Journal of Engineering and Applied Sciences 13 (12): 4403-4405, 2018
ISSN: 1816-949X

On Some Properties of Alfa Sets

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Abstract: In this study, we introduce and study the concepts of α-open set, α-continuous functions then, we also study the concepts of α-compact subsets and study some new characterizations of α-connectedness. Then we discuss the relations between the α-continuous functions and these concepts.

Key words: α-open set, α-compact, α-open cover, α-closed sets, α-continuous, concepts

INTRODUCTION

Generalized open sets play a very important role in general topology and they are now the research topics of many topologists worldwide. In this study, we discuss the properties of α-sets and α-continuous functions. All through this study (X, τ) and (Y, σ) stand for topological spaces with no separation assumed, unless otherwise stated. The closure of A and the interior of A will be denoted by Cl(A) and Int(A), respectively.

MATERIALS AND METHODS

Preliminaries; Definition 3.1: A subset A of a space X is said to be (Andrijevic, 1996; Mustafa, 2005; Calads and Jafari, 2003; Crossley and Hildebrand, 1971; Dugundji, 1996; El-Deeb *et al.*, 1983; Levine, 1963; Maheshwari and Prasad, 1972):

- Semi-open if A⊆Cl(Int(A))
- Pre open if A⊆Int(Cl(A))
- α-open if A⊆Int(Cl(Int(A))

Definition 3.2: A function f: X→Y is called (Al-Obiadi, 2005; Mashhour *et al.*, 1982):

- Semi continuous if $f^{-1}(V)$ is semi open in X for each open set V of Y
- Pre continuous if $f^{-1}(V)$ is pre open in X for each open set V of Y
- α-continuous if $f^{-1}(V)$ is α-open in X for each open set V of Y

Definition 3.3;Mustafa (2005): A space X is a α-T_2 space iff for each x, y∈X such that x≠y there are α-open sets U, V⊂X, so that, x∈U, y∈V and U∩V = ∅.

α-connectedness

Definition: A space which is a union of two disjoint non-empty α-open sets is called α-disconnected.

Equivalently: A space X is α-connectedif the only subsets of X which are both α-open and closed are ∅ and X.

Proof of equivalence: If X = A∪B with A and B α-open and disjoint then X-A = B and so, B is the complement of an α-open set and hence, is α-closed. Similarly, B is clopen.

Conversely, if A is a non-empty, proper open subset then A and X-A α-disconnect X. A subset of a topological space is called α-connected if it is connected in the subspace topology.

Theorem: The α-continuous image of a α-connected space is α-connected.

Proof: If f: X→Y is α-continuous and f(X)⊂Y is α-disconnected by α-open sets U, V in the subspace topology on f(X) then the α-open sets f-1(U) and f-1(V) would α-disconnect X.

Corollary: α-connectedness is preserved by homeomorphism.

Theorem: If A and B are α-connected and A∩B≠∅ then A∪B is α-connected.

Proof: Suppose α-open sets U and V α-disconnect A∪B. Then A∩U and A∩B would α-disconnect A and so, one of them is ∅. So, suppose A⊂U. Similarly we have either B⊂V or B⊂U. Since, B meets A the first of these is impossible and so, we have A∪B⊂U and V =∅.

RESULTS AND DISCUSSION

Covering properties

Definition 5.1: Let $\{G_\alpha: \alpha\in\Delta\}$ be a family of α-open sets of the space X. The family $\{G_\alpha: \alpha\in\Delta\}$ covers X if $X\subseteq\bigcup_{\alpha\in\Delta} G_\alpha$.

Definition 5.2: A space X is called a α-compact space if each α-open cover of X has a finite subcover for X.

Theorem 5.3: Let A be a α -compact subset of the α-T_2 space X and $\notin$A. Then there exist two disjoint$\notin$-open sets U and V containing x and respectively.

Proof: Let y$\in$A, since, X is α-T_2 space there exist two α-open sets U_x, $V_y \in X$ such that $x \in U_x$, $y \in V_y$, $U_x \cap V_y = \phi$ the family is open cover of A has a finite subcover, thus:

Theorem 5.4: If X is α-T_2 space and A is a α-open subset, if A is α-compact then A is a α-closed.

Proof: Let $x \in X$-A, by the theorem 4.3 there exist two α-open sets U and V such that $X \in U$, $A \subseteq V$, $U \cap V = \phi$, thus, $x \in U \subseteq X\text{-}V \subseteq X\text{-}A$ which implies X-A is α-open, so that, A is α-closed.

Theorem 5.5: Let A and B be a two α-compact subsets of the α-T_2 space X then there exist disjoint α-open sets U and V containing A and receptively.

