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A Modification of the Box M Statistics Using S Estimator

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Abstract: Box M statistic constructed under the multivariate normality distribution and is one of Llikelihood Ratio Test (LRT). The performance of traditional Box M statistic by utilizing classical estimators suffers from masking and swamping effects when there is outlier in data set. To ease the problem, robust estimators are suggested. A robust Box M statistic based on a S estimator, M_s suggested as the alternative to the classical Box M statistic. From the simulation study, the performance comparison of classical and M_s statistics are measured using power of test. From the results, it displayed that M_s has a competitive performance relative to the classical statistic. As a conclusion, M_s can be used for testing the equality of two difference covariance matrices or more when the data contains outlier.

Key words: Covariance matrix, S estimator, power of test, difference, covariance, matrices

INTRODUCTION

The most popular and widely used test for testing directly the equality of covariance matrices across time period is Box M statistic. This statistics is constructed by Box (1949). The formula include determinant of sample covariance matrix. In the case of high dimensional data sets p>n), determinant is one of the difficult task, since, the value will be singular and the inversion of that matrix will not exist (Yusoff and Djauhari, 2012).

On the other hand, the Generalized Variance (GV) is multivariate dispersion measure is also can be used for testing the equality of covariance matrices. The role of GV statistic is to test the homogeneity of several independent samples of covariance structure (Sharif *et al*., 2014). However, the measurement is inaccurate because the two different covariance matrices might be defined equal to each other. Furthermore, GV statistic can be implemented when the determinant of covariance matrix is nonsingular (Djauhari and Salleh, 2011). Due to that, this test is difficult to compute when the data sets are of high dimension. In year 2007, Djauhari (2007) established Vector Variance (VV) statistic as a multivariate variability measurement to assist to solve the singularity problem when dealing with high dimension data set and to overcome the problem of GV statistic. The computational time for VV statistic is exposed to be better compared to GV statistic (Sharif *et al*., 2014).

Through all of those statistics stated above, Box M statistic is commonly practiced and familiar among applied researchers because it can easily reached when they are using IBM SPSS Software. Therefore, Box M statistic is choose since, it is well recognized by applied researchers rather than GV which well known among pure statistical researcher.

To test the equality of variance-covariance matrices the hypothesis used is H_0: $\Sigma_1 = \Sigma_2, ..., = \Sigma_m$ versus H_1: $\Sigma_i \neq \Sigma_j$ for at least one pair (i, j) where i, j = 1, 2, ..., m. Thus, the M-statistic is derived as follows:

$$M = N\text{In}\left|\bar{S}\right| - \sum_{i=1}^{m} n_i \text{In}\left|S_i\right| \quad (1)$$

Where:

$\bar{S} = \frac{1}{N}\Sigma_{i=1}^{m} n_i S_i$ = The pooled sample variance-covariance matrix

S_i = The variance-covariance matrix calculated from the sample i

m = The number of subgroup where the stability of matrices is hypothesized the sample size n_i = i-; N = $n_1+n_2+, ..., nm$

Under H_0, the statistical test can be approximated either by P^2 distribution or F distribution (Box, 1949). Mardia *et al*. (1979) mentioned that the P^2 approximation will be adequate to be used in any practical determinations. Moreover, P^2 approximation is good if the number of sample sizes, n is greater than 20. Consequently, the statistical test will be rejected at significance level, α if M/b exceeds $\chi^2_{a,v}$, where:

$$b = \frac{1}{1\text{-}a};\ a = \frac{(2p^2+3p\text{-}1)}{6(p+1)(m\text{-}1)}\left(\sum_{i=1}^{m}\frac{1}{n_i} - \frac{1}{N}\right)$$

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The (1-α^{th} of Chi-squared distribution with degrees of freedom v = 1/2p(p+1)(m-1) where p is the number of variables.

Box M statistic is improved Likelihood Ratio Test (LRT) constructed under the multivariate normality distribution (Box, 1949). This test is constrained under another two assumptions which are the sample covariance matrices are independent and the sample size, n must be larger than the number of variables, p (Sharif, 2013).

