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Augmented Desirability Function for Multiple Responses with Contaminated Data

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Abstract: Quality engineering practitioners have great interest for using response surface method in a real situation. Recently, robust design has been widely used extensively for multiple responses in terms of the process location and process scale based on sample mean and sample variance, respectively. One of the methods that can be used to simultaneously, optimize multiple responses is by using the Augmented Approach to the Harrington's Desirability Function (AADF) technique by assigning weight to the location and scale in order to see the reflection the relative importance for both effects. In this technique, the AADF approach uses a dimensionality reduction approach that converts multiple predicted responses into a single response problem. Furthermore , for the regression fitting second-order polynomials model, the Ordinary Least Squares (OLS) method is usually used to acquire the sufficient response functions for the process location and scale based on mean and variance. Nevertheless, these existing procedures are easily influenced by outliers. As an alternative, we propose the uses of higher-order estimation techniques for robust MM-location, MM-scale estimator and MM regression estimator to overcome the weakness and shortcomings. The numerical results signify that the proposed approach is more efficient than the existing methods.

Key words: Augmented desirability function, higher-order estimation, MM-location, MM-scale, outlier, robust design

INTRODUCTION

Quality engineers are always searching for new idea to enhance productivity and decrease the cost of operation but at the same time maintaining the products quality. However, it can be more challenging if the products/processes have more than one quality characteristics. Most of the multi-response optimization method integrates with the Response Surface Methodology (RSM) concept to achieve the quality of product. RSM first introduced by Box and Wilson (1951) is an important tool to find the relationship between the several input variable with a response, then the optimal factor settings of design point are obtained which is can be classified into three types of quality characteristics of the response: smaller-the-better, larger-the-better and nominal-the-best . These terms are refer to for example, the larger-the-better and the smaller-the-better where the problem either to maximize or minimize the response, respectively while for nominal-the-best problem, the objectives that wish to achieve a value of desired target as possible.

In the 1980s, Taguchi (1986) was first proposed Robust Design (RD) methods to solve multi-response problems in order to improve product quality. The concept of robust design is to determine the best overall combination of optimal factor settings by minimizing signal-to-noise ratios and identifying adjustment factors which are used to tune a mean to desired target. However, several reseachers noted a few drawbacks embodied into Taguchi's approach in robust design (Easterling,1985; Vining and Myers, 1990; Myers *et al.*, 1992). As a result, many research efforts have been made to rectify these weaknesses.

Myers and Carter (1973) first introduced dual response surface approach and then were extended by Vining and Myers (1990) whereby the response functions are separately model and simultaneously optimize the process mean and variance to achieve the desired target while keeping the variance small. A quadratic (second order) polynomial model is widely incorporated to model the process location and process scale of the response variable. Along with the models parameters are usually estimated using the

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Ordinary Least Squares (OLS) method. The second-order model response function which is usually used is as follows:

$$\hat{\omega}_{\mu}(x) = \hat{\beta}_0 + \sum_{i=1}^{k} \hat{\beta}_i x_i + \sum_{i=1}^{k}\sum_{j=1}^{k} \hat{\beta}_{ij} x_i x_j \quad (1)$$

and:

$$\hat{\omega}_{\sigma}(x) = \hat{\gamma}_0 + \sum_{i=1}^{k} \hat{\gamma}_i x_i + \sum_{i=1}^{k}\sum_{j=1}^{k} \hat{\gamma}_{ij} x_i x_j \quad (2)$$

where •$_{\mu}$(x) and •.(x) are the predicted response surface model for the sample location and scale at each design point, respectively. Numerous researchers have developed extensions of these methods, contributing to the breadth of knowledge in the field (Castillo and Montgomery, 1993; Lin and Tu, 1995; Copeland and Nelson, 1996; Baba *et al.*, 2015). Later, Goethals and Cho (2012) introduced higher-order model response functions by combining the methodology of robust design for considering process variability. A higher-order model contains linear effect, cross product factor, second-order quadratic terms and all possible interaction between linear effect and second-order quadratic terms. Then, the best subset of terms for modeling is determined by using several number of evaluation criteria to find best model estimation.

