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Comparison of Geoid Undulation Values Using Different Global Geopotential Models over Egypt

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Abstract: While the most challenge research scopes of geodesy/surveying science are to identify the highly accurate three-Dimensional (3D) coordinates (ϕ, λ, h/H), the vertical positioning (h/H) are the most crucial for many applications in civil engineering projects. These projects require to convert the Ellipsoidal height (h) [observed by the Global Positioning System (GPS)] to orthometric Height (H) [observed by traditional field survey techniques such as spirit levelling]. The difference between the ellipsoidal height (h) and orthometric Height (H) called the geoid undulation or geoidal height (N). The Global Geopotential Model (GGM) is a model that is being represented by the spherical harmonic coefficient that defines the potential of gravitational in the spectral domain. Lately several GGM have been developed based on the accurate data collected by the different satellite missions. Due to the importance of Egypt location, there are many efforts for applying (as international projects) and/or modifying (as researches studies) several of these GGMs. Also, since, Egypt has area up to one million km^2 and due to the limited input data for different GGMs, therefore ,the accuracy of the GGMs are low. So, it's expected that the undulation values (N) will be different when using different GGMs. This paper aim to study the relation between the undulation values (N) using most common eight GGMs. The study will be on regular points distribution (346 point) between latitudes ϕ [22°N, 31°N] and between longitudes λ [26°E, 36°E] which cover whole Egypt territories. Generally, the maximum (N) value reach 21.47 m at [29.5°N, 33.5°E], the minimum value reach 7.23 m at [22.5°N, 36°E], the maximum difference of N calculated using deferent GGM reach 6.08 m at [28.5° N, 34°E] and the minimum difference reach 0.36 m at [26°N, 27°E].

Key words: Geoid, undulation, GGM, ellipsoidal heights, orthometric heights, Egypt

INTRODUCTION

One of the most challenge research scopes of geodesy were to identify the highly accurate three-Dimensional (3D) reference frame for the Earth system. Many researchers carried on their shoulders identifying (3D) locations of any point (ϕ, λ, h/H) on the Earth by choosing the utmost precision mathematical form which expresses the Earth's size and its geometric shape. The researchers divided the (3D) position to two components; horizontal and vertical. The vertical component is the main concern of this paper.

The vertical positioning has two main factors the height and the datum (based on the used reference surface). According to Davis *et al*. (2011), there are two reference surfaces to compute the vertical positioning; the ellipsoid and the equipotential surface. Ellipsoidal datum is based on a geometric model, an ellipsoid that approximates the Earth's surface without the topography. These ellipsoid heights are typically realized through observations from space based systems such as the Global Positioning System (GPS). Equipotential surface (the geoid) is the surface of constant gravity potential. The gravitational potential of the Earth depends on the distribution of mass density throughout the Earth. This surface includes topography and therefore, differs from a geocentric ellipsoid, up to 100 m, because of the Earth's irregular mass distribution, being higher than the ellipsoid when there is a greater mass.

So, there are two heights values for any point; ellipsoidal heights/Geodetic heights (related to the geodetic reference ellipsoid surface) and orthometric heights (related to the equipotential surfaces/The geoid). The difference between the ellipsoidal height (h) and orthometric Height (H) called the geoid undulation or geoidal height (N) which can be calculated using Eq. 1 and is shown in Fig. 1:

$$N = h - H \quad (1)$$

The ellipsoidal heights are measured by the GPS techniques and the orthometric heights are measured by traditional field survey techniques such as spirit levelling or Total station. Lately the advantages of space-based

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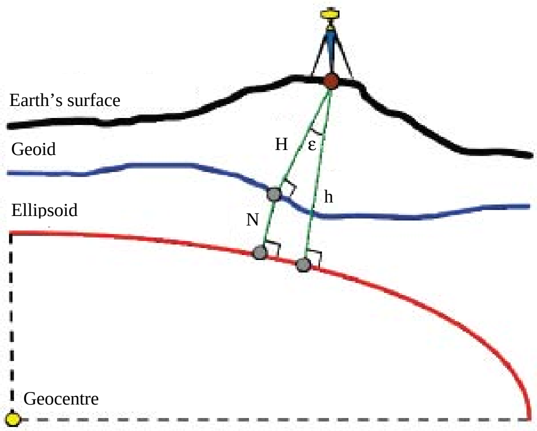

Fig. 1: The relations between ellipsoid height (h), orthometric Height (H) and geoid undulation (N) (Yilmaz *et al*., 2017)

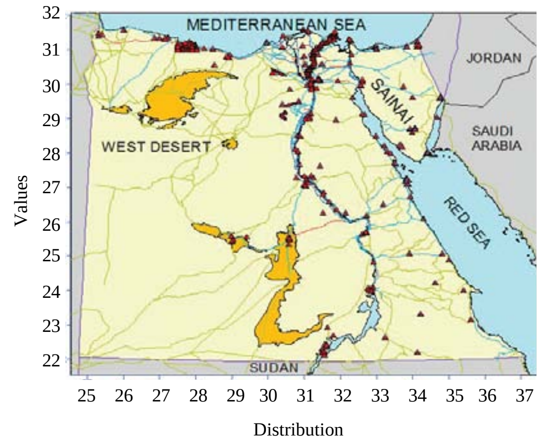

Fig. 2: Distribution of the 394 available points with observed geoid undulation (Mahmoud, 2012)

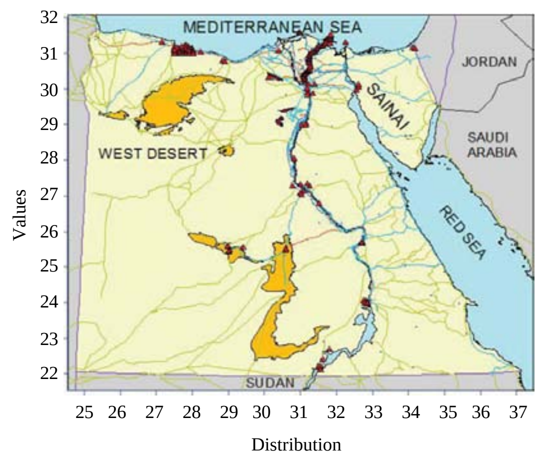

Fig. 3: Distribution of the 158 points only were accepted of the 394 available points (Mahmoud, 2012)

technique, lead to wide spread of using the GPS observations which referrer to the World Geodetic System 1984 (WGS84) ellipsoid. On other hand, most of projects require orthometric heights to be used in engineering applications and mapping processes. According to Eq. 1 geoid undulation/geoidal height (N) is required for every observed point to get its orthometric Height (H) from the ellipsoid height (h).