In practice, data that meet the assumption of normality is difficult to be found. Fail to fulfil the assumption of normality can distort Type 1 error rates (Yusof *et al.*, 2013) and make distributional behaviour totally fails. Consequently, this statistic is highly sensitive to the existence of outliers which can cause unacceptable results. Thus, a common recommendation is to use nonparametric test or performing simple transformation.

However, nonparametric test is less powerful if it compared to parametric test. Nonparametric test is call for large sample size to reject the null hypothesis and its computation is tedious and laborious (Daniel, 1990). Otherwise, simple transformation is one way to overcome the problem of outliers. Nevertheless, as identified by Wilcox (2005), simple transformations are failed to treat the outliers with efficiently. As a result, the outliers still exist and minimize the statistical power when applying simple transformations.

Moreover, there are another two alternative methods that can be used in to reduce the effect of outliers. The first method is to calculate the classical estimator after eliminating outliers from the data. The second method is by using robust estimator to replace classical estimator in decreasing the influence of outliers (Yahaya *et al.*, 2001). The robust method is aim to produce reliable parameters estimate, related tests and confidence intervals, even though data follow a given distribution correctly but conversely, only approximately in the sense would be qualified (Maronna *et al.*, 2006). Furthermore, it is vital tools in analysing data that are including a contaminated observation (Muthukrishnan and Ravi, 2016). It can be used to identify outliers and to deliver resistant results in the existence of outliers. Thus, robust method attempts to deliver a good result and therefore, would be interested in this study.

There are a lot of multivariate robust estimators of location and scatters can be found in literature review. In our previous research, S estimator is used to replace the covariance estimator (or scatter matrix) and the procedures is called M_s. The results shows that the M_s statistic performs well in terms of controlling Type 1 error.

Based on that, we found that the computation of S estimators is less complexity compared to the other robust estimators (Jeng, 2010; Salibian-Barrera and Yohai, 2006). Remarkably, S estimators have high Breakdown Point (BP) of nearly 50% (Lopuhaa, 1989). Therefore, S estimator is examined for the substitution of the sample covariance matrix.

MATERIALS AND METHODS

Modified Box M (M_s): M_s denotes as the robust Box M statistic based on S estimator. The covariance of S estimator, $S_{s(i)}$ where i = 1, 2, ..., m is used and presented into Eq. 1. Thus, the statistic for M_s is as follows:

$$M_s = N\text{In}\left|\bar{S}\right| - \sum_{i=1}^{m} n_i \text{In}\left|S_{s(i)}\right| \quad (2)$$

where, $\bar{S} = \frac{1}{N}\Sigma_{i=1}^{m} n_i S_i$ the pooled sample covariance matrix of S-estimator. In the next study, we evaluate the performance of proposed test, we use the power of test and compare classical M test and M_2.

RESULTS AND DISCUSSION

Power of test: The idea of power is used to make a comparison between different statistical testing processes where the most powerful test will have the higher number of rejection of the null hypothesis (Mittelhammer and Mittelhammer, 1996). However, this research conducting a comparison between classical M-statistic and robust M-statistic (M_s). Commonly, the power of a test or identified as 1-β is the probability of correctly rejecting the null hypothesis when it is false. It is actually referred to the sensitivity of a statistical test and the ability to detect a true. This is because the high power is very commendable. Additional importantly, highly powered research is often an able to identify small effect as statistically significant (Atiany and Sharif, 2016).

To explain the power of those test we conducted a simulation study under the alternative hypothesis H_1: $\Omega_k \neq \Omega_0$ for at least k where k = 1, 2, ..., m. We generate random data from p-variate normal distribution. In this research, the simulation is performed using MATLAB 7.8.0 (R2009a) with 10000 repetitions at significance level, α = 0.05 and the contaminated data ranging from ξ = 0, 5, 10, 15 and 20%. The shift in covariance matrix 0, 0.1, 0.3, 0.5, 0.7 and 0.8. The data set consists of different number of variables which are small (p = 3 and 5, medium p = 10 and 15 and large p = 20 and 30 as well as different size of sample, n = 5, 10, 20, 30, 40, 50 and 100 (Sharif and Atiany, 2018).

