

## PD+ and PID Control for a Mobile Manipulator of Eight Degrees of Freedom

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**Abstract:** A mobile manipulator is a mechanism constructed from a robotic manipulator arm, mounted on a mobile platform. This system combine the advantages of mobile platforms in terms of mobility and expansion of the workspace of the manipulator and the advantages of the manipulator with its operational functionalities. However, the functioning of this system is a challenge due to the many degrees of freedom and unstructured environment that it performs, so, this mechanism, require the design of controllers that allow to obtain desired behaviors in the defined paths for their operation or make corrections on the positions caused by external or internal perturbations of the mechanism. For the above, in the present research, it is calculated the PID family controllers for a mobile manipulator of 8 degrees of freedom with the kinematic decoupling.

**Key words:** Degrees of freedom, dynamics, generalized coordinates, manipulator arms, PD+ controller, PID controller, torque

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### INTRODUCTION

A mobile manipulator is defined as a mechanism constituted by the coupling between a manipulator arm and a mobile platform (Baturone, 2001). This is defined as a set of elements by prismatic or revolution type joints, resulting in a relative movement of the different elements which results in the displacement of the tool or gripper (Apostolovich, 2009). While a mobile robot is a structure that needs locomotion mechanisms for its movement through structured and unstructured environments (Romero, 2012). For the study of the behavior of mobile manipulators, there is a kinematic and dynamic model that define the characteristics of both mechanisms, so, it is possible to design controllers that allow to perform properly inspection tasks, improving its operation (Craig, 1989).

Within the variables to be considered for the management of the mobile platform, there are a workspace defined as the range of possibilities that the platform can reach in an environment, besides its controllability in which the possible trajectories are determinate. While for the study of the manipulator behaviors it is defined the kinematics that relates the space of the articular trajectories  $q_1, q_2, q_3, \dots, q_n$  with the Cartesian space disposed at the end of the mechanism. Annex, it is defined as the basis of the dynamic studies for the calculation of the forces that are required to cause movement in the system (Delgado and Bolanos, 2013; Batz, 2005).

### MATERIALS AND METHODS

**Mobile manipulator controller:** For the development of the present research it is considered the scheme of Fig. 1 Where the structure of the mobile manipulator is specified. It must be considered that for the design of the controllers of the mechanism a decoupling between the mobile platform and the manipulator is contemplated whereby the frame of reference and movement of the manipulator is considered at the beginning of the kinematic chain but not as the set of global coordinates of the hybrid platform in addition, the second derivatives of signals generated in the algorithm must be saturated due to the jerk or over acceleration effects.

**Manipulator controller:** The controller begins with the schematic of Fig. 2 where the articular configuration of a

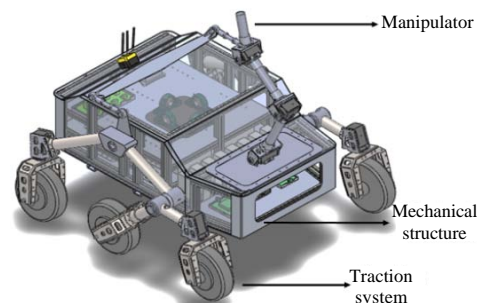


Fig. 1: Hybrid platform scheme

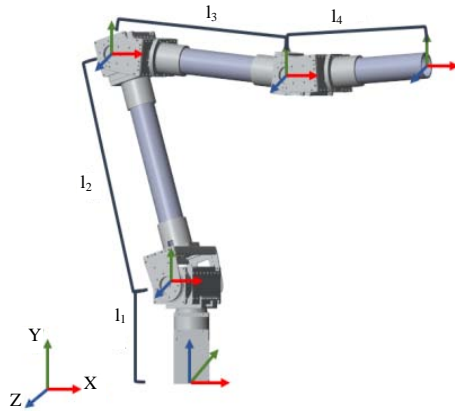


Fig. 2: Manipulator arm

manipulator arm of 4 degrees of freedom is specified. It should be noted in the system the masses are concentrated in the center of each link.

