Impact of Poisson Parameter of Impulsive Multiplicative Noise on Impairment of Single-Tone Sinusoid Signals

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Abstract: Performance of FFT frequency estimation of single tones based instantaneous frequency estimators for noisy single-tone sinusoid signal is investigated under impulsive multiplicative noise with different value of Poisson parameter as well as comparative study on the performance of the FFT frequency estimation methods under impulsive, Gaussian and uniform multiplicative noise with different values of power is presented. This study presents simulation of impulsive noise and analysis study of its damage effect on FM signalin time domain. Effect of Poisson parameter in damage effect of impulsive noise on FM signal has been investigated. Simulation results show that impulsive noise has less impairment effect at high value of Poisson parameter. FFT frequency estimation method is the best estimator under impulsive multiplicative noise in terms of minimum mean squared estimation error, especially, at high value of Poisson parameter.

Key words: Multiplicative nois, impulsive noise, FM signal, Poisson parameter, AWGN, noise

INTRODUCTION

In wireless communication systems common kinds of noise is impulsive noise which is modelled as Poisson-Gaussian Model (Ghadimi *et al.*, 2012). Source of impulsive noise is human-made (physical or industrial) source such as mechanical switches, even light switches, power lines (Ghadimi *et al.*, 2012; Gurubelli *et al.*, 2014). Unlike additive noise which effect amplitude of signal, multiplicative noise power is affecting both the amplitude and phase of the signal, therefore, it can be considered as most severedegradation noise that affects signal and system (Tuzlukov, 2002). There are many type of multiplicative noise; impulsive noise is one of these types that damages signal and weak performance and reliability of system, especially, image and communication system in spite of a high signal-to-noise ratio (Tuzlukov, 2002; Al-Mawali *et al.*, 2010). Impulsive noise has very high amplitude (100 mV) and short duration, so that, it causes great impairments and high error rate during transmission data in Power Line Communication system (PLC) according to these features where noise in network is modelled as background Gaussian noise and Poisson-Gaussian noise (impulsive noise) (Ghadimi *et al.*, 2012; Al-Mawali, 2011). Signals in most important applications such as communication, biomedical, sonar and radar are nonstationary signals (Konig and Bohme, 1996). Frequency Modelled (FM) signals are nonstationary signal which their frequency content is varying with timeand defined by Instantaneous Frequency (IF), therefore, IF estimation of signal under noise is considered major problem in important fields (Cohen, 1995).

Liu *et al.* (2016) proposed estimation methods to estimate carrier frequency of signal under impulsive noise, this proposal method exceed the conventional DFT method.

Gurubelli *et al.* (2014) proposed effective and robust frequency estimator which estimated exact frequency of signal under impulsive noise without iteration search.

Few literatures focus on multiplicative noise, especially when it modelled as impulse process.This study analyses the effects of impulsive multiplicative noise on FM signals.

Problem formulation: In this study the noisy signal can be modelled as:

$$y(t) = n(t)\sin\left[\phi(t)\right] + \in(t) \tag{1}$$

Where:

n(t)	=	Multiplicative noise
$\phi(t)$	=	The initial phase
$\in(t)$	=	An Additive White Gaussian Noise (AWGN) with zero mean and variance
σ_a^2	=	With n(t) and $\in(t)$ considered as independent processes

First, consider n(t) as white Gaussian noise with zero mean and variance σ_m^2. And then n(t) consider as impulsive noise with Poisson-Gaussian Model. Linear Frequency Modulation (LFM) law of signal model in Eq. 1 has been modelled as (Boashash *et al.*, 1990):

$$s(t) = e^{\left\{j2\pi\left(f_0 t + \frac{a}{2}t^2\right)\right\}} \tag{2}$$

Where:
α = The linear modulation index
f_0 = The initial frequency (Hz)

In this study, Gaussian, impulsive and uniform models have been considered. Since, impulsive noise sources are human-mansources environment, therefore, it non Gaussian Model, so, it can be modelled as Poisson-Gaussian noise (Gurubelli *et al.*, 2014) as:

$$i_k = b_k \cdot g_k \qquad (3)$$

where, b_k Poisson process that is modeling the arrival time of impulsive noise at instant k with parameter λ which denote the rate of unit per second. A random variable X is said to be Poisson, if its PDF is given by Hogg and Craig (1978):

$$P(X = x) = e - \lambda \frac{\lambda^x}{x!}; x = 1, 1, 2, \dots, \qquad (4)$$

$$\in \{x\} = \lambda; \operatorname{var}(x) = \lambda$$

where, $P(X = x)$ Is the probability of event of arrivals in unit time, thereby when represents the time count of arrival of impulsive noise, then it distributed with above Poisson PDF.