From Table 1 and 2 in appendix we can conclude that the larger the sample size n, the more powerful of the test because the values fall within the power interval is increase. When n =10, 20, 30, 40, 50 and 100, there are 46, 70, 75, 80, 80 and 85 out of 720 values fall within the power interval, respectively. From the results, we found that when the sample size is large, the value is power in a smaller covariance shift. Meanwhile, when the sample size is small, the value is power in a larger covariance shift only.

From both Table 3 and 4, there are 300 conditions involved in assessing the power of test for medium number of variables (p = 10 and 15). For p, there are 100 conditions of M-statistic and 107 conditions of M_s statistic that fall within the power interval. While, for p = 15 there are 103 conditions of M-statistic and 110 conditions of M_s statistic that fall within the power interval. In summary, we can conclude that M_s statistic is more powerful compared to M-statistic.

For large number of variables p = 20 and 30, from Table 5 and 6, there are 180 conditions involved in evaluating the power of test for large number of variables (p = 20 and 30). For p = 20, there are 67 conditions of M-statistic and 70 conditions of M_s statistic that fall within the power interval. Whereas, for p = 30, there are 68 conditions of M-statistic and 75 conditions of M_s statistic that fall within the power interval. In summary, we can conclude that M_s statistic is dominated the M-statistic.

CONCLUSION

For testing two or several covariance matrices Box M statistic is known as a test widely used for this purpose, under situations of non-normality, this test is known to underperform. To produce active approaches regardless of the situations other test statistics are suggested. In this research, we suggested other processes to the Box M statistic by using a robust estimator known as the S estimator for scatter matrix. The S estimator has the properties such as the affine equivariant and a high BP and has a better calculation. The performance of the recommended robust test by using the S estimator (M_s was compared with the Box M statistic in terms of the power of test. The result study displayed that M_s statistic performs well in terms of power of test. As a conclusion, M_s statistic is more power than M-statistic.

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Table 1: Power of test for variable, p = 5

		n = 10		n = 20		n = 30		n = 40		n = 50		n = 100	
Values	t	M	M_s	M	M_s	M	M_s	M	M_s	M	M_s	M	M_2
0	0.0	0.0456	0.0540	0.0491	0.0508	0.0569	0.0486	0.0492	0.0538	0.0445	0.0490	0.0558	0.0474
	0.1	0.2852	0.0692	0.0808	0.0860	0.0962	0.1172	0.0964	0.1410	0.1268	0.1764	0.2080	0.3394
	0.3	0.3186	0.2162	0.4024	0.5132	0.4240	0.7656	0.7626	0.9102	0.8742	0.9652	0.9940	1
	0.5	0.4519	0.6554	0.9377	0.9782	0.9943	0.9993	0.9994	1	0.9999	1	1	1
	0.7	0.6264	0.9856	0.9998	1	1	1	1	1	1	1	1	1
	0.8	0.8230	0.9988	1	1	1	1	1	1	1	1	1	1
0.05	0.0	0.0637	0.0392	0.1006	0.0334	0.1060	0.0348	0.1265	0.0371	0.0942	0.0346	0.0548	0.0492
	0.1	0.0702	0.0492	0.1274	0.0470	0.1408	0.0606	0.1052	0.0712	0.1280	0.1878	0.2144	0.3456
	0.3	0.1490	0.1198	0.3514	0.2708	0.4452	0.5812	0.7618	0.6612	0.8640	0.9682	0.9928	1
	0.5	0.3965	0.4394	0.8609	0.9040	0.9727	0.9912	0.9942	0.9993	0.9983	1	1	1
	0.7	0.8878	0.9506	0.9994	1	1	1	1	1	1	1	1	1
	0.8	0.9930	0.9994	1	1	1	1	1	1	1	1	1	1
0.1	0.0	0.0500	0.0376	0.0521	0.0304	0.0516	0.0336	0.0564	0.0297	0.0421	0.0314	0.0536	0.0430
	0.1	0.0500	0.0492	0.0600	0.0412	0.0658	0.0512	0.0930	0.0634	0.1146	0.1802	0.2162	0.3422
	0.3	0.0818	0.1032	0.2122	0.2434	0.3500	0.5390	0.7626	0.6152	0.8546	0.9616	0.9904	1
	0.5	0.2580	0.3970	0.7795	0.8732	0.9515	0.9878	0.9897	0.9989	0.9958	1	1	1
	0.7	0.8206	0.9446	0.9996	1	1	1	1	1	1	1	1	1
	0.8	0.9908	0.9986	1	1	1	1	1	1	1	1	1	1
0.15	0.0	0.0337	0.0400	0.0443	0.0310	0.0399	0.0320	0.0373	0.0308	0.0223	0.0318	0.0520	0.0454
	0.1	0.0428	0.0488	0.0408	0.0468	0.0514	0.0506	0.0996	0.0642	0.1218	0.1800	0.2078	0.3396
	0.3	0.0604	0.0942	0.1848	0.2306	0.2970	0.6238	0.7660	0.5938	0.8528	0.9616	0.9936	1
	0.5	0.2206	0.3868	0.7566	0.8786	0.9444	0.9834	0.9898	0.9989	0.9955	1	1	1
	0.7	0.8088	0.9456	0.9998	1	1	1	1	1	1	1	1	1
	0.8	0.9898	0.9984	1	1	1	1	1	1	1	1	1	1
0.2	0.0	0.0362	0.0330	0.0400	0.0302	0.0367	0.0308	0.0359	0.0313	0.0212	0.0302	0.0582	0.0478
	0.1	0.0358	0.0438	0.0454	0.0404	0.0438	0.0468	0.1124	0.0614	0.1212	0.1686	0.2216	0.3574
	0.3	0.0636	0.0948	0.1780	0.2256	0.2892	0.5032	0.7708	0.5786	0.8590	0.9616	0.9904	1
	0.5	0.2082	0.3882	0.7490	0.8686	0.9440	0.9866	0.9892	0.9985	0.9968	1	1	1
	0.7	0.8054	0.9332	1	0.9998	1	1	1	1	1	1	1	1
	0.8	0.9856	0.9998	1	1	1	1	1	1	1	1	1	1