**Inverse dynamics:** After specifying the manipulator configuration to be worked on, the La Grange formulation of Eq. 1 is considered in which the system dynamics is determined to calculate and control the torque required by each joint per its masses, lengths and viscous friction (Ogata, 2004):

$$M(q)\ddot{q} + V(q, \dot{q}) + G(q) + B(\dot{q}) = \tau \quad (1)$$

For the calculations of the controller the system is expressed as observed in Eq. 2 that determines the operation of the system as a function of the torque required by each joint:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix} = \begin{bmatrix} A & B & C & D \\ E & F & G & H \\ I & J & K & L \\ M & N & O & P \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{bmatrix} + \begin{bmatrix} Q \\ R \\ S \\ T \end{bmatrix} + \begin{bmatrix} U \\ V \\ W \\ X \end{bmatrix} + \begin{bmatrix} v_1 & 0 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & 0 & v_3 & 0 \\ 0 & 0 & 0 & v_4 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} \quad (2)$$

**Direct dynamic:** The direct dynamic model shows the temporal evolution of articular coordinates as well as their derivatives as a function of the forces and pairs involved. To obtain it, the inverse model of Eq. 2 the Cramer rule is applied and the substitution of the null cofactors is used. For the management of the matrices in the

calculation of the determinants it is used Eq. 3. Therefore, in solving Cramer's rule is obtained Eq. 2 from 4-7:

$$\begin{aligned} Z_1 &= A\ddot{\theta}_1 \\ Z_2 &= F\ddot{\theta}_2 + G\ddot{\theta}_3 + H\ddot{\theta}_4 \\ Z_3 &= J\ddot{\theta}_2 + K\ddot{\theta}_3 + L\ddot{\theta}_4 \\ Z_4 &= N\ddot{\theta}_2 + O\ddot{\theta}_3 + P\ddot{\theta}_4 \end{aligned} \quad (3)$$

$$\ddot{\theta}_1 = \frac{\begin{bmatrix} Z_1 & 0 & 0 & 0 \\ Z_2 & F & G & H \\ Z_3 & J & K & L \\ Z_4 & N & O & P \end{bmatrix}}{\begin{bmatrix} A & 0 & 0 & 0 \\ 0 & F & G & H \\ 0 & J & K & L \\ 0 & N & O & P \end{bmatrix}} = \frac{Z_1}{A} \quad (4)$$

$$\ddot{\theta}_2 = \frac{\begin{bmatrix} A & Z_1 & 0 & 0 \\ 0 & Z_2 & G & H \\ 0 & Z_3 & K & L \\ 0 & Z_4 & O & P \end{bmatrix}}{\begin{bmatrix} A & 0 & 0 & 0 \\ 0 & F & G & H \\ 0 & J & K & L \\ 0 & N & O & P \end{bmatrix}} = \frac{(GLZ_4 - HKZ_3 - HOZ_2 + HOZ_3 + KPZ_2 - LOZ_2)}{(FKP - GJP + GLN - HKN - FLO + HJO)} \quad (5)$$

$$\ddot{\theta}_3 = \frac{\begin{bmatrix} A & 0 & Z_1 & 0 \\ 0 & F & Z_2 & H \\ 0 & J & Z_3 & L \\ 0 & N & Z_4 & P \end{bmatrix}}{\begin{bmatrix} A & 0 & 0 & 0 \\ 0 & F & G & H \\ 0 & J & K & L \\ 0 & N & O & P \end{bmatrix}} = \frac{(FLZ_4 - HJZ_3 - FPZ_3 + HNZ_3 + JPZ_2 - LNZ_2)}{(FKP - GJP + GLN - HKN - FLO + HJO)} \quad (6)$$

$$\ddot{\theta}_4 = \frac{\begin{bmatrix} A & 0 & 0 & Z_1 \\ 0 & F & G & Z_2 \\ 0 & J & K & Z_3 \\ 0 & N & O & Z_4 \end{bmatrix}}{\begin{bmatrix} A & 0 & 0 & 0 \\ 0 & F & G & H \\ 0 & J & K & L \\ 0 & N & O & P \end{bmatrix}} = \frac{(FKZ_4 - GJZ_3 - GNZ_3 + FOZ_3 + KNZ_2 - JOZ_2)}{(FKP - GJP + GLN - HKN - FLO + HJO)} \quad (7)$$

This model allows to obtain the current position outputs of the mechanism which the comparison of data

that determines the system error signal is performed. This signal is taken as input parameter of the controller to calculate the control signal to be considered by each articulator pairs of the manipulator.