The Global Geopotential Model (GGM) is a model that is being represented by the spherical harmonic coefficient that defines the potential of gravitational in the spectral domain (Tugi *et al*., 2016). Over the last two decades, several GGM have been developed based on the accurate data collected by the satellite missions CHAllenging Minisatellite Payload (CHAMP), Gravity Recovery And Climate Experiment (GRACE), Global Ocean Circulation Experiment (GOCE) and LAser GEOdynamic Satellite (LAGEOS). The GGM data which published in public domain ((http://icgem.gfz-potsdam.de/ICGEM/) by the International Centre of Global Earth Models (ICGEM), can be classified to satellite-only model (consists of only artificial satellite-based gravity observation such as from CHAMP, GRACE and GOCE) and combined model (combined together with other gravimetry data from terrestrial and/or the airborne, satellite altimetry and topography or bathymetry data such as from LAGEOS) (Tugi *et al*., 2016).

Lately, in Egypt the researchers are interested to find the best GGM which fit Egypt such as Mahmoud (2012), quantify the GGM precision in Egypt such as Al-Krargy *et al.* (2015), developed a national geoid model for Egypt such as Dawod (1998, 2008), Abd-Elmotaal (2008), Rabah and Kaloop (2013), Al-Krargy *et al.* (2014) and evaluation the GGM performance generally such as Krynski and Kloch (2009), Foerste *et al.* (2009) and Yilmaz *et al.* (2016). There is no research to study the relationship of undulation values (N) obtained using different GGMs. So, this paper aim is to study the relation between the collected undulation values (N) using eight most common GGMs. The study will be on points distributed evenly across Egypt.

Literature review: In this section, some important studies related to this work will be highlighted. According to Mahmoud (2012), twenty five GGMs were evaluated with consideration variability of data input type which varied from terrestrial gravity data, satellite tracking data and altimetry data. The research used 394 ground stations distributed over Egypt as shown in Fig. 2 with known geoid shift values (N) coming from observed GPS and levelling surveys. Only 158 stations were accepted and 236 stations were rejected as shown in Fig. 3 (Mahmoud, 2012). Undulation value (N) was calculated

Table 1: Most recent GGMs used by Al-Krargy *et al.* (2015)

Models	Years	Degree	Data type
EGM2008	2008	2190	S (GRACE), G, A
EIGEN-5C	2008	360	S (GRACE, CHAMP), G, A
EGM96	1996	360	S, G, A
EIGEN-6C4	2014	2190	S (GOCO, GRACE, LAGEOS), G, A
GO_CONS_GCF_2_DIR_R5	2014	300	S (GOCO, GRACE, LAGEOS)
GO_CONS_GCF_2_TIM_R5	2014	280	S (GOCO)
DGM-1S	2012	250	S (GOCO, GRACE)

S = Satellite tracking data, G = Terrestrial gravity data, A =Altimetry data and CHAMP, GRACE and LAGEOS are gravity satellite missions

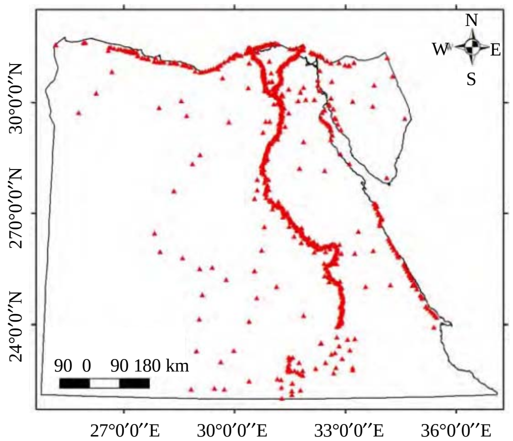

Fig. 4: GPS/levelling points used by Al-Krargy *et al.* (2015)

for each ground station once using GGM and another from GPS/leveling. Then, the difference (ΔN) was calculated as GGM error at each ground station and the standard deviations (σ) of (ΔN) for each data set were calculated. By analyzing the results, the author recommended ten GGMs as best models fit Egypt window and classified them to three accuracy levels as follows (Mahmoud, 2012); EGM 2008 (up to 2190 degree) and EIGEN-CG01C (360) as a first level. EIGEN-CG03C (360), EIGEN-05C (360), EIGEN-GL04C (360), EGM2008 (360) and GGM03C (360) are classified as a second level. The third level are GGM02C (200), EIGEN GRACE-02S (150) and EGM96 (360).

Also, Al-Krargy *et al.* (2015) compared most- recent seven GGMs as shown in Table 1 and evaluate their accuracy when using precise local geodetic datasets in Egypt. They used 1074 GPS/Levelling stations as shown in Fig. 4, also 941 observed terrestrial gravity points. He arranged the GGMs according to the accuracy of geoid undulations as shown respectively in Table 1.

MATERIALS AND METHODS

Data over Egypt and GGM methodology: According to the previous analysis of related research for using different GGMs over Egypt, this study will exposed to the relationships for several recommended GGMs by other researchers. Eight from most common GGMs used in Egypt were selected and are as shown in Table 2. Also, this study will use 346 equally distributed points located between latitudes (ϕ) [22°N, 31°N] and between longitudes (λ) [26°E, 36°E] to cover whole Egypt as shown in Fig. 5.

Since, the study of relationships in this paper will be based on the geoid undulation values (N) which can be calculated by several GGMs, then the basic method which uses by GGMs to calculate the geoid undulation values (N) will be briefly described. According to Al-Krargy *et al.* (2015), the geoid undulation (N) can be computed from gravity data by the well-known Stoke's formula as shown in Eq. 2. Practically the gravity datasets of the whole Earth are not available, therefore the gravity anomalies (Δg) are divided to components as shown in Eq. 3. Finally, the final geoid undulation (N) can be calculated using three components as shown in Eq. 4:

$$N = \frac{R}{4\pi\gamma}\iint_{\sigma} S(\Psi)\Delta g d\sigma \tag{2}$$

Where:

- R = The mean Earth radius
- γ = The normal gravity on the reference ellipsoid
- $d\sigma$ = An infinitesimal surface element on the unit sphere σ
- $S(\Psi) = \sum_{n=2}^{\infty}\left(2n+1/n-1\right) P_n(\cos\psi)$ = The Stoke's function which can be expressed as a series of Legendre polynomial Pn(cos Ψ)
- Δg = The gravity anomaly and can be written as:

$$\Delta g = \Delta g_{REF} + \Delta g_F + \Delta g_h \tag{3}$$

Where:

- Δg_{REF} = The gravity anomalies of a reference gravity field represented by a GGM
- Δg_F = The free-air gravity anomalies
- Δg_h = The effect of topography

$$N = N_{REF} + N_{\Delta g} + N_h \tag{4}$$

Table 2: Eight selected GGMs in this study

Model	Years	Degree	Data type
EGM96	1996	360	S, G, A
EIGEN-CG01C	2004	360	S(CHAMP, GRACE), G, A
GGM03C	2009	360	S(GRACE), G, A
EIGEN-CG03C	2005	360	S(CHAMP, GRACE), G, A
EIGEN-GL04C	2006	360	S(GRACE, LAGEOS), G, A
EIGEN-5C	2008	360	S(GRACE, LAGEOS), G, A
EGM 2008	2008	360	S(GRACE), G, A
EGM2008	2008	2190	S(GRACE), G, A