Table 2: Power of test for variable, p = 10

Values	t	n = 10 M	n = 10 M_s	n = 20 M	n = 20 M_s	n = 30 M	n = 30 M_s	n = 40 M	n = 40 M_s	n = 50 M	n = 50 M_s	n = 100 M	n = 100 M_s
0	0.0	0.05	0.0460	0.0486	0.0450	0.0516	0.0466	0.0588	0.0476	0.0558	0.0428	0.0514	0.0468
	0.1	0.065	0.0629	0.1002	0.1150	0.1332	0.1696	0.1736	0.2392	0.2202	0.3014	0.4564	0.6760
	0.3	0.1498	0.2408	0.6448	0.7522	0.8966	0.9636	0.9812	0.9958	0.9978	0.9990	1	1
	0.5	0.4888	0.7437	0.9982	0.9992	1	1	1	1	1	1	1	1
	0.7	0.9808	0.9979	1	1	1	1	1	1	1	1	1	1
	0.8	1	1	1	1	1	1	1	1	1	1	1	1
0.05	0.0	0.0486	0.0478	0.0554	0.043	0.0568	0.0496	0.0530	0.0528	0.0574	0.0428	0.0608	0.0474
	0.1	0.0498	0.0634	0.0966	0.1032	0.1236	0.1694	0.1680	0.2290	0.2186	0.3156	0.4630	0.6784
	0.3	0.1418	0.2476	0.6382	0.7456	0.8976	0.9608	0.9826	0.9950	0.9980	0.9996	1	1
	0.5	0.4786	0.7487	0.9980	0.9994	1	1	1	1	1	1	1	1
	0.7	0.9804	0.9978	1	1	1	1	1	1	1	1	1	1
	0.8	1	1	1	1	1	1	1	1	1	1	1	1
0.1	0.0	0.0476	0.0456	0.0548	0.043	0.0502	0.0464	0.0588	0.0458	0.0536	0.0480	0.0548	0.0544
	0.1	0.0566	0.0652	0.0964	0.1044	0.1200	0.1668	0.1702	0.2404	0.2216	0.3012	0.4502	0.6776
	0.3	0.1362	0.2508	0.6336	0.7504	0.8940	0.9598	0.9818	0.9956	0.9976	0.9996	1	1
	0.5	0.4992	0.7450	0.9974	0.9992	1	1	1	1	1	1	1	1
	0.7	0.9824	0.9974	1	1	1	1	1	1	1	1	1	1
	0.8	1	1	1	1	1	1	1	1	1	1	1	1
0.15	0.0	0.0536	0.0480	0.0552	0.0466	0.0526	0.0480	0.0560	0.0426	0.0552	0.0444	0.0588	0.0512
	0.1	0.057	0.0646	0.0888	0.1008	0.1238	0.1650	0.1786	0.2314	0.2172	0.3072	0.4600	0.6830
	0.3	0.1436	0.2452	0.6404	0.7604	0.8990	0.9616	0.9806	0.9956	0.9962	0.9992	1	1
	0.5	0.4936	0.7416	0.9982	0.999	1	1	1	1	1	1	1	1
	0.7	0.983	0.9972	1	1	1	1	1	1	1	1	1	1
	0.8	1	1	1	1	1	1	1	1	1	1	1	1
0.2	0.0	0.0508	0.0466	0.0486	0.0466	0.0546	0.0492	0.0568	0.0450	0.0528	0.0452	0.0528	0.0494
	0.1	0.0538	0.0682	0.0960	0.1136	0.1240	0.1704	0.1658	0.2366	0.2198	0.3064	0.4584	0.6760
	0.3	0.147	0.2448	0.6360	0.752	0.9016	0.9628	0.9826	0.9932	0.9970	0.9994	1	1
	0.5	0.4944	0.7482	0.9968	0.9992	1	1	1	1	1	1	1	1
	0.7	0.983	0.9970	1	1	1	1	1	1	1	1	1	1
	0.8	1	1	1	1	1	1	1	1	1	1	1	1