**PD+controller design:** The position control of manipulator arm can be developed using simple techniques such as the PD controller whose control law given by Eq. 8 ensures compliance of the position control objective for dynamic models that do not have the vector of pairs Gravitational  $G(q)$  (Nino, 2013). It should be noted that, although, the manipulator has 5 degrees of freedom, only four are controllable because the last degree of freedom is configured manually:

$$k_v \dot{e} + k_p e = \tau \quad (8)$$

where,  $\dot{e} = \dot{q}_d - \dot{q}$  and  $e = q_d - q$ . The tuning of the constants is trivial because it is sufficient to select matrices  $k_v$  and  $k_p$  symmetric and defined positive. However, this controller does not guarantee the operation for the model to work because in its dynamics are contemplated inertial matrices, centrifugal and Coriolis forces, gravitational forces and frictional forces. Due to the above, a PD+ controller must be developed to satisfy the position control objective for any robot manipulator with  $n$  degrees of freedom. This requires the prior knowledge of the dynamic model in Eq. 1 and its control law expressed in Eq. 9 (Urias, 2005):

$$M(q)\ddot{q}_d + V(q, \dot{q})\dot{q}_d + G(q) + B(\dot{q}) + k_v \dot{e} + k_p e = \tau \quad (9)$$

Replacing the torque  $\tau$  of Eq. 1 in 9, it is obtained Eq. 10 which simplified as shown in Eq. 11 describes the dynamics of the error that is used as control for manipulator arm of  $n$  degrees of freedom:

$$M(q)\ddot{q}_d + V(q, \dot{q})\dot{q}_d + G(q) + B(\dot{q}) + k_v \dot{e} + k_p e = \tau \quad (10)$$

$$\ddot{e} + \left( \frac{V(q, \dot{q}) + K_v}{M(q)} \right) \dot{e} + \left( \frac{K_p}{M(q)} \right) e = \tau \quad (11)$$

For the development of the manipulator controller of 4 degrees of freedom is propose as control constants  $k_p$  in Eq. 12 and  $k_v$  in Eq. 13:

$$k_p = \begin{bmatrix} 150 & 0 & 0 & 0 \\ 0 & 160 & 0 & 0 \\ 0 & 0 & 180 & 0 \\ 0 & 0 & 0 & 350 \end{bmatrix} \quad (12)$$

$$k_v = \begin{bmatrix} 12 & 0 & 0 & 0 \\ 0 & 13 & 0 & 0 \\ 0 & 0 & 14 & 0 \\ 0 & 0 & 0 & 16 \end{bmatrix} \quad (13)$$

For the simulation of the system, the schematic of Fig. 3 presents the layout of the controller as well as the model of the manipulator described in B4DOF-R. Also, the control signal must be saturated at the output because of the torque limit stipulated by the actuators that are the joints of the mechanism.

**Mobile manipulator controller:** For the calculation of the controller, the first step is obtaining the mathematical models that describe the operation of the motors to perform the traction movements of the mobile platform to research. For this, the mathematical structure of a permanent magnet brushless DC motor is not totally