Within the last decade, various new methods and techniques have been introduced through literature to solve multi-response problems that take into account the location and scale effect. Chiao and Hamada (2001) proposed an optimization scheme to simultaneously optimize correlated multiple responses that met respective specifications. Peterson (2004) developed optimization method by incorporated the variance-covariance structure data with the model parameter uncertainty based on Bayesian reliability technique. Lee and Kim (2007) suggested took the average of the existing desirability values on the basis of the probability distribution of the predicted response using expected desirability function approach. Nevertheless, most of the existing methods have its own shortcoming. Recently, Chen *et al.* (2013) proposed a natural extension of desirability function to optimize the multiple responses by imposing relative weights on process location and scale to reflects the relative importance for both effects. The performance of this method was reported to be more effective, compared to the traditional approach.

The traditional approach gives good parameter estimates and accurate optimal settings when the responses are normally, distributed and no outliers in the data sets. Often, however in real situations many distributions of response variable are (considerably) not normal which is due to the presence of outliers. If this assumption is violated in serious manner, the optimum response is not reliable as it is based on traditional approach which is not resistant to outliers. Thus, a new approach needs to be proposed.

The aim of this study is to propose using robust location and scale estimator namely MM-estimator introduced by Yohai (1987) which is more resistant to departures of outliers compared to classical mean and variance. Since, the OLS is not resistant to outliers, we suggest using alternative robust MM-estimator with higher order model which has a very high efficiency to estimate the parameters of the process location and scale.

MATERIALS AND METHODS

Robust location and scale: Let Y_{ij} represents the jth response at the jth design point where i = 1, 2, ..., n and j = 1, 2, ..., m. Suppose that replicates are taken at each of the design points. The most popular estimators of the location and scale parameters are mean and variance, respectively. At the design point, we have the sample mean and sample variance as follows:

$$\bar{Y} = \frac{1}{m}\sum_{j=1}^{m} Y_{ij} \text{ and } S_i^2 = \frac{1}{m\text{-}1}\sum_{j=1}^{m}\left(Y_{ij}\text{-}\bar{Y}\right)^2 \quad (3)$$

These estimators are known to be easily affected by outliers. In other words, replacing one out observations with large value can negatively affect the value of the sample mean and variance.

Tukey (1960) pointed out that this estimator can be heavily influenced by any single outlier for example, if Y_i goes to ±•, then $\bar{Y}$ goes to ±•. Since, the resulting optimum responses are inefficiently determined by the sample variance and mean in this study proposes the use of another outlier-resistant estimator for estimating the location and scale of the response value. This estimator was proposed by Yohai (1987) and was called as the MM-estimator. It is not only highly efficient and robust but it also has high breakdown property. In addition, the MM-estimator refers to the fact that more than one M-estimation procedures are used to calculate the final estimates. Consider that the following location-scale model: let x_1, x_2, ..., x_n be observation on the real line satisfying:

$$x_i = \mu+\sigma\varepsilon_i \quad (4)$$

This is the model where •$_i$, i = 1, 2, ..., n is independent and known as identically distributed (i.i.d) observation with variance = 1. The interest of the model is in estimating μ and the scale •.

In this research, Yohai (1987) MM-estimator was adapted to estimate the MM-location and MM-scale. The procedures for finding MM-estimator are summarized as follows:

Step 1: The initial consistent estimator of the location μ_0 and scale •$_0$ was computed with 50% possibility of high breakdown point which is S-estimate introduced by Yohai (1987).

Step 2: An M-estimation of the scale of the residuals from the initial S-estimates was computed.

Step 3: An M-estimation of the location and scale as in Eq. 5 was computed where • functioned as a very small (often zero) weight to sufficiently large residuals:

$$\sum_{i=1}^{n}\psi\left(\frac{x_i-\mu_0}{\sigma_0}\right)=0 \quad (5)$$

Modeling robust location and scale effects: The second-order model polynomial models are not always sufficed for estimation due to considering variance response for multiple response surface design. Then, the estimation parameters for higher-order model response surface designs have been proposed by Goethals and Cho (2012). For each response surface design, the combinations of terms which are significant to the regression are remained for further analysis. Several evaluation criteria will be used in order to identify which subset of terms is the best combination. Three usual criteria will be applied to analyze the model with • parameters which are the coefficient of determination R^2, the adjusted coefficient of determination R^2_{adj} and the Mean Square Error MSE,. The best model will be selected based on highest value R^2 and R^2_{adj} with the smallest MSE.