And g_k is Gaussian process that is used to model the amplitude of impulsive noise with zero mean and variance (power) σ^2, so, the total power of impulsive noise is (Al-Mawali *et al.*, 2010):

$$n_p = \frac{\sigma^2}{\lambda} \qquad (5)$$

Gaussian process can be simulated by using Probability Density Function (PDF) with zero mean and variance (power) σ^2 as follows (Hussain *et al.*, 2011):

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} e^{-n^2/2\sigma^2} \qquad (6)$$

In this study, uniform noise also consider by using its Probability Density Function (PDF) as:

$$p(x) = \frac{1}{b-a}; -\infty < a < b < \infty$$

The mean and variance of this distribution are given by:

$$\mu = \in \{x\} = \frac{a+b}{2}; \operatorname{var} = \in \{X-\mu^2\} = \frac{(b-a)^2}{12}$$

MATERIALS AND METHODS

Estimation methods: In this study, Maximum Likelihood (ML) estimator using Fast Fourier Transform (FFT) within terpolated peak estimation based frequency estimation is used to estimate frequency of noisy signal under three kinds of multiplicative noise (Gaussian, uniform and impulsive) where estimated frequency can be given by the peak of the Fourier transform occurs (Rife and Boorstyn, 1974):

$$f_{ML} = \arg(\max |X(f)|) \qquad (7)$$

where, f_{ML} was estimated frequency, arg return the index of peak of X(f) and X(f) is the Fourier transform of the single-tone signal X(t), computed from the sampled version of the input signal X(n) of length N by the DFT as:

$$X(k) = \frac{1}{\sqrt{n}} \sum_{n=0}^{N-1} x(n) e^{\left(\frac{2\pi kn}{n}\right)}; \ 0 \le k \le N \qquad (8)$$

Signal to Noise Ratio (SNR) of noisy FM signal: Since, the multiplicative noise power is affecting both the amplitude and phase of the signal, the Signal-to-Noise Ratio (SNR) of noisy FM under effect of both AWGN and MN is defined as follows:

$$P_{mn} = (p_n + p_m), \text{then } SNR = \frac{p_x}{p_{mn}} \qquad (9)$$

Where:
p_x = The signal power
p_m = The MN power
p_n = The additive noise power

The relative squared-error under each SNR and MN power is calculated as follow:

$$e = |((F_0 - f_0)/f_0)|^2 \qquad (10)$$

Where:
F_0 = Estimated Frequency
f_0 = Actual frequency of noiseless signal

Simulation hypothesis: We simulated the above algorithms with signal model under Multiplicative Noise (MN) as per Eq.1 using MATLAB. Linear Frequency Modulation (LFM) signal is simulated as follows:

$$y(t) = n(t)\sin\left\{j2\pi\left(f_0 t + \frac{a}{2}t^2\right)\right\} + \in(t) \qquad (11)$$

Where:

n(t) = Multiplicative zero-mean noise

$\in$(t) = Additive White Gaussian Noise (AWGN) with zero mean

α = Slope of IF low of the signal

The simulated signal has total time length L = 10 sec, the sampling interval is T = 0.001 sec and the number of samples is given by N = $[L/T_s]$. The signal amplitude is A = 1 V, ω_0 is angler frequency $\omega_0 = 2\pi f_0$ where f_0 = 23 Hz. First, MN has been modelled as zero-mean Gaussian, then it has been modelled as impulsive processesas per Eq. 3 and 4 with Poisson parameter λ = 20. Monte Carlo simulations were performed with M = 20 realizations.