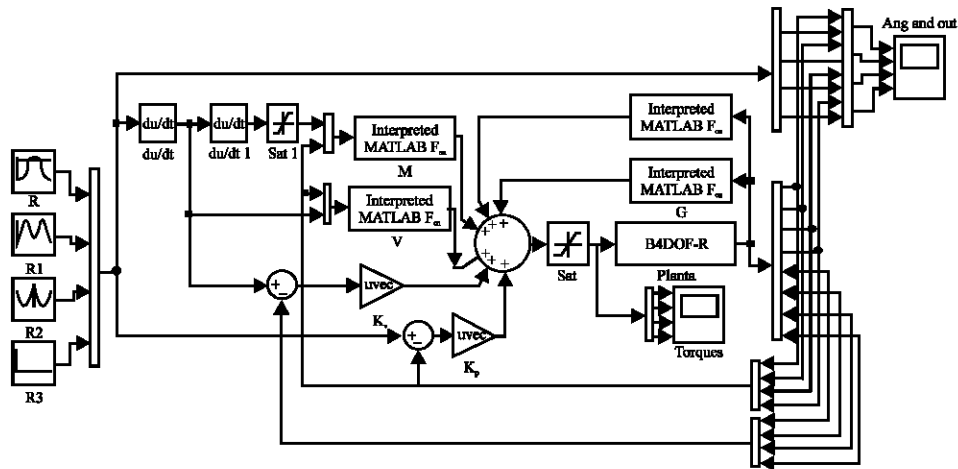


Fig. 3: PD+ controller scheme

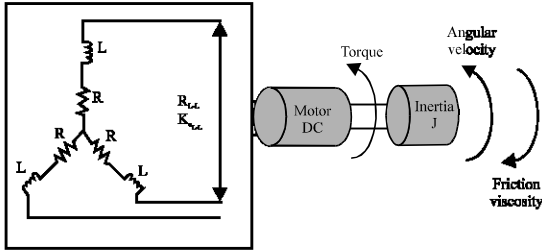


Fig. 4: Brushless motor scheme

different from the conventional DC motor model but it has a difference in the motor phases which particularly affect the resistivity and inductivity of Brushless modeling as shown in Fig. 4 (Shao, 2003; Becerra-Vargas *et al.*, 2014). Then, to the develop the model, the mechanical time counter of the system per Eq. 14 and the electric time constant with Eq. 15 is defined:

$$\tau_m = \sum \frac{RJ}{K_e K_m} = \frac{J \Sigma R}{K_e K_m} \quad (14)$$

$$\tau_e = \sum \frac{L}{R} = \frac{L}{\Sigma R} \quad (15)$$

But having a symmetrical and three-phase configuration, the mechanical and electrical constants are expressed with Eq. 16 and 17:

$$\tau_m = \frac{J(3R)}{K_e K_m} \quad (16)$$

$$\tau_e = \frac{L}{3R} \quad (17)$$

Considering the phase effect is defined Eq. 18 which allows it to define the final mechanical time constant to be considered in the mathematical model:

$$\tau_m = \frac{J(3R)}{\left(\frac{K_{e,l-1}}{\sqrt{3}}\right) K_m} \quad (18)$$

There is also the respective relationship between the voltage constant and the mechanical constant determined by  $K_e = K_m 0.0605$ . To finally obtain the transfer function is modeled all parameters of the Brushless DC motor in Eq. 19:

$$G(s) = \frac{\left(\frac{1}{K_e \tau_m \tau_e}\right)}{s^2 + \frac{1}{(\tau_e)}s + \frac{1}{(\tau_m \tau_e)}} \quad (19)$$

Table 1: Brushless golden motor specifications

Parameters	Units	Values
R	$\Omega$	1.05
L	H	$1.95 \times 10^{-3}$
J	$\text{kg} \cdot \text{m}^2$	$8.5 \times 10^{-6}$
B	$\text{Nmsec/rad}$	0
$\tau_m$	Sec	$13.5 \times 10^{-3}$
$\tau_e$	Sec	$1.85 \times 10^{-3}$

For this control a Brushless motor is implemented and its specifications are shown in Table 1 to finally obtain Eq. 20, characteristic of this type of motors (Shao, 2003):

$$G(s) = \frac{1312800}{s^2 + 540.5405s + 4004} \quad (20)$$

The system is discretized with a frequency of 50 Hz which define as sampling period  $t_m = 0.002$  sec to obtain Eq. 21:

$$G(z) = \frac{1.865z + 1.302}{z^2 - 1.243z + 0.3392} \quad (21)$$

**PID controller design:** A PID controller with filter bases its operation on three control laws: proportional, integral and derivative. The proportional action which determines the magnitude of the error and the speed which reference is reached, the derivative action determines the speed of compensation the error and finally, the integral action eliminate the error in permanent state (Kuo, 1995).