S = Satellite tracking data, G = Terrestrial gravity data, A = Altimetry data and CHAMP, GRACE and LAGEOS are gravity satellite missions

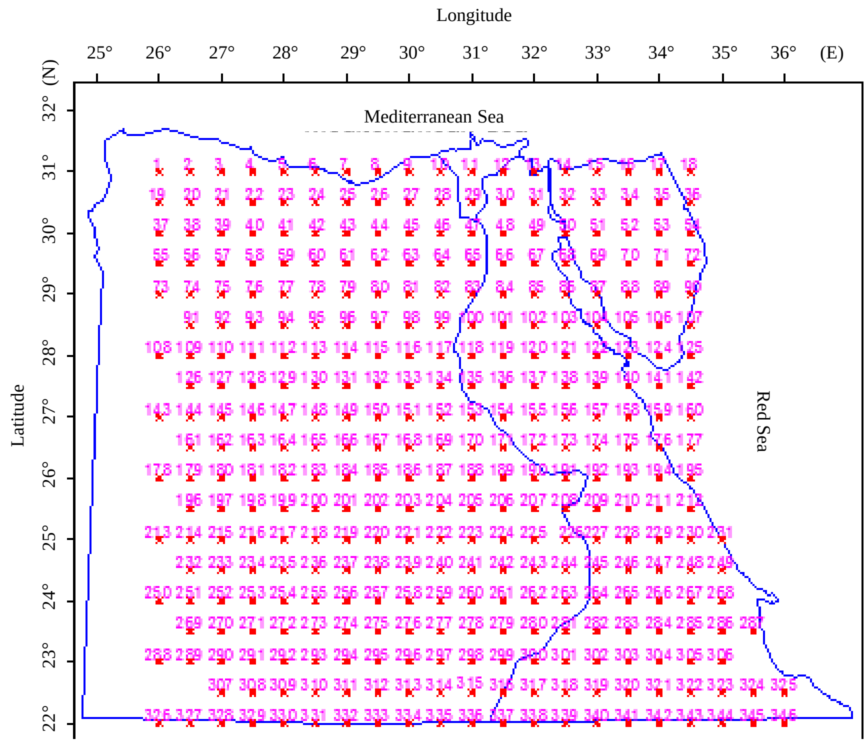

Fig. 5: The 346 equally distributed points over Egypt

Where:

N_{REF} = The contribution of reference gravity field

$N_{\Delta g}$ = The contribution of reduced gravity anomalies

N_h = The contribution of topography

Also, another researcher (Mahmoud, 2012) mentioned the main concept to calculate the geoid undulation values (N) by GGMs as following; "the geo-potential models are available with coefficients (C nm) and (S nm) complete up to degree and order (n, m). So, the geoid undulations above the reference (normal) ellipsoid are computed from the fully normalized spherical harmonic coefficients to degree (n) using the truncated formula" as shown in Eq. 5 (Mahmoud, 2012):

$$N = (GM/r\gamma)\sum_{n=2}^{n=max}\left(\frac{a}{r}\right)^n - \sum_{m=0}^{n}\left[\left(\bar{C}_{nm}\cos m\lambda\right)+\left(\bar{S}_{nm}\sin m\lambda\right)\right]\bar{P}_{nm}\sin\phi \qquad (5)$$

Where:

G = The Newtonian gravitational constant

M = The Mass of the Earth

$\bar{C}_{nm}, \bar{S}_{nm}$ = The fully normalized geo-potential coefficients of degree and order (n, m)

$\bar{P}_{n,m}$ = The fully normalized associated legendre function of degree and order (n, m)

α = The semi-major axis

n = The degree of the geo-potential model

γ = The normal gravity on the reference ellipsoid

r = The radial distance from Earth's mass center

ϕ, λ = The geocentric latitude and longitude

RESULTS AND DISCUSSION

The test in this study has been carried out as follows; The 346 points are distributed equally as an array to cover whole Egypt with 0.5° interval of both latitudes (ϕ) and longitudes (λ). The distance between points is 50 Km, approximately. The latitudes (ϕ) of the points start from 22°N to 31°N and longitudes (λ) starts from 26°E to 36°E (Fig. 5).

Eight GGMs, as shown in Table 2 have been chosen in this research to calculate eight undulation values (N) of each point for the 346 points. All these data are available from the author but due to the limited space only the results with interval of one degree are shown in Table 3. The point number (ID) in Table 3 shows the point number as original data shown in Fig. 5.

To avoid the prolongation in the narrative Fig. 6-15 show the eight undulation values calculated for every one degree latitude and 0.5° longitude. While the conclusion of this research will be based on the analysis of that data for all 346 points which used in this study with interval 0.5° in both latitude and longitude.

Table 3: Calculated undulation values using eight GGMs for 186 points only as an example