Table 3: Power of test for variable, p = 15

Values	t	n = 20 M	n = 20 M_s	n = 30 M	n = 30 M_s	n = 40 M	n = 40 M_s	n = 50 M	n = 50 M_s	n = 100 M	n = 100 M_s
0	0.0	0.0486	0.0450	0.0445	0.0490	0.0588	0.0476	0.0516	0.0466	0.0514	0.0468
	0.1	0.1002	0.1250	0.1268	0.1764	0.1736	0.2392	0.1332	0.6960	0.4564	0.6760
	0.3	0.6448	0.7622	0.8742	0.9652	0.9812	0.9958	0.8966	0.9636	1	1
	0.5	0.9982	0.9992	0.9999	1	1	1	1	1	1	1
	0.7	1	1	1	1	1	1	1	1	1	1
	0.8	1	1	1	1	1	1	1	1	1	1
0.05	0.0	0.0554	0.0430	0.0942	0.0346	0.0530	0.0528	0.0568	0.0496	0.0608	0.0474
	0.1	0.0966	0.1032	0.1280	0.1878	0.1680	0.2290	0.1236	0.6943	0.4630	0.6784
	0.3	0.6382	0.7456	0.8640	0.9682	0.9826	0.9950	0.8976	0.9608	1	1
	0.5	0.9980	0.9994	0.9983	1	1	1	1	1	1	1
	0.7	1	1	1	1	1	1	1	1	1	1
	0.8	1	1	1	1	1	1	1	1	1	1
0.1	0.0	0.0548	0.0430	0.0421	0.0314	0.0588	0.0478	0.0502	0.0464	0.0548	0.0533
	0.1	0.0964	0.1044	0.1146	0.1802	0.1702	0.2404	0.1200	0.4668	0.4502	0.6776
	0.3	0.6336	0.7504	0.8546	0.9616	0.9818	0.9956	0.8940	0.9598	1	1
	0.5	0.9974	0.9992	0.9958	1	1	1	1	1	1	1
	0.7	1	1	1	1	1	1	1	1	1	1
	0.8	1	1	1	1	1	1	1	1	1	1
0.15	0.0	0.0552	0.0466	0.0223	0.0318	0.056	0.0426	0.0526	0.0480	0.0588	0.0512
	0.1	0.0898	0.1008	0.1218	0.18	0.1786	0.2314	0.1238	0.4565	0.4600	0.6830
	0.3	0.6404	0.7604	0.8528	0.9616	0.9806	0.9956	0.8990	0.9616	1	1
	0.5	0.9982	0.9990	0.9955	1	1	1	1	1	1	1
	0.7	1	1	1	1	1	1	1	1	1	1
	0.8	1	1	1	1	1	1	1	1	1	1
0.2	0.0	0.0486	0.0466	0.0212	0.0302	0.0568	0.0450	0.0546	0.0492	0.0528	0.0494
	0.1	0.0960	0.1136	0.1212	0.1686	0.1658	0.2366	0.1240	0.4704	0.4584	0.6760
	0.3	0.6360	0.7520	0.859	0.9616	0.9826	0.9932	0.9016	0.9628	1	1
	0.5	0.9968	0.9992	0.9968	1	1	1	1	1	1	1
	0.7	1	1	1	1	1	1	1	1	1	1
	0.8	1	1	1	1	1	1	1	1	1	1