RESULTS AND DISCUSSION

From simulation results of implementing Maximum Likelihood (ML) estimator, using Fast Fourier Transform (FFT) within terpolated peak estimation using complex single-tone sinusoid affected by additive Gaussian and zero mean multiplicative in MATLAB. Comparative study of above estimation method performance in term of Mean Square Error (MSE) using three models of multiplicative noise (Gaussian, impulsive and uniform) is present with different value of Poisson parameter as shown in Fig. 13-18.

Effect of impulsive noise on FM signal in time domain with different value of arrival time is shown as in Fig. 1-12. Figure 1-4 show one realization of impulsive noise with different value of λ (Poisson parameter) in time domain. Figure 5-8 show noisy FM signal under additive Gaussian noise and multiplicative impulsive noise with different value of λ (Poisson parameter) in time domain. Figure 9 and 10 show one realization of Gaussian noise at power -3 and 3 dB, respectively. Figure 11 and 12 show noisy FM signal under multiplicative Gaussian noise at power = -3 and 3 dB, respectively in time domain.

It is clear that impulsive noise has less destructive effect at high value of λ. Since, impulsive noise is modelled as Poisson-Gaussian representation, it clear from results in Fig. 1-4 that amplitude of impulsive noise increase and be more damage at high power of Gaussian noise. Nevertheless, at low value of λ and high power of Gaussian noise the impulsive noise effect approaches to the Gaussian noise as evidenced by simulation results in Fig. 1-12.

Figure 13-15 show the estimated frequency versus SNR using interpolated FT peak for various multiplicative noise models (Gaussian, impulsive and uniform) with
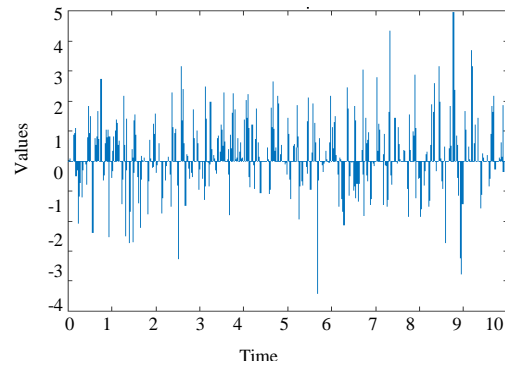

Fig. 1: Time domain representation of one realization of impulsive noise with λ = 5; One realization of impulsive noise with Poisson parameter a = 5 and Gaussian noise power = -3 dB
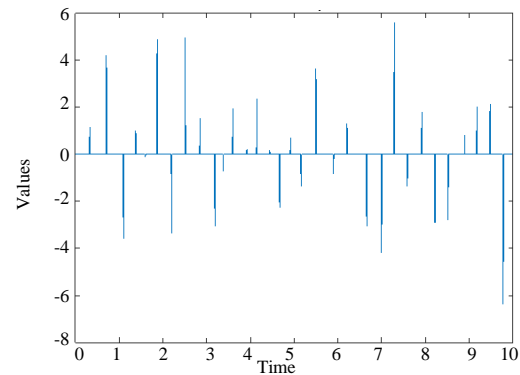

Fig. 2: Time domain representation of one realization of impulsive noise with λ = 30; One realization of impulsive noise with Poisson parameter a = 30 and Gaussian noise power = -3
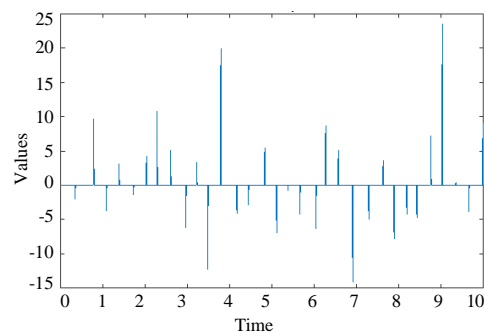