To perform the controller design, the model describing the behavior of the motors for traction velocity in discrete time is used where by the poles must be calculated for the controller design, considering Eq. 22, expressed in Cartesian form as shown in Eq. 23:

$$|z| = e^{-T \zeta \omega_n} \quad (22)$$

$$\angle z = T \omega_d$$

$$z = re^{\pm j\theta} \quad (23)$$

When the poles of the system are specified, the desired polynomial is calculated as shown in Eq. 24 and 25, determined by a desired settling time of 0.05 and a damping coefficient of 0.9. Finally, the magnitudes of each power with the characteristic polynomial of the model are matched and thus determine the control variables for each element:

$$P_d = (z - re^{-j\theta})(z - re^{+j\theta})(z - e^{-10T\zeta\omega_n}) \quad (24)$$

$$z = 1 - 1.25065z^{-1} + 0.42282z^{-2} + 0.000039z^{-4} \quad (25)$$

The PID controller in its structure must consider the implementation of a filter to match the number of unknowns and equations of the system as observed in Eq. 26:

$$C(z) = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 - z^{-1}} \cdot \frac{1}{1 + s_0 z^{-1}} \quad (26)$$

The characteristic polynomial of the system is obtained by multiplying Eq. 21 with 26, then it must be matched with Eq. 25 to obtain the values of the unknowns of the controller as observed in Eq. 27:

$$\begin{aligned} q_0 &= 0.320525 \\ q_1 &= 0.371383 \\ q_2 &= 0.102665 \\ q_2 &= 0.394968 \end{aligned} \quad (27)$$

## RESULTS AND DISCUSSION

**Manipulator results:** To verify the correct operation of the previously designed controller, the values of Fig. 5 which represent the angular paths of each joint are taken as desired positions of the system.

Knowing the desired trajectories of the system, it proceeds with the verification of the controller. So, in Fig. 6-10 the blue signal that determines the desired condition of each joint and the purple signal that describes the behavior of the manipulator under the behavior of the signal generated by the controller are shown.

