 931-946.
- Connolly, C.I., J.B. Burns and R. Weiss, 1990. Path planning using Laplace's equation. Proceedings of the IEEE International Conference on Robotics and Automation, Volume 3, May 13-18, 1990, Cincinnati, OH., pp: 2102-2106.
- Dahalan, A.A., A. Saudi, J. Sulaiman and W.R.W. Din, 2017. Numerical evaluation of mobile robot navigation in static indoor environment via EGAOR Iteration. *J. Phys. Conf. Ser.*, 890: 012064-012064.
- Hachour, O., 2008. Path planning of autonomous mobile robot. *Int. J. Syst. Appl. Eng. Dev.*, 2: 178-190.
- Hadjidimos, A., 1978. Accelerated over-relaxation method. *Math. Comput.*, 32: 149-157.
- Khatib, O., 1985. Real-time obstacle avoidance for manipulators and mobile robots. Proceedings of the 1985 IEEE International Conference on Robotics and Automation Vol. 2, March 25-28, 1985, IEEE, St. Louis, Missouri, pp: 500-505.
- Koditschek, D.E., 1987. Exact robot navigation by means of potential functions: Some topological considerations. Proceedings of the IEEE International Conference on Robotics and Automation, Volume 4, March 1987, Raleigh, NC., pp: 1-6.
- Liang, X., H. Wang, D. Li and C. Liu, 2014. Three-dimensional path planning for unmanned aerial vehicles based on fluid flow. Proceedings of the 2014 IEEE Conference on Aerospace, March 1-8, 2014, IEEE, Montana, USA., ISBN:978-1-4799-1619-1, pp: 1-13.
- Masoud, A.A. and A. Al-Shaikh, 2015. Time-sensitive, sensor-based, joint planning and control of mobile robots in cluttered spaces: A harmonic potential approach. Proceedings of the 2015 IEEE 54th Annual Conference on Decision and Control (CDC'15), December 15-18, 2015, IEEE, Osaka, Japan, ISBN:978-1-4799-7884-7, pp: 2761-2766.
- Motonaka, K., K. Watanabe and S. Maeyama, 2014. 3-dimensional kinodynamic motion planning for an X4-Flyer using 2-dimensional harmonic potential fields. Proceedings of the 14th International Conference on Control, Automation and Systems (ICCAS'14), October 22-25, 2014, IEEE, Seoul, South Korea, ISBN:978-8-9932-1506-9, pp: 1181-1184.
- Pedersen, M.D. and T.I. Fossen, 2012. Marine vessel path planning and guidance using potential flow. *IFAC. Proc.*, 45: 188-193.
- Sasaki, S., 1998. A practical computational technique for mobile robot navigation. Proceedings of the IEEE International Conference on Control Applications, Volume 2, September 1-4, 1998, Trieste, pp: 1323-1327.
- Saudi, A. and J. Sulaiman, 2013. Robot path planning using Laplacian Behaviour-Based Control (LBBC) via half-sweep SOR. Proceedings of the 2013 International Conference on Technological Advances in Electrical, Electronics and Computer Engineering (TAECE'13), May 9-11, 2013, IEEE, Konya, Turkey, ISBN:978-1-4673-5612-1, pp: 424-429.
- Saudi, A., J. Sulaiman and M.H.A. Hijazi, 2014. Fast robot path planning with Laplacian Behaviour-Based Control via four-point explicit decoupled group SOR. *Res. J. Appl. Sci.*, 9: 354-360.
- Shi, C., M. Zhang and J. Peng, 2007. Harmonic potential field method for autonomous ship navigation. Proceedings of the 7th International Conference on ITS Telecommunications (ITST'07), June 6-8, 2007, IEEE, Sophia Antipolis, France, ISBN:1-4244-1177-7, pp: 1-6.
- Vallve, J. and J. Andrade-Cetto, 2015. Potential information fields for mobile robot exploration. *Rob. Auton. Syst.*, 69: 68-79.