Table 4: Power of test for variable, p = 15

		n = 20		n = 30		n = 40		n = 50		n = 100	
Values	t	M	M_s	M	M_s	M	M_s	M	M_s	M	M_s
0	0.0	0.0445	0.049	0.0516	0.0466	0.0588	0.0476	0.0516	0.0466	0.0514	0.0468
	0.1	0.1268	0.1764	0.1332	0.1696	0.1736	0.2392	0.1332	0.5696	0.564	0.676
	0.3	0.8742	0.9652	0.8966	0.9636	0.9812	0.9958	0.8966	0.9636	1	1
	0.5	0.9999	1	1	1	1	1	1	1	1	1
	0.7	1	1	1	1	1	1	1	1	1	1
	0.8	1	1	1	1	1	1	1	1	1	1
0.05	0.0	0.0942	0.0346	0.0568	0.0496	0.053	0.0528	0.0568	0.0496	0.0608	0.0474
	0.1	0.128	0.1878	0.1236	0.1694	0.168	0.229	0.1236	0.5694	0.5463	0.6784
	0.3	0.864	0.9682	0.8976	0.9608	0.9826	0.995	0.8976	0.9608	1	1
	0.5	0.9983	1	1	1	1	1	1	1	1	1
	0.7	1	1	1	1	1	1	1	1	1	1
	0.8	1	1	1	1	1	1	1	1	1	1
0.1	0.0	0.0421	0.0314	0.0502	0.0464	0.0588	0.0478	0.0502	0.0464	0.0548	0.0533
	0.1	0.1146	0.1802	0.12	0.1668	0.1702	0.2404	0.12	0.5668	0.5024	0.6776
	0.3	0.8546	0.9616	0.894	0.9598	0.9818	0.9956	0.894	0.9598	1	1
	0.5	0.9958	1	1	1	1	1	1	1	1	1
	0.7	1	1	1	1	1	1	1	1	1	1
	0.8	1	1	1	1	1	1	1	1	1	1
0.15	0.0	0.0223	0.0318	0.0526	0.048	0.056	0.0426	0.0526	0.048	0.0588	0.0512
	0.1	0.1218	0.18	0.1238	0.165	0.1786	0.2314	0.1238	0.6165	0.46	0.683
	0.3	0.8528	0.9616	0.899	0.9616	0.9806	0.9956	0.899	0.9616	1	1
	0.5	0.9955	1	1	1	1	1	1	1	1	1
	0.7	1	1	1	1	1	1	1	1	1	1
	0.8	1	1	1	1	1	1	1	1	1	1
0.2	0.0	0.0212	0.0302	0.0546	0.0492	0.0568	0.045	0.0546	0.0492	0.0528	0.0494
	0.1	0.1212	0.1686	0.124	0.1704	0.1658	0.2366	0.124	0.7014	0.4584	0.676
	0.3	0.859	0.9616	0.9016	0.9628	0.9826	0.9932	0.9016	0.9628	1	1
	0.5	0.9968	1	1	1	1	1	1	1	1	1
	0.7	1	1	1	1	1	1	1	1	1	1
	0.8	1	1	1	1	1	1	1	1	1	1