Fig. 3: Time domain representation of one realization of impulsive noise with λ = 30; One realization of impulsive noise with Poisson parameter a =30 and Gaussian noise power = 3 dB

different powers and λ = 30. It is clear that FT method hold at SNR = 0 dB in case of impulsive noise with p_m = 30 while in case of uniform and Gaussian noise it hold at SNR more than 10 dB.
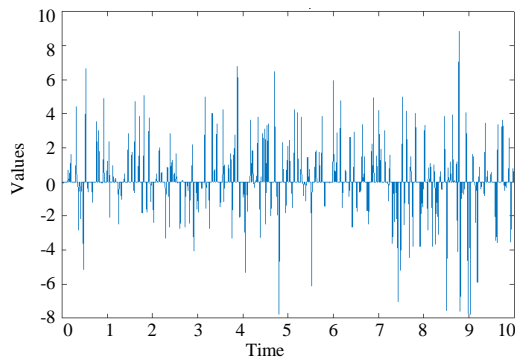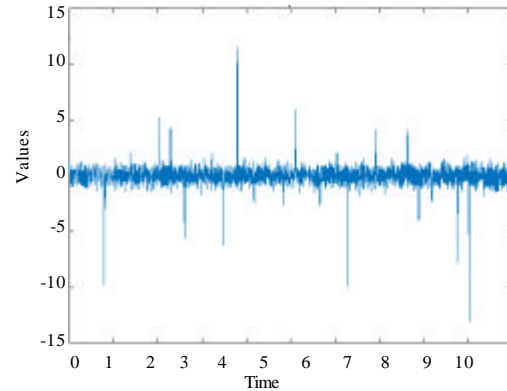
Fig. 4: Time domain representation of one realization of impulsive noise with λ = 5; One realization of impulsive noise with Poisson parameter a = 5 and Gaussian noise power = 3 dB

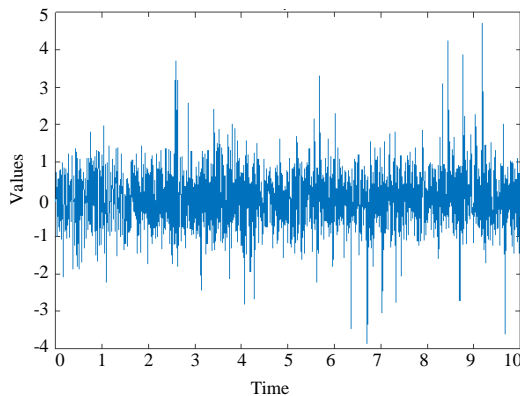
Fig. 5: Noisy FM signal under impulsive noise with λ = 5 and Gaussian noise power = -3; Noisy FM signal under impulsive noise with Poisson parameter a = 5 and AWGN power = -3 dB
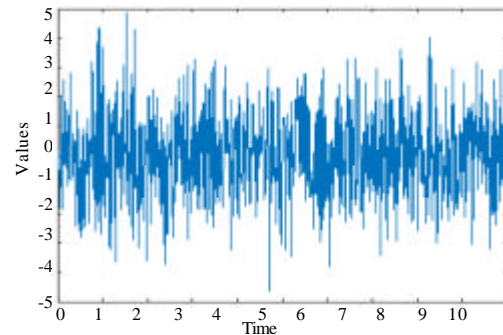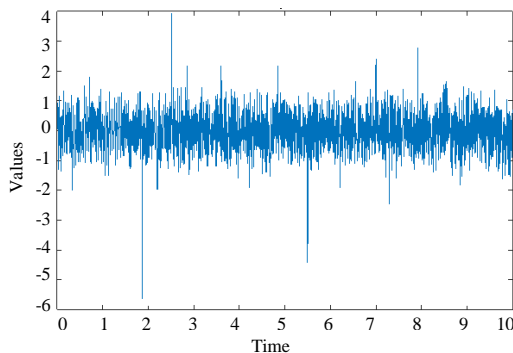

Fig. 6: Noisy FM signal under impulsive noise with λ = 30 and Gaussian noise power = -3; Noisy FM signal under impulsive noise with Poisson parameter a = 30 and AWGN power = -3 dB

Fig. 7: Noisy FM signal under impulsive noise with λ = 30 and Gaussian noise power = 3 dB; Noisy FM signal under impulsive noise with Poisson parameter a = 30 and AWGN power = 3 dB