			Calculated (N) values (m)								
Point (ID)	Latitude (Deg.)	Longitude (Deg.)	EGM96 (1)	EIGEN-CG01C (2)	GGM03C (3)	EIGEN-CG03C (4)	EIGEN-GL04C (5)	EIGEN-5C (6)	EGM 2008 (360) (7)	EGM 2008 (2190) (8)	Max. diff. (m)
1	31	26.0	21.39	20.84	21.25	21.31	21.29	21.34	21.23	20.85	0.55
2	31	26.5	20.93	20.38	20.79	20.84	20.89	21.06	20.95	20.61	0.68
3	31	27.0	19.84	19.25	19.77	19.64	19.78	19.99	19.66	19.25	0.75
4	31	27.5	18.50	17.77	18.30	18.07	18.29	18.36	18.21	17.96	0.73
5	31	28.0	17.08	16.30	16.89	16.53	16.78	16.74	16.89	16.58	0.78
6	31	28.5	16.01	15.38	15.87	15.59	15.76	15.72	15.77	15.50	0.63
7	31	29.0	15.29	14.65	15.07	14.91	14.92	14.96	14.86	14.72	0.64
8	31	29.5	15.43	14.81	15.18	15.14	15.05	15.20	15.02	14.62	0.81
9	31	30.0	15.74	15.23	15.44	15.61	15.47	15.52	15.44	15.01	0.73
10	31	30.5	15.79	15.23	15.36	15.61	15.44	15.45	15.55	15.01	0.77
11	31	31.0	16.02	15.36	15.59	15.71	15.58	15.59	15.80	15.34	0.67
12	31	31.5	16.36	15.49	15.96	15.77	15.76	15.70	16.03	15.64	0.86
13	31	32.0	16.81	15.74	16.34	15.97	16.11	16.16	16.37	15.94	1.07
14	31	32.5	17.07	16.14	16.62	16.34	16.55	16.54	16.55	16.31	0.94
15	31	33.0	17.79	17.22	17.25	17.41	17.58	17.47	17.20	16.80	0.98
16	31	33.5	17.79	17.41	17.27	17.58	17.64	17.55	17.30	17.01	0.78
17	31	34.0	17.54	17.25	17.30	17.38	17.23	17.66	17.20	17.02	0.64
18	31	34.5	18.44	18.34	18.19	18.44	18.15	18.87	18.17	17.62	1.24
37	30	26.0	21.06	20.64	20.87	21.02	20.81	20.94	21.05	20.56	0.50
38	30	26.5	20.02	19.57	19.86	19.97	19.80	19.85	20.01	19.59	0.45
39	30	27.0	18.64	18.13	18.41	18.47	18.43	18.42	18.38	17.95	0.69
40	30	27.5	17.78	17.16	17.40	17.41	17.52	17.51	17.47	17.02	0.76
41	30	28.0	17.31	16.64	16.91	16.83	16.99	16.95	17.00	16.57	0.73
42	30	28.5	17.24	16.52	16.86	16.74	16.83	16.88	16.85	16.37	0.88
43	30	29.0	17.13	16.36	16.72	16.68	16.67	16.58	16.65	16.50	0.77
44	30	29.5	16.90	16.15	16.51	16.59	16.52	16.36	16.58	16.30	0.75
45	30	30.0	16.69	15.92	16.28	16.44	16.38	16.11	16.47	16.08	0.77
46	30	30.5	16.59	15.90	16.20	16.41	16.40	15.98	16.29	15.79	0.80
47	30	31.0	15.97	15.23	15.68	15.64	15.74	15.76	15.74	15.49	0.74
48	30	31.5	16.25	15.54	15.98	15.82	15.99	16.39	16.16	15.96	0.85
49	30	32.0	16.79	16.44	16.75	16.62	16.77	17.17	16.70	16.45	0.74
50	30	32.5	16.13	16.20	16.50	16.36	16.40	16.53	16.30	15.99	0.53
51	30	33.0	17.38	18.16	18.08	18.36	18.24	18.17	18.08	17.73	0.97
52	30	33.5	18.21	19.57	19.41	19.81	19.58	19.56	19.06	18.64	1.60
53	30	34.0	17.02	18.52	18.59	18.77	18.52	18.50	18.34	18.14	1.76
54	30	34.5	17.04	18.38	18.29	18.61	18.45	18.20	18.58	18.20	1.57
73	29	26.0	18.17	17.69	18.01	18.11	17.94	17.96	17.82	17.33	0.85
74	29	26.5	17.21	16.69	17.05	17.15	17.03	16.97	17.11	16.66	0.55
75	29	27.0	16.30	15.72	16.01	16.12	16.16	15.96	15.96	15.64	0.66
76	29	27.5	15.73	15.07	15.40	15.37	15.51	15.24	15.31	14.97	0.76
77	29	28.0	15.76	14.97	15.30	15.20	15.31	15.13	15.35	14.97	0.79
78	29	28.5	16.23	15.37	15.76	15.61	15.57	15.62	15.59	15.18	1.05
79	29	29.0	16.69	15.75	16.17	16.08	15.89	16.05	15.98	15.53	1.16
80	29	29.5	16.87	15.91	16.28	16.35	16.07	16.23	16.31	15.95	0.96
81	29	30.0	16.24	15.29	15.73	15.78	15.57	15.73	15.55	15.17	1.07
82	29	30.5	15.64	14.77	15.21	15.22	15.22	15.29	14.91	14.43	1.21
83	29	31.0	15.59	14.97	15.45	15.33	15.55	15.44	15.19	14.75	0.83
84	29	31.5	16.16	15.99	16.53	16.28	16.57	16.52	16.18	15.70	0.87
85	29	32.0	16.55	17.11	17.60	17.42	17.57	17.36	17.51	16.72	1.05
86	29	32.5	15.10	16.77	16.94	17.20	17.11	16.45	16.99	16.98	2.10
87	29	33.0	14.03	16.78	17.12	17.40	17.11	17.17	16.73	15.95	3.37
88	29	33.5	15.35	18.75	19.63	19.54	19.25	19.48	19.40	18.95	4.28