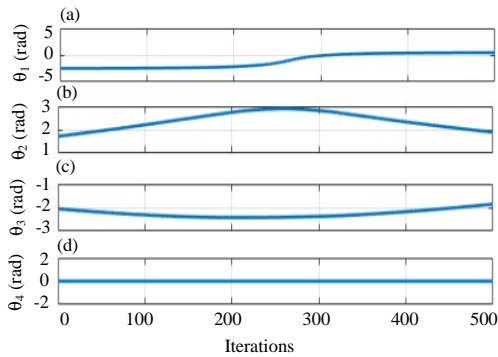


Fig. 5: Angular joint variability

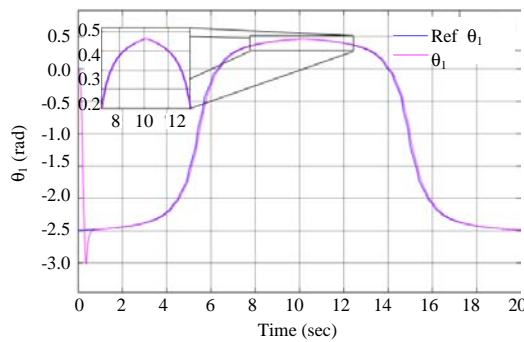


Fig. 6: Control over joint 1

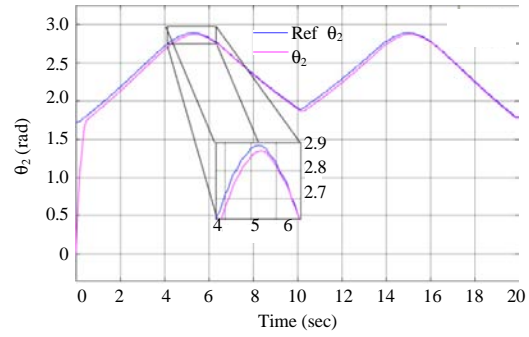


Fig. 7: Control over joint 2

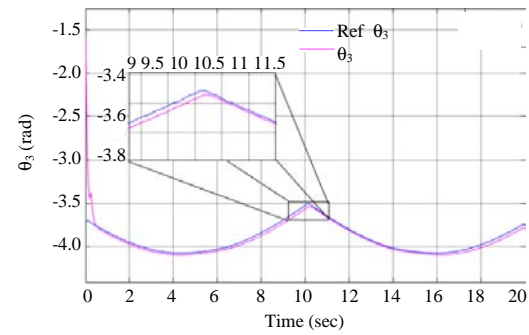


Fig. 8: Control over joint 3

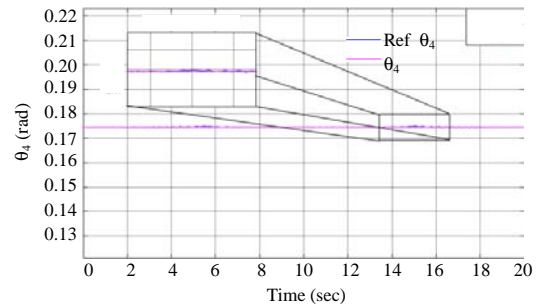


Fig. 9: Control over joint 4

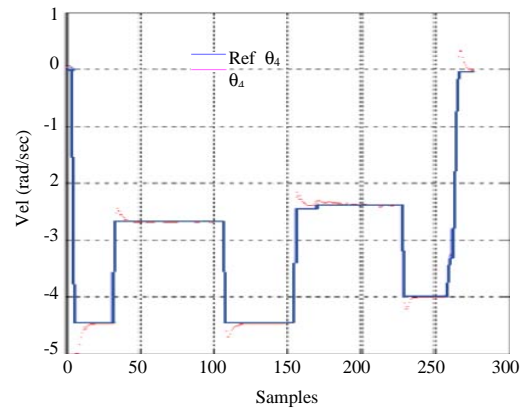


Fig. 10: Mobile platform behavior

**Mobile platform results:** To verify the correct operation of the controller, a random path defined by the blue route of Fig. 10 is proposed and its respective behavior after the control signal is applied to the red path.

## CONCLUSION

From the results obtained in the previous study, it is observed that the PD+ controller reaches the initial reference at times under the second when its starting position is at zero angle. This because when controlling directly the error generated by the current position of the manipulator and the desired position of the manipulator, the control signals act with high torques considering the dynamics of error. Another variable to review is a system that follows the reference behaves without oscillations and its position error does not exceed  $1.10^{-5}$  radians, this due to the compensation of the vector of the inertia matrix, the vector of centrifugal and Coriolis forces, the vector of gravitational forces and the vector of frictional forces presented in Eq. 10.

On the other hand, about the speed control performed for the traction of the mobile platform it is observed that the control signal leads to a behavior with over-impulse to reach the desired reference this because the calculation performed contemplated a damping coefficient of 0.9 but still achieves the desired behavior in about 10 samples.

## ACKNOWLEDGEMENT

The research for this study was supported by Nueva Granada Military University, through the project ING-IMP-2138.

## NOMENCLATURE

$\xi$	= Damping coefficient
$e$	= Derivative of error
$\ddot{e}$	= Double derivative of error
$K_2$	= Electrical motor constant
$e$	= Error
$q$	= Generalize coordinates
$\ddot{q}$	= Generalize coordinates of acceleration
$\dot{q}$	= Generalize coordinates of velocity
$G(q)$	= Gravitational forces vector
$L$	= Inductor
$M(q)$	= Inertial Matrices
$B$	= Input Matrix
$K_m$	= Mechanical motor constant
$w_n$	= Natural frequency
$C$	= Output matrix
$K_p$	= Proportional constant

$R$	= Resistance
$T$	= Sample Time
$A$	= System matrix
$\tau$	= Torque
$v(q\dot{q})$	= Vector of centrifugal and coriolis forces
$B(\dot{q})$	= Vector friction forces
$K_v$	= Velocity constant

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