In practice, the fitted responses for higher-order models mentioned above are often estimated by the OLS method. Data analysis based on the least squares estimator is less efficient and not reliable when outliers are present in the data (Riazoshams *et al*., 2010). To remedy this problem, robust regression technique has been considered to dampen the effects of the outliers. In this study, the MM-regression estimator was used to estimate the parameters of the model location and scale instead of the OLS method.

Robust location and scale desirability function: The desirability function was first introduced by Derringer and Suich (1980) is one of the most widely used methods in optimizing multiple responses. Recently, Chen *et al*. (2013) transformed each standard deviation into an individual desirability function and namely as Augmented Approach to the Harrington's Desirability Function (AADF). In this study, AADF has been used to transform the higher-order fitted models to individual MM-scale and MM-location, d_{si} and $d_{\mu i}$, respectively. For the MM-location effect, quality characteristics can be classified into three types of desirability functions: Smaller-The-Better (STB), Larger-The-Better (LTB) and Nominal-The-Best (NTB) depends on the objective function of the process mean whether to achieve the specified desired target maximize or minimize, respectively.

The individual desirability function for the NTB type when the response is maximized is defined as follows:

$$d_{\mu_i}=\begin{cases}\left(\dfrac{\hat{\omega}_\mu-\mu_{min}}{\tau_\mu-\mu_{min}}\right)^{r_1} & \text{for } \mu_{min}\le\hat{\omega}_\mu\le\tau_\mu\\ \left(\dfrac{\mu_{max}-\hat{\omega}_\mu}{\mu_{max}-\tau_\mu}\right)^{r_2} & \text{for } \tau_\mu\le\hat{\omega}_\mu\le\mu_{max}\end{cases} \quad (6)$$

where •$_\mu$ is the higher-order predicted from the location model for the response, μ_{max}, μ_{min} and •$_\mu$ are upper and lower limits and the target for the response •$_\mu$. The weights r_1>0 and r_2>0 are a user-selected shape parameter. By definition the individual desirability lies between 0-1, i.e.,0 (0• $d_{\mu i}$• 1), respectively. For the LTB and STB type, the desirability function is defined as in Eq. 7 and 8 respectively:

$$d_{\mu_i}=\begin{cases}1 & \text{for } \hat{\omega}_\mu\le\mu_{min}\\ \left(\dfrac{\mu_{max}-\hat{\omega}_\mu}{\mu_{max}-\mu_{min}}\right)^{r} & \text{for } \mu_{min}<\hat{\omega}_\mu<\mu_{max}\\ 0 & \text{for } \hat{\omega}_\mu\ge\mu_{max}\end{cases} \quad (7)$$

$$d_{\mu_i}=\begin{cases}0 & \text{for } \hat{\omega}_\mu\le\mu_{min}\\ \left(\dfrac{\hat{\omega}_\mu-\mu_{min}}{\mu_{max}-\mu_{min}}\right)^{r} & \text{for } \mu_{min}<\hat{\omega}_\mu<\mu_{max}\\ 1 & \text{for } \hat{\omega}_\mu\ge\mu_{max}\end{cases} \quad (8)$$

Next, the suitable individual location desirability function have been selected then, we combined all individual location desirability functions values d = ($d_{\mu 1}$,$d_{\mu 2}$, ...,$d_{\mu m}$) into an overall desirability function D using geometric mean as follows:

Table 1: Experimental design for the chemical filtration process

	Coded units			Quality characteristics of interest								
Run	Tempt X_1	Pressure X_2	Humidity X_3	Filtration time (sec) Y_1 (3 replications)			Filtration volume (mL) Y_2 (3 replications)			Filtration purity (%) Y_3 (3 replications)		
1	-1	-1	-1	3.86	4.03	3.92	9.70	9.79	9.73	93.09	92.99	93.03
2	1	-1	-1	3.12	3.07	3.02	9.96	9.95	9.93	93.76	93.83	93.81
3	-1	1	-1	2.82	2.79	2.87	9.94	9.96	9.97	94.33	94.35	94.30
4	1	1	-1	1.07	0.97	0.99	10.00	9.97	9.89	95.64	95.76	95.72
5	-1	-1	1	1.56	1.54	1.53	9.87	9.89	10.01	94.18	94.13	94.16
6	1	-1	1	0.54	0.52	0.58	10.10 **(20.00)**	10.04	10.03	96.31	96.23	96.27
7	-1	1	1	0.85	0.82	0.71	10.08	10.11	10.09	95.83	96.01	96.04
8	1	1	1	0.01	0.02	0.16	10.16	10.19	10.22	96.86	96.55	97.23
9	-1.682	0	0	1.30	1.26	1.32	9.78	9.87	10.01	93.59	93.73	93.76
10	1.682	0	0	2.07	2.14	2.11	10.02	10.15	9.92	94.94	94.88	94.90
11	0	-1.682	0	0.60	0.63	0.68	9.80	10.04	9.98	93.41	93.28	93.59
12	0	1.682	0	2.03	2.08	2.04	10.10	9.99	10.01	95.39	95.42	95.36
13	0	0	-1.682	2.12 **(10.0)**	1.79	2.16	10.12	10.01	9.86	94.37	95.17	94.64
14	0	0	1.682	2.80	2.52	2.42	10.10	9.97	9.85	95.36	95.63	94.99
15	0	0	0	2.19	2.02	2.14	10.08	9.99	10.13	95.76	94.93	95.43
16	0	0	0	1.96	1.77	2.19	9.98	10.11	9.78	94.12	94.20	95.13