Table 3: Continue

			Calculated (N) values (m)								
Point (ID)	Latitude (Deg.)	Longitude (Deg.)	EGM96 (1)	EIGEN-CG01C (2)	GGM03C (3)	EIGEN-CG03C (4)	EIGEN-GL04C (5)	EIGEN-5C (6)	EGM 2008 (360) (7)	EGM 2008 (2190) (8)	Max. diff. (m)
89	29	34.0	14.09	17.57	18.65	18.43	18.35	17.88	19.20	18.60	5.11
90	29	34.5	11.15	13.88	14.87	14.68	14.91	14.82	15.53	15.76	4.61
108	28	26.0	15.81	14.84	15.53	15.49	15.40	15.39	15.25	14.83	0.98
109	28	26.5	15.70	14.79	15.42	15.42	15.48	15.45	15.31	14.94	0.91
110	28	27.0	15.46	14.54	15.12	15.08	15.34	15.26	15.31	14.96	0.92
111	28	27.5	15.26	14.31	14.84	14.75	15.13	15.03	14.95	14.66	0.96
112	28	28.0	14.97	13.89	14.49	14.28	14.62	14.59	14.32	13.97	1.08
113	28	28.5	15.16	14.01	14.59	14.42	14.53	14.64	14.55	14.22	1.15
114	28	29.0	15.51	14.38	14.87	14.85	14.66	14.81	14.88	14.47	1.13
115	28	29.5	15.39	14.30	14.76	14.83	14.41	14.53	14.71	14.42	1.09
116	28	30.0	15.02	14.05	14.25	14.57	14.16	14.16	14.54	14.14	0.97
117	28	30.5	14.35	13.53	13.79	13.96	13.82	13.72	14.15	13.82	0.82
118	28	31.0	14.31	13.78	14.23	14.10	14.31	14.33	14.47	14.03	0.68
119	28	31.5	14.72	14.73	15.08	14.99	15.38	15.62	15.25	14.64	0.98
120	28	32.0	14.58	15.29	15.87	15.57	15.86	16.20	15.88	15.51	1.61
121	28	32.5	14.77	16.47	16.95	16.85	16.86	16.84	17.14	16.43	2.37
122	28	33.0	13.95	16.77	16.84	17.29	17.02	16.47	16.84	16.86	3.34
123	28	33.5	12.23	15.67	15.69	16.28	15.97	15.77	15.90	15.36	4.04
124	28	34.0	11.25	14.60	15.24	15.19	15.10	16.03	15.91	16.05	4.80
125	28	34.5	11.04	13.71	14.55	14.20	14.39	14.96	14.47	13.86	3.92
143	27	26.0	14.97	14.05	14.56	14.58	14.35	14.51	14.77	14.47	0.92
144	27	26.5	15.08	14.30	14.75	14.78	14.60	14.72	14.77	14.34	0.77
145	27	27.0	14.92	14.15	14.56	14.56	14.52	14.64	14.53	14.16	0.77
146	27	27.5	14.80	13.90	14.25	14.27	14.39	14.38	14.40	14.21	0.89
147	27	28.0	14.59	13.54	14.00	13.94	14.19	13.93	13.70	13.21	1.38
148	27	28.5	14.21	13.07	13.72	13.57	13.80	13.55	13.09	12.79	1.42
149	27	29.0	14.08	13.02	13.69	13.62	13.65	13.58	13.53	13.13	1.06
150	27	29.5	14.08	13.25	13.73	13.90	13.68	13.69	13.91	13.44	0.83
151	27	30.0	13.78	13.11	13.49	13.73	13.41	13.45	13.71	13.39	0.67
152	27	30.5	13.55	12.99	13.34	13.53	13.35	13.37	13.68	13.29	0.69
153	27	31.0	13.26	12.78	13.28	13.23	13.35	13.27	13.21	13.07	0.57
154	27	31.5	12.87	12.42	13.19	12.82	13.18	13.06	12.84	12.54	0.77
155	27	32.0	13.45	13.19	14.00	13.60	13.94	13.78	13.58	13.07	0.93
156	27	32.5	14.15	14.17	14.96	14.62	14.68	14.66	14.19	13.92	1.04
157	27	33.0	14.69	15.05	15.62	15.49	15.24	15.58	15.43	14.81	0.93
158	27	33.5	14.75	15.44	15.38	15.80	15.44	15.79	16.06	16.03	1.30
159	27	34.0	13.11	13.96	13.77	14.20	14.01	13.90	13.96	13.56	1.09
160	27	34.5	12.27	12.70	12.61	12.85	12.94	12.43	12.41	12.31	0.67
178	26	26.0	14.26	14.16	14.48	14.59	14.77	14.39	14.75	14.47	0.61
179	26	26.5	13.81	13.85	14.15	14.25	14.22	14.09	14.40	14.09	0.59
180	26	27.0	13.38	13.36	13.66	13.71	13.50	13.72	13.47	13.37	0.36
181	26	27.5	13.25	12.99	13.33	13.33	13.10	13.33	13.06	12.54	0.79
182	26	28.0	13.14	12.47	13.00	12.87	12.82	12.86	12.93	12.67	0.67
183	26	28.5	12.98	12.11	12.91	12.63	12.78	12.69	12.78	12.41	0.87
184	26	29.0	12.91	12.09	12.95	12.72	12.93	12.87	12.67	12.40	0.86
185	26	29.5	13.20	12.61	13.31	13.27	13.36	13.39	13.15	12.81	0.78
186	26	30.0	13.53	13.18	13.68	13.78	13.70	13.65	13.61	13.24	0.60
187	26	30.5	13.21	12.94	13.35	13.44	13.30	13.17	13.21	12.90	0.54
188	26	31.0	13.13	12.73	13.24	13.19	13.14	12.95	13.14	12.71	0.53
189	26	31.5	13.15	12.50	13.12	13.03	13.14	12.98	12.82	12.60	0.65
190	26	32.0	13.04	12.08	12.95	12.75	12.88	12.88	12.59	12.13	0.95
191	26	32.5	13.17	12.01	12.92	12.80	12.80	12.77	12.49	12.20	1.17
192	26	33.0	13.74	12.52	13.43	13.35	13.16	13.26	12.68	12.25	1.49
193	26	33.5	14.70	13.37	14.30	14.13	13.93	14.23	14.16	13.56	1.33
194	26	34.0	14.39	13.09	13.79	13.72	13.71	14.12	14.31	14.10	1.31
195	26	34.5	12.89	11.56	12.22	12.11	12.35	12.57	12.61	12.00	1.33
213	25	26.0	14.09	14.68	15.46	15.53	15.86	15.51	15.48	15.16	1.77
214	25	26.5	13.49	14.36	15.14	15.12	15.29	15.19	15.00	14.74	1.80
215	25	27.0	13.41	14.26	14.74	14.86	14.78	14.93	14.44	14.16	1.52
216	25	27.5	13.41	13.91	14.41	14.40	14.17	14.45	14.17	13.89	1.04
217	25	28.0	13.61	13.64	14.24	14.14	13.99	14.22	14.39	13.94	0.79
218	25	28.5	13.96	13.63	14.33	14.25	14.24	14.39	14.56	14.27	0.94
219	25	29.0	13.56	13.00	13.86	13.75	13.86	13.82	14.04	13.80	1.04
220	25	29.5	13.21	12.66	13.44	13.41	13.47	13.39	13.49	13.18	0.83