Fig. 8: Noisy FM signal under impulsive noise with λ = 5 and Gaussian noise power = 3; Noisy FM signal under impulsive noise with Poisson parameter a = 5 and AWGN power = 3 dB
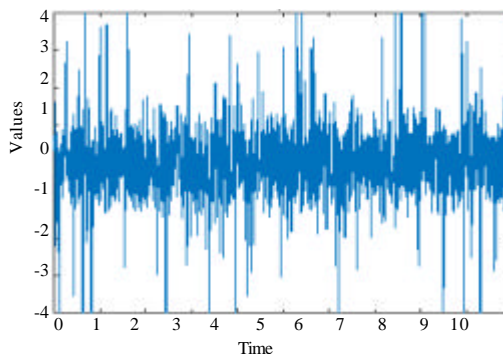

Fig. 9: Time domain representation of one realization of Gaussian noise at power = -3 dB different powers and λ = 30; One realization of Gaussian noise at powe r = -3 dB

Figure 16-18 show the estimated frequency versus SNR using interpolated FT peak for various multiplicative noise models (Gaussian, impulsive and uniform) with

Note that, under impulsive noise better frequency estimate is obtained at lower SNR under the same
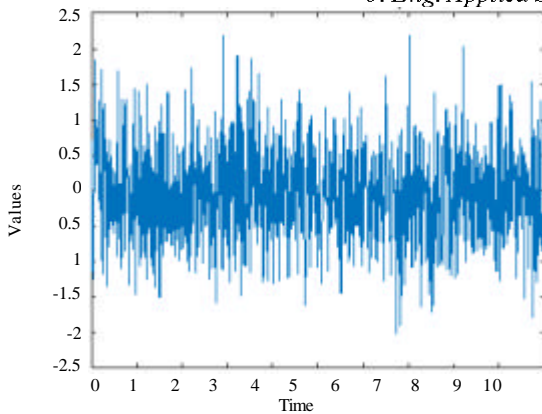
Fig. 10: Time domain representation of one realization of Gaussian noise at power = 3 dB; One realization of Gaussian noise at power = -3 dB
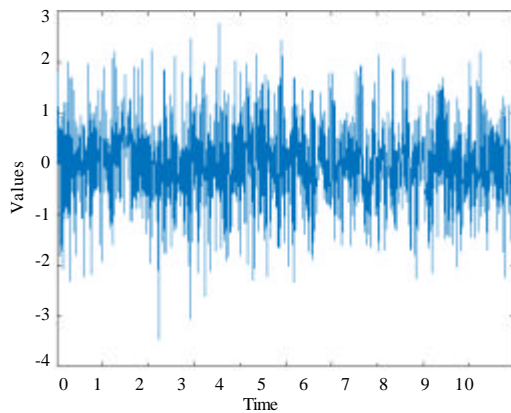

Fig. 11: Noisy FM signal under Gaussian noise at power = -3 dB; Noisy FM signal under Gaussian noise at power = -3 dB
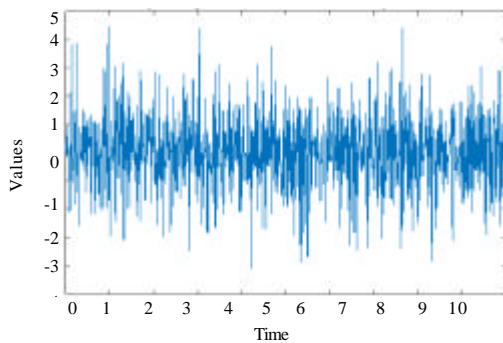

Fig. 12: Noisy FM signal under Gaussian noise at power = 3dB; Noisy FM signal under Gaussian noise at power = 3 dB

multiplicative noise power especially at high value of Poisson parameter (low value of arrival time of impulse), high signal-to-noise ratios and $p_m = 30$.
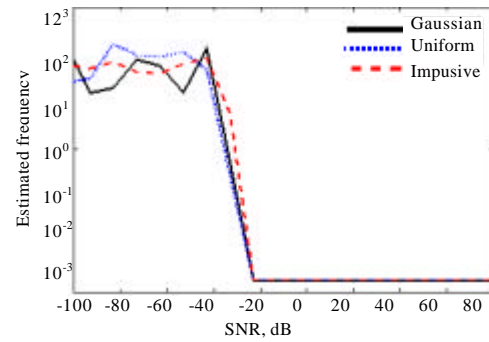