Bold values when compared with proposed estimation approach

Table 2: Quality characteristics goals and specifications

Quality characteristics	Goal	Specifications	Target or acceptable region
Filtration time (sec) Y_1	Minimize	Y_1• 7	•$_1$ = 0
Filtration volume (mL) Y_2	Nominal (target)	9.5• Y_2• 10.5	•$_2$ = 10.0
Filtration purity (%) Y_3	Maximize	Y_3• 90	•$_3$ = 100

$$D = \left(d_{\mu1}.d_{\mu2}, ..., d_{\mu m}\right)^{1/m} \quad (9)$$

where 0• D• 1. The higher value D indicates a more desirable is the overall product and the high values of the d_is result in high value of D.

The STB type is considered as individual scale desirability, since, it is desirable to minimize the variation. The desirability function is defined as follows:

$$d_{\sigma_i} = \begin{cases} 1 & \text{for } \hat{\omega}_\sigma \le \sigma_{min} \\ \left(\dfrac{\sigma_{max} - \hat{\omega}_\sigma}{\sigma_{max} - \sigma_{min}}\right)^r & \text{for} \sigma_{min} < \hat{\omega}_\sigma < \sigma_{max} \\ 0 & \text{for } \hat{\omega}_\sigma \ge \sigma_{max} \end{cases} \quad (10)$$

where 0• d$_{•i}$• 1. Then, • . is the higher-order predicted from the scale model for the response, •$_{max}$ and •$_{min}$ are upper and lower limits and ith weights r>0 is a user-selected shape parameter. After that an overall dispersion desirability function is obtained by combining all individual scale desirability function into geometric mean which is defined as follows:

$$S = \left(d_{\sigma1}.d_{\sigma2}, ..., d_{\sigma m}\right)^{1/m} \quad (11)$$

where 0• S• 1, respectively.

The overall desirability function for location effects D and overall dispersion desirability function S are combining as defined as follows:

$$DS_\lambda = D^\lambda S^{1-\lambda} = \left(d_{\mu1}.d_{\mu2}, ..., d_{\lambda m}\right)^{\lambda/m}.\left(d_{\sigma1}.d_{\sigma2}, ..., d_{\sigma m}\right)^{1-\lambda/m} \quad (12)$$

where 0• • • 1 is a user-selected weight that reflects the relative importance of optimizing D and where 0• DS• 1.

RESULTS AND DISCUSSION

Numerical example: This example is taken from the case study performed by Kovach and Cho (2008). The aim of this experiment was to analyze the effects of the filtration time (Y_1) measured in seconds, the filtration volume (Y_2) measured in milliliters and the filtration purity (Y_3) measured as a percentage on the chemical filtration process based on temperature (X_1), pressure (X_2) and humidity (X_3). At three design points, the Central Composite Design (CCD) consisting 16 runs with three replicates were considered as shown in Table 1. Two contaminated data points (in bold) were observed. The associated target values, goals and specifications for the characteristics are shown in Table 2.

Since, unusual observations occurred in the data series, the outlier-resistant estimator was more suitable to be used in order to find the optimal operating conditions.