Table 3: Continue

			Calculated (N) values (m)								
Point (ID)	Latitude (Deg.)	Longitude (Deg.)	EGM96 (1)	EIGEN-CG01C (2)	GGM03C (3)	EIGEN-CG03C (4)	EIGEN-GL04C (5)	EIGEN-5C (6)	EGM 2008 (360) (7)	EGM 2008 (2190) (8)	Max. diff. (m)
221	25	30.0	13.29	12.92	13.41	13.52	13.45	13.41	13.29	13.16	0.60
222	25	30.5	13.16	12.88	13.19	13.28	13.15	13.19	12.93	12.40	0.88
223	25	31.0	13.01	12.67	12.98	12.94	12.87	13.06	13.29	13.04	0.62
224	25	31.5	13.03	12.49	12.92	12.82	12.87	13.07	14.05	13.38	1.56
225	25	32.0	12.98	12.32	12.81	12.85	12.92	12.99	14.03	13.62	1.70
226	25	32.5	12.41	11.70	12.27	12.45	12.42	12.29	12.60	12.15	0.90
227	25	33.0	12.14	11.37	12.08	12.24	12.09	11.89	11.63	11.21	1.04
228	25	33.5	12.62	11.83	12.56	12.66	12.54	12.32	12.00	11.60	1.06
229	25	34.0	13.46	12.54	13.32	13.24	13.29	13.08	12.92	12.52	0.95
230	25	34.5	13.59	12.53	13.24	13.11	13.35	13.24	13.23	12.88	1.06
231	25	35.0	12.03	10.90	11.33	11.48	11.75	11.66	11.60	11.30	1.13
250	24	26.0	14.92	16.27	17.55	17.28	17.44	17.13	17.02	16.74	2.63
251	24	26.5	14.49	16.30	17.43	17.11	17.18	17.18	16.38	15.97	2.95
252	24	27.0	14.16	15.75	16.64	16.29	16.33	16.37	15.75	15.58	2.49
253	24	27.5	13.98	15.07	15.60	15.44	15.55	15.43	15.26	14.85	1.63
254	24	28.0	13.81	14.43	14.89	14.84	14.93	14.84	14.96	14.74	1.14
255	24	28.5	13.65	13.70	14.34	14.34	14.44	14.28	14.64	14.24	0.99
256	24	29.0	13.51	13.19	13.98	14.09	14.08	14.03	14.32	14.09	1.13
257	24	29.5	13.17	12.65	13.55	13.66	13.50	13.56	13.67	13.25	1.02
258	24	30.0	12.68	12.20	12.95	13.07	12.84	12.89	13.03	12.68	0.87
259	24	30.5	12.30	11.89	12.56	12.47	12.34	12.48	12.23	11.86	0.70
260	24	31.0	12.11	11.77	12.46	12.11	12.25	12.46	11.98	11.64	0.82
261	24	31.5	12.00	11.72	12.44	12.00	12.34	12.59	12.12	11.75	0.87
262	24	32.0	11.94	11.77	12.39	12.19	12.48	12.60	12.19	11.79	0.83
263	24	32.5	11.45	11.44	12.00	12.09	12.11	12.10	11.71	11.38	0.74
264	24	33.0	10.77	10.79	11.32	11.53	11.30	11.24	11.14	11.02	0.76
265	24	33.5	10.85	10.84	11.35	11.50	11.19	11.15	11.41	10.97	0.66
266	24	34.0	11.72	11.50	12.02	11.95	11.80	11.82	11.89	11.64	0.52
267	24	34.5	12.12	11.57	12.13	11.85	11.90	11.95	12.01	11.51	0.62
268	24	35.0	11.86	10.96	11.43	11.22	11.28	11.43	11.81	11.57	0.91
288	23	26.0	15.41	17.26	18.16	17.95	18.40	18.26	17.60	17.32	2.99
289	23	26.5	15.10	17.06	17.52	17.65	17.51	17.82	16.78	16.49	2.72
290	23	27.0	14.42	15.77	15.97	16.15	15.93	16.10	15.96	15.60	1.74
291	23	27.5	13.63	14.23	14.51	14.47	14.60	14.56	14.95	14.65	1.32
292	23	28.0	13.64	13.87	14.14	14.16	14.41	14.38	14.55	14.26	0.91
293	23	28.5	13.63	13.39	13.97	13.94	14.23	13.96	14.27	13.92	0.89
294	23	29.0	13.13	12.55	13.46	13.43	13.51	13.34	13.63	13.27	1.08
295	23	29.5	12.63	12.02	12.94	13.09	12.88	12.85	12.98	12.66	1.07
296	23	30.0	11.64	11.12	12.06	12.13	11.77	11.84	11.92	11.58	1.01
297	23	30.5	10.85	10.60	11.42	11.37	11.16	11.20	11.22	10.91	0.82
298	23	31.0	10.48	10.46	11.20	10.99	11.13	10.95	11.21	10.84	0.75
299	23	31.5	10.24	10.39	11.15	10.88	11.25	10.94	11.30	10.93	1.06
300	23	32.0	10.12	10.31	11.09	10.99	11.30	10.95	11.29	11.02	1.19
301	23	32.5	9.82	9.93	10.79	10.86	10.85	10.74	11.05	10.71	1.24
302	23	33.0	9.91	9.87	10.65	10.94	10.66	10.76	11.25	10.97	1.38
303	23	33.5	10.17	9.69	10.53	10.64	10.37	10.51	10.85	10.53	1.16
304	23	34.0	11.36	10.44	11.13	11.13	11.15	11.05	11.33	11.05	0.92
305	23	34.5	11.97	10.74	11.40	11.20	11.52	11.28	11.61	11.21	1.23
306	23	35.0	10.96	9.51	10.31	9.96	10.31	10.29	10.05	9.78	1.45
326	22	26.0	15.85	15.67	16.69	16.50	16.86	16.61	16.95	16.58	1.28
327	22	26.5	15.33	15.09	15.86	16.03	15.52	15.73	16.07	15.80	0.99
328	22	27.0	14.22	13.74	14.34	14.58	13.84	14.22	14.70	14.39	0.97
329	22	27.5	13.78	12.97	13.57	13.62	13.34	13.67	13.96	13.62	0.99
330	22	28.0	13.45	12.46	13.16	13.00	13.09	13.06	13.34	13.09	0.99
331	22	28.5	12.81	11.67	12.63	12.27	12.63	12.44	12.59	12.35	1.14
332	22	29.0	12.40	11.17	12.23	11.96	12.17	12.13	12.22	11.83	1.23
333	22	29.5	11.89	10.87	11.79	11.79	11.68	11.81	11.79	11.47	1.02
334	22	30.0	11.36	10.60	11.33	11.46	11.17	11.50	11.31	11.08	0.90
335	22	30.5	10.78	10.47	10.96	11.11	10.92	11.05	11.00	10.67	0.64
336	22	31.0	10.35	10.46	10.83	10.90	10.96	10.83	10.82	10.52	0.61
337	22	31.5	10.12	10.43	10.87	10.87	11.07	10.96	10.72	10.44	0.96
338	22	32.0	9.66	9.93	10.69	10.60	10.68	10.81	10.38	10.07	1.16
339	22	32.5	9.56	9.51	10.60	10.50	10.29	10.68	10.18	9.86	1.17
340	22	33.0	9.62	9.26	10.14	10.44	10.01	10.22	10.03	9.59	1.18

Table 3: Continue

Point (ID)	Latitude (Deg.)	Longitude (Deg.)	Calculated (N) values (m) EGM96 (1)	EIGEN-CG01C (2)	GGM03C (3)	EIGEN-CG03C (4)	EIGEN-GL04C (5)	EIGEN-5C (6)	EGM 2008 (360) (7)	EGM 2008 (2190) (8)	Max. diff. (m)
341	22	33.5	9.75	8.85	9.66	9.96	9.78	9.55	9.80	9.59	1.11
342	22	34.0	10.73	9.47	10.26	10.31	10.53	10.23	10.22	9.91	1.26
343	22	34.5	11.24	9.71	10.54	10.30	10.72	10.72	10.37	10.21	1.54
344	22	35.0	10.98	9.36	10.14	9.89	10.41	10.58	10.42	9.93	1.63
345	22	35.5	9.94	8.53	9.37	9.24	9.47	9.51	9.63	9.85	1.42
346	22	36.0	8.87	7.61	8.56	8.57	8.43	8.25	8.46	7.87	1.27