Fig. 13: Interpolated FT peak method for various multiplicative noise models with power = -3 dB and $\lambda = 30$; Ft est. freq. vs. SNR with multip. noise; pm (dB) = -30: N = 50001
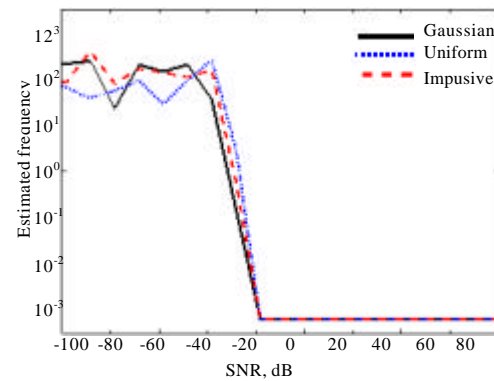

Fig. 14: Interpolated FT peak method for various multiplicative noise models with power = 0 dB and $\lambda = 30$; Ft est. freq. vs. SNR with multip. noise; pm (dB) = 0: N = 50001
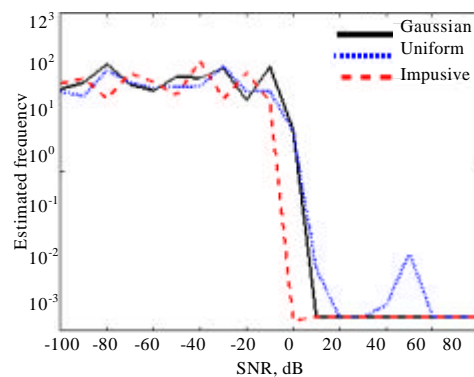

Fig. 15: Interpolated FT peak method for various multiplicative noise models with power = 30 dB and $\lambda = 30$; Ft est. freq. vs. SNR with multip. noise; pm (dB) = 30: N = 50001
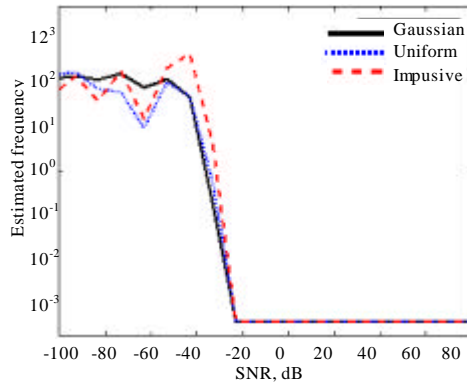
Fig. 16: Interpolated FT peak method for various multiplicative noise models with power = -30dB and λ = 3; Ft est. Freq. vs. SNR with multip. noise; pm (dB) = -30: N = 50001

Fig. 17: Interpolated FT peak method for various multiplicative noise models with power = 0 dB and λ = 3; Ft est. freq. vs. SNR with multip. noise; pm (dB) = 0: N = 50001

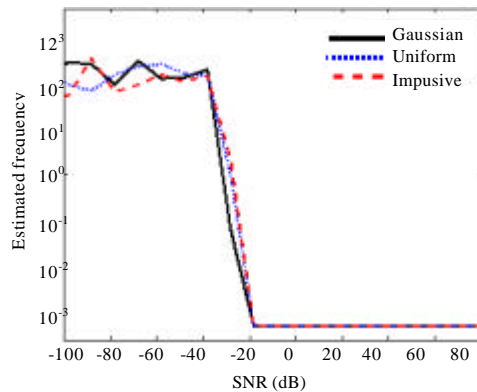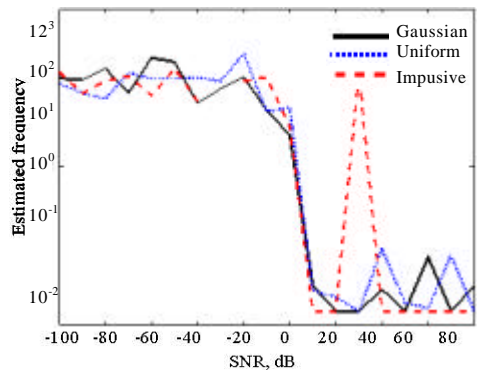Fig. 18: Interpolated FT peak method for various multiplicative noise models with power = 30 dB and λ = 3; Ft est. freq. vs. SNR with multip. noise; pm (dB) = 30: N = 50001