Table 3: Mean, variance, MM-location and MM-scale calculations

	Mean and variance calculations						MM-location and MM-scale calculations					
	Y_1		Y_2		Y_3		Y_1		Y_2		Y_3	
Run	$\bar{y}_1$	σ_1^2	$\bar{y}_2$	σ_2^2	$\bar{y}_3$	σ_3^2	MMl_1	MMs_1	MMl_2	MMs_2	MMl_3	MMs_3
1	3.94	0.0074	9.74	0.0021	93.04	0.0025	3.93	0.0041	9.74	0.0010	93.04	0.0018
2	3.07	0.0025	9.95	0.0002	93.80	0.0013	3.07	0.0028	9.95	0.0001	93.80	0.0005
3	2.83	0.0016	9.96	0.0002	94.33	0.0006	2.83	0.0010	9.96	0.0001	94.33	0.0005
4	1.01	0.0028	9.95	0.0032	95.71	0.0037	0.99	0.0005	9.96	0.0010	95.71	0.0018
5	1.54	0.0002	9.92	0.0057	94.16	0.0006	1.54	0.0001	9.88	0.0005	94.16	0.0005
	-	-	10.06	0.0014	-	-	-	-	10.04	0.0292	-	-
6	0.55	0.0009	-	-	96.27	0.0016	0.55	0.0005	-	-	96.27	0.0018
	-	-	**(13.36)**	**(33.1000)**	-	-	-	-	**(10.04)**	**(0.0293)**	-	-
7	0.79	0.0054	10.09	0.0002	95.96	0.0129	0.81	0.0010	10.09	0.0001	96.03	0.0010
8	0.06	0.0070	10.19	0.0009	96.88	0.1159	0.02	0.0001	10.19	0.0010	96.88	0.1097
9	1.29	0.0009	9.89	0.0134	93.69	0.0082	1.29	0.0005	9.88	0.0092	93.74	0.0010
10	2.11	0.0012	10.03	0.0133	94.91	0.0009	2.11	0.0010	10.03	0.0114	94.91	0.0005
11	0.64	0.0016	9.94	0.0156	93.43	0.0242	0.64	0.0010	9.95	0.0041	93.43	0.0193
12	2.05	0.0007	10.03	0.0034	95.39	0.0009	2.04	0.0001	10.01	0.0005	95.39	0.0651
13	2.02	0.0412	-	-	-	-	1.98	0.1563	-	-	-	-
			10.00	0.0170	94.73	0.1656	-	-	9.99	0.0138	94.72	0.0832
	(4.65)	**(21.5)**	-	-	-	-	**(1.98)**	**(1.624)**	-	-	-	-
14	2.58	0.0388	9.97	0.0156	95.33	0.1032	2.56	0.0114	9.97	0.0164	95.33	0.0832
15	2.12	0.0076	10.07	0.0050	95.37	0.1746	2.12	0.0029	10.07	0.0029	95.38	0.1243
16	1.97	0.0442	9.96	0.0276	94.48	0.3152	1.97	0.0412	9.96	0.0193	94.16	0.0073

Bold values when compared with proposed estimation approach

Table 4: Summary of model selection for location model

Parameters	Model	v	R^2_v (%)	$R^2_{adj,v}$ (%)	MSE_v (%)
$\bullet_{\mu 1}$	Traditional design (2nd order)	10	38.9	52.7	1.640
	Higher-order design (3rd order) based on sample mean	14	98.9	94.7	0.057
	Higher-order design (3rd order) based on MM-location	15	99.9	98.9	0.011
$\bullet_{\mu 2}$	Traditional design (2nd order)	10	71.8	29.5	0.007
	Higher-order design (3rd order) based on sample mean	8	94.3	89.3	0.001
	Higher-order design (3rd order) based on MM-location	9	94.3	87.8	0.001
$\bullet_{\mu 3}$	Traditional design (2nd order)	10	82.5	56.2	0.513
	Higher-order design (4th order) based on sample mean	10	96.2	90.5	0.111
	Higher-order design (4th order) based on MM-location	9	93.2	85.4	0.175

Then, the sample mean $\bar{Y}$, sample standard deviation?, MM-location estimator (MMl) and MM-scale estimator (Mms) were computed at design points shown in Table 3. These estimates were computed using R language. What is immediately clear form Table 3 is process location and process scale based on sample mean and sample variance are very sensitive to the contaminated data points denoted in bold when compared with the proposed estimation approach using MM-location and MM-scale.