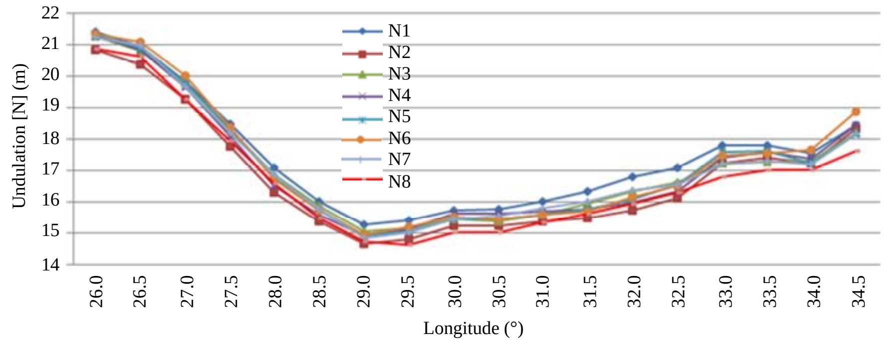

Fig. 6: (N) Values for latitude (ϕ) = 31°N

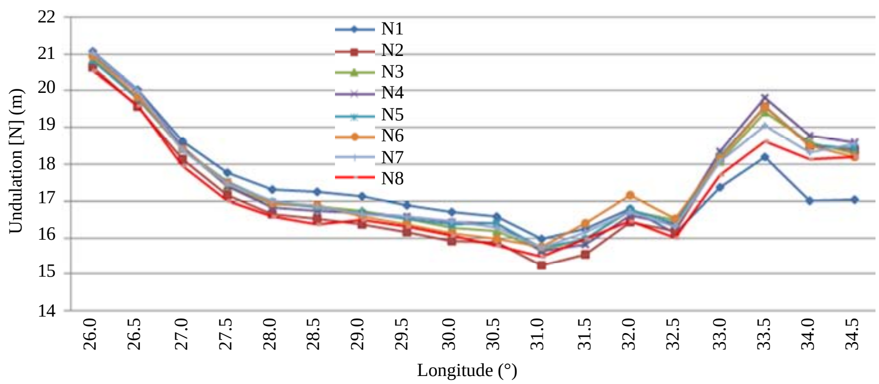

Fig. 7: (N) Values for latitude (ϕ) = 30°N

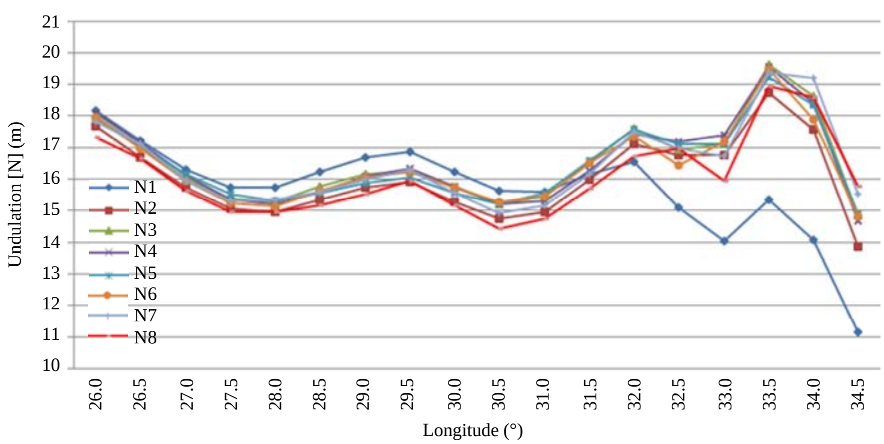

Fig. 8: (N) Values for latitude (ϕ) = 29°N

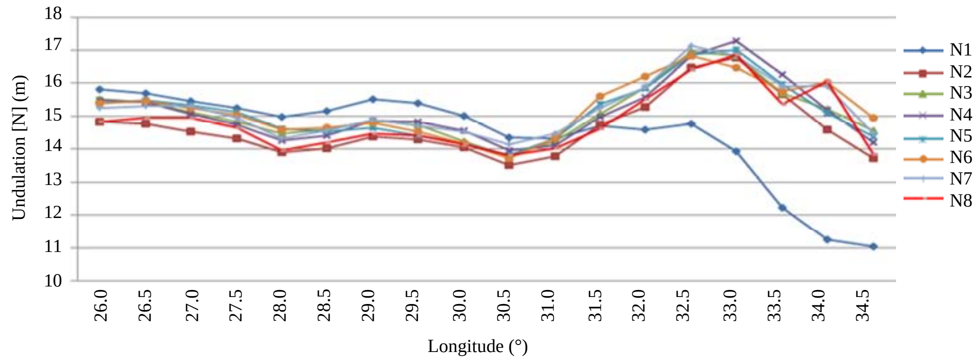

Fig. 9: (N) Values for latitude (ϕ) = 28°N

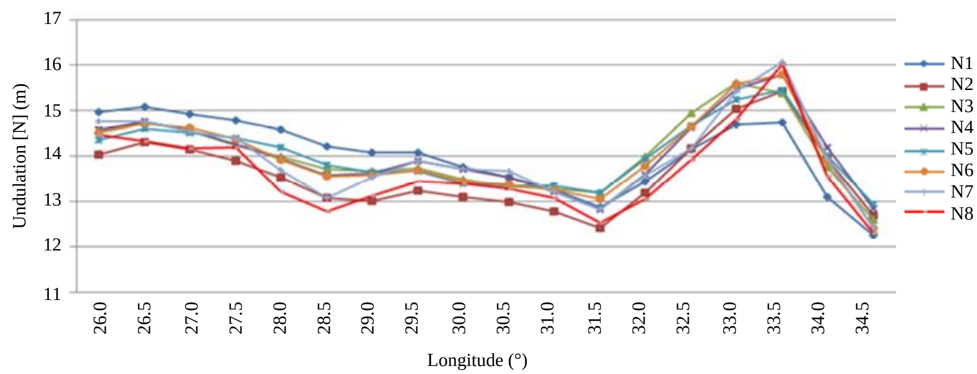

Fig. 10: (N) Values for latitude (ϕ) = 27°N

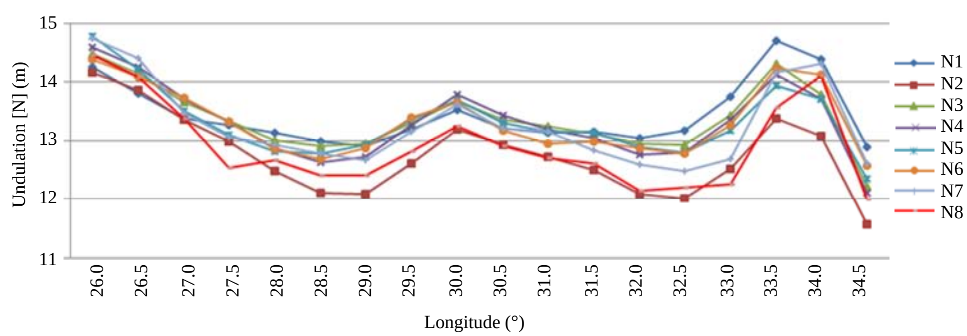

Fig. 11: (N) Values for latitude (ϕ) = 26°N

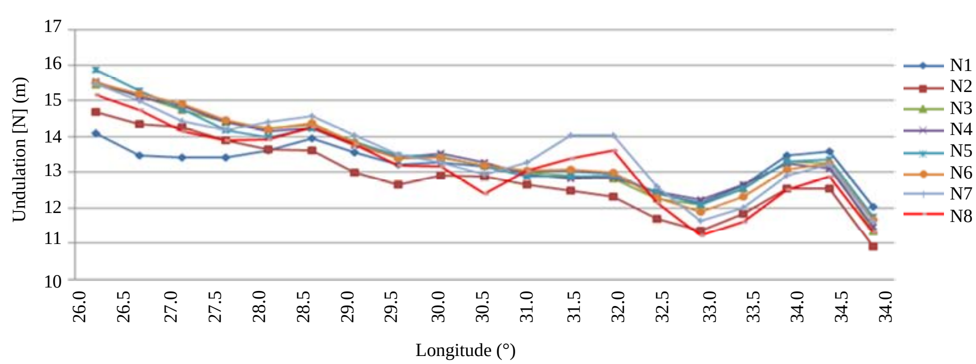

Fig. 12: (N) Values for latitude (ϕ) = 25°N

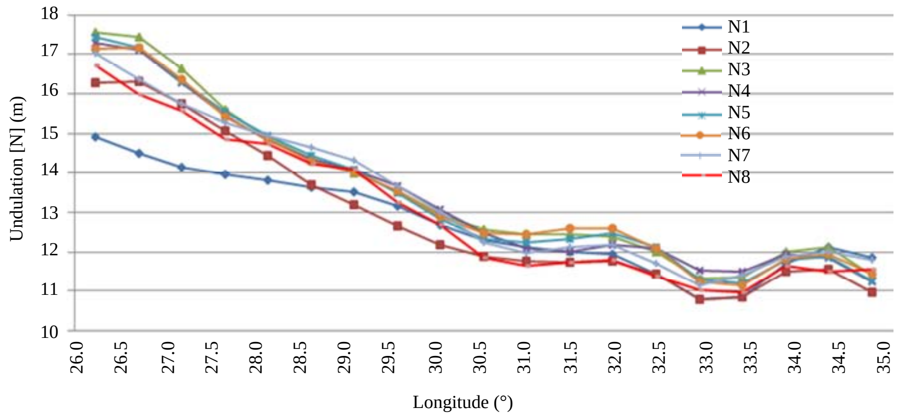

Fig. 13: (N) Values for latitude (ϕ) = 24°N

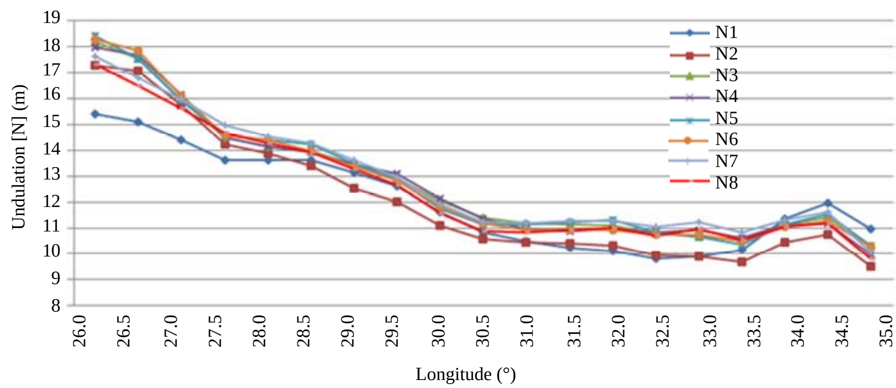

Fig. 14: (N) Values for latitude (ϕ) = 23°N

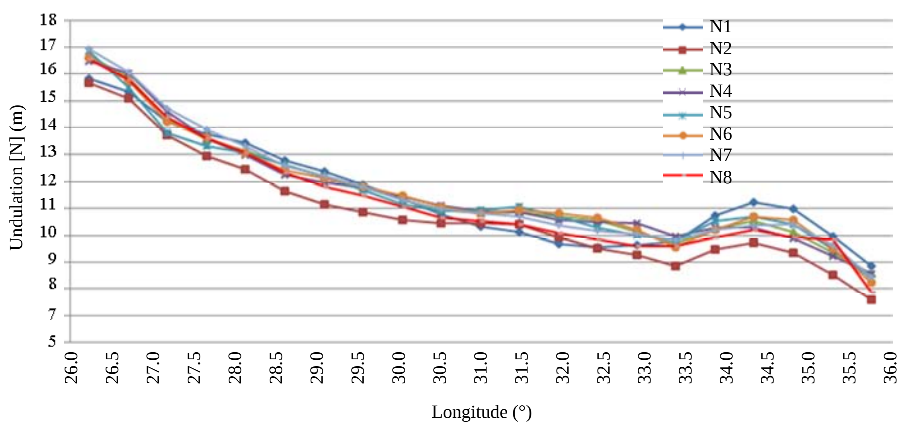

Fig. 15: (N) Values for latitude (ϕ) = 22°N

CONCLUSION

From the tables and figures it can be seen the following; the values of the undulation (N) as expected changes in Egypt from East to West and also from North to South. They have the maximum value = 21.47 m at point number 70 [29.5°N, 33.5°E] by using GGM03C and its minimum value = 7.23 m at point number 325 [22.5°N, 36°E] using EIGEN-CG01C.

Calculating (N) using the eight suggested GGM gave different values of (N). The differences ranges from 0.36-6.08 m. The maximum difference at point number 106

[28.5°N, 34°E]. This point located in the mountain chain beside Catherins Monastery in Sinai. On the other hand, the minimum difference at point number 180 [26°N, 27°E] which is located in semi-flat area named the Dune Corridor Camp in Western Desert.

The values of (N) and differences between these values calculated in North Delta of Egypt is relatively small, (N) ranges from 14.69 m (at point number 29) to 16.59 m (at point number 46) and the differences range from 0.67 (at point number 11) to 0.86 m (at point number 12).