CONCLUSION

This study presented a study on the impairment effect of Impulsive Multiplicative Noise (MN). Poisson parameter of impulsive noise is important factor that effect on damaged effect of impulsive noise on signals as well as on performance of frequency estimation method where the low value of Poisson parameter increase the severe effect of noise and damage of signal, thereby impulsive noise effect be closed to Gaussian noise. Impulsive noise is less destructive than Gaussian or uniform noise with the same power and high value of Poisson parameter.

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REFERENCES

Al-Mawali, K., 2011. Techniques for broadband power line communications: Impulsive noise mitigation and adaptive modulation. Ph.D Thesis, RMIT University, Melbourne, Australia.

Al-Mawali, K.S., F.S. Al-Qahtani and Z.M. Hussain, 2010. Adaptive power loading for OFDM-based power line communications impaired by impulsive noise. Proceedings of the 2010 IEEE International Symposium on Power Line Communications and Its Applications (ISPLC'10), March 28-31, 2010, IEEE, Rio de Janeiro, Brazil, ISBN:978-1-4244-5009-1, pp: 178-182.

Boashash, B., P. O'Shea and M.J. Arnold, 1990. Algorithms for instantaneous frequency estimation: A comparative study. Proceedings of the Olukomaiya34th Annual International Technical Symposium on Optical and Optoelectronic Applied Science and Engineering Vol. 1348, November 1, 1990, International Society for Optics and Photonics, San Diego, California, USA., pp: 126-149.

Cohen, L., 1995. Time-Frequency Analysis. Prentice Hall, Upper Saddle River, New Jersey, USA., ISBN:9780135945322, Pages: 299.

Ghadimi, S., J. Hussian, T.S. Sidhu and S. Primak, 2012. Effect of impulse noise on wireless relay channel. Wirel. Sens. Netw., 4: 167-172.

Gurubelli, Y.R., S. Koneti, U. Bollampalli, S. Kolloju and S.R. Vempati *et al.*, 2014. Robust frequency estimation of sampled sinusoidal signals with impulsive noise. Proceedings of the 5th International Conference on Intelligent Systems, Modelling and Simulation (ISMS'14), January 27-29, 2014, IEEE, Langkawi, Malaysia, ISBN:978-1-4799-3858-2, pp: 704-707.

Hogg, R.V. and A.T. Craig, 1978. Introduction to Mathmatical Statistics. 4th Edn., Macmillan Publishers, Basingstoke, England, UK., ISBN:9780023557101, Pages: 438.

Hussain Z.M., A.Z. Sadik and P. O'Shea, 2011. Digital Signal Processing: An Introduction with MATLAB and Applications. Springer, Berlin, Germany, ISBN:978-3-642-15590-1, Pages: 350.

Konig, D. and J.F. Bohme, 1996. Wigher-ville spectral analysis of automotive signals captured at knock. Appl. Sig. Process., 3: 54-64.

Liu, Y., Y. Tie, S. Na and S. Tan, 2016. Carrier frequency estimation with cyclostationary signals in impulsive noise. Intl. J. Sig. Image Process. Pattern Recognit., 9: 89-102.

Rife, D. and R. Boorstyn, 1974. Single tone parameter estimation from discrete-time observations. IEEE. Trans. Inf. Theor., 20: 591-598.

Tuzlukov, V.P., 2002. Signal Processing Noise. CRC Press, Boca Raton, Florida, USA., Pages: 662