In this example, the multiple characteristics were involved. The second-order model polynomial models will not suffice for estimation due to considering variance response for this problem (Goethals and Cho, 2012). Then, higher-order response surface designs are developed for sample mean, sample variance, MM-location and MM-scale measures. Using R-software, third and fourth order response surface are considered and only combinations of terms which are significant to the regression are remained for analysis. To identify the best model for approximating the samples mean, sample variance, MM-location and MM-scale, R^2_{adj} and has been used. The best model will be selected based on highest value R^2_{adj} with the smallest MSE_v.

For brevity, only the comparisons between higher-order and second-order model based on location mean is shown Table 4. Based upon the results in Table 4, it shows that for sample location based on sample mean and sample MM-location, the models are chosen with 14 (Y_1), 8 (Y_2) and 10 (Y_3) parameters and 15 (Y_1), 9 (Y_2) and 9 (Y_3), respectively, since, they keeping the highest values for R^2_{adj} and attain optimal values for the evaluation criteria. Based upon the results, it is suggested that higher-order model is more precise and efficient compared to second-order model, since, it's achieve the criteria value.

After that Table 5 is constructed to determine the factor settings which are maximize the composite desirability. Note that for brevity, optimization scheme for location and scale based on mean and variance without outlier is shown in Table 5.

Table 5: Optimization scheme for location and scale based on mean and variance without outlier

Maximize	$Ds. = D^{\bullet} S^{1-\bullet} = (d_{\mu 1}.\ d_{\mu 2}.\ d_{\mu 3})^{\bullet/3}.\ (d_{\bullet 1}.\ d_{\bullet 2}.\ d_{\bullet 3})^{1-\bullet/3}$
Satisfy	Constraint: The type of STB (Y_1), NTB (Y_2) and LTB (Y_3) for mean and MM-location response (see equations below)
Given	Higher-order fitted response surface functions (see below)
Find	Optimal factor settings $x^* = (x^*_1, x^*_2, x^*_3)$

Satisfy — Constraint:

The type of STB (Y_1), NTB (Y_2) and LTB (Y_3) for mean and MM-location response

$$d_{\hat{\omega}\mu 1} = \begin{cases} 0 & \text{if } \hat{\omega}_{\mu 1}(x) > 7 \\ \left(\dfrac{\hat{\omega}_{\mu 1}(x) - 7}{0 - 7}\right) & \text{if } 0 \le \hat{\omega}_{\mu 1}(x) \le 7 \end{cases}$$

$$d_{\hat{\omega}\mu 2} = \begin{cases} 0 & \text{if } \hat{\omega}_{\mu 2}(x) < 9.5 \text{ or } \hat{\omega}_{\mu 2}(x) > 10.5 \\ \left(\dfrac{\hat{\omega}_{\mu 2}(x) - 9.5}{10.0 - 9.5}\right) & \text{if } 9.5 \le \hat{\omega}_{\mu 2}(x) \le 10.0 \\ \left(\dfrac{\hat{\omega}_{\mu 2}(x) - 10.5}{10.0 - 10.5}\right) & \text{if } 10.0 \le \hat{\omega}_{\mu 2}(x) \le 10.5 \end{cases}$$

$$d_{\hat{\omega}\sigma i} = \begin{cases} 0 & \text{if } \hat{\omega}_{\mu 3}(x) > 90 \\ \left(\dfrac{\hat{\omega}_{\mu 3}(x) - 90}{90 - 100}\right) & \text{if } 90 \le \hat{\omega}_{\mu 3}(x) \le 100 \end{cases}$$

The type of STB for all three sample variance and MM-scale response

$$d_{\hat{\omega}\mu 3} = \begin{cases} 0 & \text{if } \hat{\omega}_{\mu 3}(x) > 90 \\ \left(\dfrac{\hat{\omega}_{\mu 3}(x) - 90}{90 - 100}\right) & \text{if } 90 \le \hat{\omega}_{\mu 3}(x) \le 100 \end{cases}$$

Given — Higher-order fitted response surface functions:

- $\bullet_{\mu 1}(x) = 2.0490 + 0.2438X_1 + 0.4191X_2 + 0.1665X_3 - 0.0863X_1X_2 + 0.1213X_2X_3 + 0.2413X_2X_3 - 0.1317X^2_1 - 0.2542X^2_2 + 0.0804X^2_3 - 0.9704X^2_1X_2 - 1.1552X^2_1X_3 - 0.7950X^2_2X_1 + 0.1513X_1X_2X_3$
- $\bullet_{\mu 1}(x) = 9.9948 + 0.0494X_1 + 0.0277X_2 - 0.0308X_1X_2 - 0.0127X^2_1 + 0.03809X^2_1X_2 + 0.0833X^2X_3 + 0.02167X_1X_2X_3$
- $\bullet_{\mu 1}(x) = 94.9775 + 0.3607X_1 + 0.6525X_2 + 0.1784X_3 - 0.2395X^2_1 - 0.2012X^2_2 + 0.6212X^2_1X_3 + 0.2864X^2_1X_1 + 0.4802X^2_1X^2_2 - 0.2263X_1X_2X_3$
- $\bullet_{\bullet 1}(x) = 0.0259 - 0.0088X^2_1 - 0.0088X^2_2 + 0.00498X^2_3 - 0.0099X^2_1X^2_2$
- $\bullet_{\bullet 1}(x) = 0.0153 - 0.0036X_2 - 0.0021X^2_2 + 0.0030X^2_1X_2 - 0.0115X^2_1X^2_2$
- $\bullet_{\bullet 1}(x) = 0.2470 - 0.0899X^2_1 - 0.0871X^2_2 - 0.0440X^2_3$

Find — Optimal factor settings $x^* = (x^*_1, x^*_2, x^*_3)$

Table 6: Comparison of solutions for data without outliers

Methods	x^*	$\bullet_{\mu}^{*} d(\bullet_{\mu}^{*})$	$\bullet_{\bullet}^{2*} d(\bullet_{\bullet}^{2*})$	•	D	S	DS
Traditional approach with location mean and scale variance	(0.518, -0.982, 1.618)	(0.944, 10.033, 95.946) **(0.865, 0.933, 0.595)**	(0.016, 0.007, 0.002) **(0.838, 0.931, 0.982)**	0.9	0.783	0.915	0.796
Higher-order design with location mean and scale variance	(0.618, 1.318, -1.082)	(0.676, 9.997, 95.995) **(0.903, 0.995, 0.599)**	(0.007, 0.0009, 0.009) **(0.934, 0.991, 0.902)**	0.9	0.814	0.942	0.806
Higher-order design with location MMl and scale MMs	(-1.282, -0.082, 0.918)	(0.015, 10.003, 95.109) **(0.998, 0.995, 0.510)**	(0.001, 0.006, 0.003) **(0.986, 0.944, 0.971)**	0.9	0.798	0.967	0.813

Two contaminated ata points (in bold)

The comparison based on second-order model polynomial models, the technique used by Gothels and Cho (2012) and the proposed desirability function approach is employed and presented in Table 6. Note that the optimization method for the location and scale in Table 6 is using the AADF proposed by Chen *et al*. The result in Table 6 shows that using OLS along with location and scale based on mean and variance produced (x^*_1, x^*_2, x^*_3) = (0.618, 1.318, -1.082) with overall DS = 0.813. This solution is different what is obtained using MM-estimation where (x^*_1, x^*_2, x^*_3) = (-1.282, 0.082, 0.918) with overall DS = 0.813. Based on the result of the overall value DS, the MM-estimation with higher-order model clearly produced better result compared to the OLS based on mean and variance with higher-order model.

Table 7: Comparison of solutions for data with outliers

Method	x^*	•$_{\mu}^{*}$d(•$_{\mu}^{*}$)	•.^{2*}d(•.^{2*})	•	D	S	DS
Traditional approach with location mean and scale variance	(-1.282, -0.382, -1.182)	(4.872, 9.829, 93.403) **(0.513, 0.659, 0.340)**	(34.308, 0.0853, 0.045) **(0.714, 0.147, 0.549)**	0.9	0.487	0.386	0.4757
Higher-order design with location mean and scale variance	(-0.182, 1.618, -0.082)	(2.398, 11.202, 95.351) **(0.760, 0.966, 0.535)**	(1.164, 7.910, 0.031) **(0.990, 0.209, 0.694)**	0.9	0.732	0.524	0.708
Higher-order design with location MMl and scale MMs	(-1.182, -0.082, 1.018)	(0.125, 10.006, 95.278) **(0.982, 0.987, 0.528)**	(0.052, 0.010, 0.004) **(0.850, 0.898, 0.961)**	0.9	0.800	0.748	0.801

Two contaminated ata points (in bold)

To see the effect of outliers on the performance of the proposed desirability function approach, two contaminated ata points (in bold) are purposely introduced and presented in Table 1. It can be observed in Table 7 that in the presence of outliers in the data set, the traditional method failed to determine the correct optimal solution. In the case that the optimal solution is obtained, the result is misleading. However, using the MM-estimation with higher-order model, the results are closed to the results as in the clean dataset. It is clearly show that the proposed method will give more accurate results in the presence of outliers.