The values of (N) in South of Egypt decrease by about 7 m going from West to east of Egypt for example at latitude 22°N [N≈15.9 and 8.9 m East at points number 326 (longitude 26°E) and 346 (longitude 36°E), respectively] as shown in Table 3. The differences between (N) calculated using different GGM are relatively small [about 0.61 m at point number 336 and 1.63 at point number 344] with maximum value of about 3 m and the large differences between EGM 96 and EGM 2008.

The most popular two models in Egypt and around the world are using EGM 96 and EGM 2008 (2190). Generally, the differences between the maximum and minimum (N) values which calculated by different eight GGM for every point of 346 points over Egypt proved that; 197 points <1 m, 1 m<118 points <2 m and 31 points>2 m (up to the maximum difference = 6.08 m).

The larger differences are located at two zones; first zone between latitudes (27°N, 30°N) and longitudes (32°E, 34.5°E) with maximum difference equal 6.08 m at point number 106 (28.5°N, 34°E). Second zone is between latitudes (23°N and 25°N) and longitudes (26°E and 28°E).

RECOMMENDATIONS

The future recommendation of this paper is to study more deeply the main reasons of significant differences values between these eight GGM based on the location of different points and also study a relationship to convert the (N) values from one model to another in Egypt.

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My special thanks and deep gratitude should be expressed to Prof. Dr. Mahmoud El Nokrashy O. Ali (Prof. of surveying, Civil Engineering Department, College of Engineering, Al-Azhar University, Cairo, Egypt) for his advices, discussions and valuable remarks.

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