Table 5: Power of test for variable, p = 20

		n = 40		n = 50		n = 100	
Values	t	M	M_s	M	M_s	M	M_s
0	0.0	0.0588	0.0476	0.0445	0.049	0.0528	0.0558
	0.1	0.1736	0.2392	0.1268	0.7164	0.9944	0.9976
	0.3	0.9812	0.9958	0.8742	0.9652	1	1
	0.5	1	1	1	1	1	1
	0.7	1	1	1	1	1	1
	0.8	1	1	1	1	1	1
0.05	0.0	0.0530	0.0528	0.0942	0.0346	0.0500	0.052
	0.1	0.1680	0.2290	0.3128	0.8978	0.9898	0.997
	0.3	0.9826	0.9950	0.8640	0.9682	1	1
	0.5	1	1	1	1	1	1
	0.7	1	1	1	1	1	1
	0.8	1	1	1	1	1	1
0.1	0.0	0.0588	0.0478	0.0421	0.0314	0.0409	0.0502
	0.1	0.1702	0.2404	0.4146	0.8002	0.9872	0.9974
	0.3	0.9818	0.9956	0.8546	0.9616	1	1
	0.5	1	1	1	1	1	1
	0.7	1	1	1	1	1	1
	0.8	1	1	1	1	1	1
0.15	0.0	0.0560	0.0426	0.0223	0.0318	0.0499	0.0508
	0.1	0.3786	0.2314	0.5218	0.8312	0.9897	0.9968
	0.3	0.9806	0.9956	0.8528	0.9616	1	1
	0.5	1	1	1	1	1	1
	0.7	1	1	1	1	1	1
	0.8	1	1	1	1	1	1
0.2	0.0	0.0568	0.045	0.0212	0.0302	0.050	0.0506
	0.1	0.3658	0.2366	0.5212	0.6786	0.996	0.9978
	0.3	0.9826	0.9932	0.8590	0.9616	1	1
	0.5	1	1	1	1	1	1
	0.7	1	1	1	1	1	1
	0.8	1	1	1	1	1	1

Table 6: Power of test for variable, p = 30

		n = 40		n = 50		n = 100	
Values	t	M	M_s	M	M_s	M	M_s
0	0.0	0.0538	0.0466	0.0445	0.049	0.0558	0.0528
	0.1	0.2898	0.6967	0.4268	0.7564	0.9976	0.9944
	0.3	0.9868	0.9636	0.8742	0.9652	1	1
	0.5	1	1	1	1	1	1
	0.7	1	1	1	1	1	1
	0.8	1	1	1	1	1	1
0.05	0.0	0.0568	0.0496	0.0942	0.0346	0.052	0.05
	0.1	0.1236	0.6943	0.4028	0.8978	0.997	0.9898
	0.3	0.8976	0.9608	0.864	0.9682	1	1
	0.5	1	1	0.9983	1	1	1
	0.7	1	1	1	1	1	1
	0.8	1	1	1	1	1	1
0.1	0.0	0.0502	0.0464	0.0421	0.0314	0.0502	0.0409
	0.1	0.12	0.6668	0.6146	0.8502	0.9974	0.9872
	0.3	0.894	0.9598	0.8546	0.9616	1	1
	0.5	1	1	1	1	1	1
	0.7	1	1	1	1	1	1
	0.8	1	1	1	1	1	1
0.15	0.0	0.0526	0.048	0.0223	0.0318	0.0508	0.0499
	0.1	0.1238	0.6235	0.5418	0.7845	0.9968	0.9897
	0.3	0.899	0.9616	0.8528	0.9616	1	1
	0.5	1	1	1	1	1	1
	0.7	1	1	1	1	1	1
	0.8	1	1	1	1	1	1
0.2	0.0	0.0546	0.0492	0.0212	0.0302	0.0506	0.05
	0.1	0.124	0.7804	0.5212	0.6876	0.9978	0.996
	0.3	0.9016	0.9628	0.859	0.9616	1	1
	0.5	1	1	1	1	1	1
	0.7	1	1	1	1	1	1
	0.8	1	1	1	1	1	1

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