CONCLUSION

Robust design has been widely used extensively for multiple responses in terms of the process location and process scale based on sample mean and sample variance respectively. Then for the regression fitting, the Ordinary Least Square (OLS) method is usually used to acquire the sufficient response functions for the process location and scale based on mean and variance. Nevertheless, these existing procedures are easily influenced by outliers. Our proposed approach uses higher-order estimation techniques for robust MM-location, MM-scale estimator and MM regression estimator to overcome the weakness and shortcomings. The AADF method uses to simultaneously optimize the process location and scale based on MM-location and MM-scale. The numerical results have shown sthat the proposed approach works better than the traditional approach in terms of having highest overall desirability function value. Moreover, the proposed approach also, performs well when the outliers exist in the data series.

REFERENCES

Baba, I., H. Midi, S. Rana and G. Ibragimov, 2015. An alternative approach of dual response surface optimization based on penalty function method. Math. Prob. Eng., 2015: 1-6.

Box, G.E.P. and K.B. Wilson, 1951. On the experimental attainment of optimum conditions. J. Roy Stat. Soc., 13: 1-45.

Castillo, E.D. and D.C. Montgomery, 1993. A nonlinear programming solution to the dual response problem. J. Qual. Technol., 25: 199-204.

Chen, H.W., H. Xu and W.K. Wong, 2013. Balancing location and dispersion effects for multiple responses. Qual. Reliab. Eng. Intl., 29: 607-615.

Chiao, C.H. and M. Hamada, 2001. Analyzing experiments with correlated multiple responses. J. Qual. Technol., 33: 451-465.

Copeland, K.A. and P.R. Nelson, 1996. Dual response optimization via direct function minimization. J. Qual. Technol., 28: 331-336.

Derringer, G. and R. Suich, 1980. Simultaneous optimization of several response variables. J. Qual. Technol., 12: 214-219.

Easterling, R.G., 1985. Discussion of off-line quality control, parameter design and the Taguchi method. J. Qual. Technol., 17: 191-192.

Goethals, P.L. and B.R. Cho, 2012. Extending the desirability function to account for variability measures in univariate and multivariate response experiments. Comput. Ind. Eng., 62: 457-468.

Kovach, J. and B.R. Cho, 2008. Solving multiresponse optimization problems using quality function-based robust design. Qual. Eng., 20: 346-346.

Lee, M.S. and K.J. Kim, 2007. Expected desirability function: Consideration of both location and dispersion effects in desirability function approach. Qual. Technol. Quant. Manage., 4: 365-377.

Lin, D.J.K. and W.Z. Tu, 1995. Dual response surface optimization. J. Qual. Technol., 27: 34-39.

Myers, R.H. and W.H. Carter, 1973. Response surface techniques for dual response systems. Technometrics, 15: 301-317.

Myers, R.H., A.I. Khuri and G. Vining, 1992. Response surface alternatives to the Taguchi robust parameter design approach. Am. Statistician, 46: 131-139.

Peterson, J.J., 2004. A posterior predictive approach to multiple response surface optimization. J. Qual. Technol., 36: 139-153.

Riazoshams, H., H.B. Midi and O.S. Sharipov, 2010. The performance of robust two-stage estimator in nonlinear regression with autocorrelated error. Commun. Stat. Simulation Comput., 39: 1251-1268.

Taguchi, G., 1986. Introduction to Quality Engineering: Designing Quality into Products and Processes. University of Michigan, Ann Arbor, Michigan, ISBN:9789283310846, Pages: 191.

Tukey, J. W., 1960. A survey of Sampling from Contaminated Distributions. In: Contributions to Probability and Statistics, Olkin, I. (Ed.)., Stanford University Press, Stanford, California, pp: 448-485.

Vining, G.G. and R.H. Myers, 1990. Combining Taguchi and response surface philosophies: A dual response approach. J. Qual. Technol., 22: 38-45.

Yohai, V.J., 1987. High breakdown-point and high efficiency robust estimates for egression. Ann. Stat., 15: 642-656