Proof: Let b$\in$B, since, A is a α-compact subset and α-open in X there exist two α-open sets U_b, V_b such that $U_b \cap V_b = \phi$; $b \in V_b$, $A \subseteq U_b$, so, $\beta = \{B \cap V_b; b \in B\}$ is a α-open cover of B, since, B is α-compact subset there exist finite subcover $\{B \cap V_{in}; 1 \leq i \leq n\}$ from β. Let:

$$U = \bigcap_{i=1}^{n} U_{b_i}, V = \bigcup_{i}^{n} V_{b_i}$$

Thus:

$$A \subseteq U, B \subseteq V, U \cap V = \phi$$

Theorem 5.6: Let f: (X, τ)$\rightarrow$(Y, ρ) be a α-continuous surjection open function, if X is a α-compact then Y is a α-compact.

Proof: Let $\beta = \{V_\alpha: \alpha \in \Delta\}$ be a α-open cover of Y, then $L = \{f^1(V_\alpha): \alpha \in \Delta\}$ is a α-open cover of X. Since, X is a α-compact space there exist a finite subcover from L to the space X. Such that:

$$X \subseteq \bigcup_{i=1}^{n} f^{-1}(V_{\alpha i})$$

Thus:

$$Y = f(X) \subseteq f\left(\bigcup_{i=1}^{n} f^{-1}(V_{\alpha i})\right) = f\left(f^{-1}\left(\bigcup_{i=1}^{n}(V_{\alpha i})\right)\right) = \bigcup_{i=1}^{n}(V_{\alpha i})$$

Hence:

$$Y \subseteq \bigcup_{i=1}^{n}(V_{\alpha i})$$

This shows Y is a α-compact.

Corollary 5.7: α-compactness is a topological property

Proof: The proof from Theorem 4.5.

Definition 5.8: A family of sets β has "finite intersection property" if every finite subfamily of β has a nonempty intersection.

Theorem 5.9: A topological space is α-compact if and only if any collection of its α-closed sets having the finite intersection property has non-empty intersection.

Proof: Suppose X is α-compact, i.e., any collection of α-open subsets that cover X has a finite collection that also cover X. Further, suppose $\{G_\alpha: \alpha \in \Delta\}$ is an arbitrary collection of α-closed subsets with the finite intersection property. We claim that:

$$\bigcap_{\alpha \in \Delta} G_\alpha \neq \phi$$

Is non-empty. Suppose otherwise, i.e., suppose:

$$\bigcap_{\alpha \in \Delta} G_\alpha = \phi$$

Then:

$$\bigcup_{\alpha \in \Delta}(X\text{-}G_\alpha) = X\text{-}\left(\bigcap_{\alpha \in \Delta} G_\alpha\right) = X\text{-}\phi = X$$

Since, each Gα is α-closed, the collection $\{X\text{-}G_\alpha: \alpha \in \Delta\}$ is an α-open cover for X. By compactness there is a finite subcover L such that:

$$X = \bigcup_{i=1}^{n}(X\text{-}G_{\alpha i})$$

But then:

$$\bigcap_{i=1}^{n} G_{\alpha_i} = \bigcap_{i=1}^{n}\left(X\text{-}(X\text{-}G_{\alpha_i})\right) = X\text{-}\left(\bigcup_{i=1}^{n}(X\text{-}G_{\alpha_i})\right) = X\text{-}X = \phi$$

which contradicts the finite intersection property of $\{G_\alpha: \alpha \in \Delta\}$. Conversely, take the hypothesis that every family of a α-closed sets in X having the finite intersection property has a nonempty intersection. We are to show X is α-compact. Let $\{G_\alpha: \alpha\in\Delta\}$ be any α-open cover of X. Then $\{X\ G_\alpha: \alpha\in\Delta\}$ is a family of α-closed sets such that:

$$\bigcap X\text{-}G_\alpha = X\text{-}\left(\bigcup_{\alpha\in\Delta} G_\alpha\right) = X\text{-}X = \phi$$

Consequently, our hypothesis implies the family $\{X\text{-}G_\alpha: \alpha\in\Delta\}$ does not have the finite intersection property. Therefore, there is some finite subcollection $\{X\text{-}G_{\alpha i}: i = 1, 2, 3, ..., n\}$ such that:

$$\bigcap_{i=1}^{n} X\text{-}G_{\alpha_i} = \phi$$

And hence:

$$X = \bigcup_{i=1}^{n} G_{\alpha_i} = \bigcup_{i=1}^{n}\left(X\text{-}\left(X\text{-}G_{\alpha_i}\right)\right) = X\text{-}\left(\bigcap_{i=1}^{n}\left(X\text{-}G_{\alpha_i}\right)\right) = X\text{-}\phi = X$$

Thus:

$$X = \bigcup_{i=1}^{n} G_{\alpha_i}$$

Implying X is α-compact.

CONCLUSION

In this study, the relations between the α-continuous functions and their concepts are discussed clearly.

ACKNOWLEDGEMENT

The resarcher acknowledges Applied Science Private University, Amman, Jordan, for the fully financial support granted of this research.

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