

Cognitive Analysis and Predicting Mathematics Training Outcomes of BA (Economics) Students

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Abstract: The study applies fuzzy cognitive analysis to the mechanism of developing personal cognitive styles of learners (Bachelor students majoring in Economics) in their study of mathematics. The approach is based on the method of illustrative training and modeling. The technology of fuzzy cognitive maps has enabled the authors to define the indices that characterize the impact of environmental components on the results of student's personal cognitive styles development.

Key words: Individualized training, personal cognitive style, illustrative training and modeling, fuzzy cognitive map, fuzzy

INTRODUCTION

The current stage of reforms in the Russian system of higher education is accompanied by an enhancing role of mathematical knowledge in economic education. Educational practice has convincingly shown that now it is impossible to become a qualified specialist in Economics, meeting the modern demands and requests of information society without acquiring basic mathematical knowledge and skills. By now, the volume of scientific papers that study theoretical and applied aspects of improving the system of economic education has largely increased. A lot of studies are aimed at the mechanism that enables mathematical knowledge to influence the development of professional competences of an economic university graduate (Smirnova and Nadezhdin, 2016). The concept of personal experience funding based on regular use of illustrative training and modeling is one of the upcoming trends in providing fundamental training to future economists (Abaturova *et al.*, 2004; Smimov, 2012). This concept is based on the idea of targeted complex development of a student's personality. Stage-by-stage improvement of a student's Personal Cognitive Style (PCS) with account for interdisciplinary dialogue and integration of mathematical, economic and information culture is one of the key components of personal experience funding.

The goal of the study is to make a cognitive analysis of the Development Mechanism of Personal Cognitive

Style (PCS) of a learner's (student's) personality during the study of mathematics. This analysis is based on illustrative training and modeling, identification and quantitative evaluation of the influence of key components (hereinafter referred to as concepts) in the system of individual training methodology based on mathematical apparatus of fuzzy cognitive maps.

As soon as the system of higher education moved to the educational paradigm based on the development of a learner's personality with its set of professional competences, creativity and readiness for social integration, the didactic tasks of the principal educational program had to be corrected to intentionally develop special mathematical skills in learners. Usually, Methodological System of Individualized Education (MSIE) for future economists is developed by traditional pedagogical methods. The cybernetic nature of MSIE which is the object of this study, together with the growing demands of business environment, constantly stimulates the search for new methods of formal description, analysis and improvement of the mechanism of individualization of the future economist's training.

The method of fuzzy cognitive modeling is one of the most productive approaches to the analysis of weakly structured systems and processes (Erokhin *et al.*, 2006; Silov, 1995; Nadezhdin, 2016). The information about an explored system is represented as a set of concepts (factors) and a cause-and-effect network to connect them (this network is called a cognitive map). The cognitive

map (CP) reflects an expert's (or a group of expert's) subjective ideas of the laws and regularities typical for the modeled system. In fact, creating the CP of the modeled system means removing the uncertainty through collection (extraction) and formalization of expert knowledge about this system. The created cognitive model is used with the methods of analytical processing focused on studying the structure of the system and predicting its behavior in case of various controlling influences. The final goal is to find the optimal controlling strategies.

MATERIALS AND METHODS

This study uses Silov (1995)'s Fuzzy Cognitive Maps (FCM). These cognitive maps provide a highly objective mathematical description and proper accuracy of process modeling in comparison with the classic sign cognitive maps because they account for consolidated opinion of several experts and lead to generation and selection or controlling strategies as well as their dynamic modeling. In general, a cognitive map is a cause and effect semantic network established in the form of an oriented graph (Erokhin *et al.*, 2006):

$$G = \langle E, W \rangle$$

Where:

$E = \{e_1, e_2, \dots, e_n\}$ = A variety of factors (called concepts)
 W = A binary relation on multitude E
 which assigns a set of relations
 between its elements

We call this cause-and-effect network the cognitive map of a modeled system. Elements e_i and e_j are considered bound by relationship W which is indicated as $e_i W e_j$, if the change of concept value e_i (cause) results in the change of concept value e_j (effect). Pursuant to the terms of cognitive modeling, it means that concept e_i makes an appropriate impact on concept e_j . Let us consider the impact positive ("enhancement") if the increase of the concept-cause value results in the increase of concept-result value and vice versa, negative ("inhibition") if the concept-effect value decreases simultaneously with the decrease of the concept-cause value.

The concept of Silov's fuzzy cognitive map is a broadened classic concept of CM based on the assumption that mutual influences between concepts can vary in intensity and their intensity can change with time. Therefore, we introduce the index of influence intensity

and move from classic relationship to fuzzy relationship W , the elements $w_{i,j}$ of which describe the direction and degree of intensity (weight) of the influence between concepts e_i and e_j :

$$w_{i,j} = w(e_i, e_j)$$

where, w is the normed index of influence intensity (characteristic function of relation W) which has a number of special qualities (Silov, 1995). Among these qualities, we specify the following: $w_{i,j}$ takes the value from interval $[-1; 1]$ with intermediate degree of positive or negative impact.

A fuzzy cognitive map reflects the studied object as a weighted-oriented graph, the tops of which correspond to the elements of multitude E (concepts) and the arcs correspond to non-zero elements of relationships W , i.e. to cause-and-effect relationships. Each arc has its weight assigned by the corresponding value $w_{i,j}$. Relation W is represented in the form of cognitive matrix $W = \{w_{i,j}, i, j = \overline{1, n}\}$ ($n \times n$) (n is the number of concepts in the system) which will be interpreted as the adjacency matrix of this graph. The current state of system is defined by a set of values of all FCM concepts.

Let us accept the condition that an expert analysis of a subject area enabled to identify a number of factors (concepts) which make the strongest impact on the development of PCS of a future bachelor in economics. The creation of the cognitive model of the mechanism of developing PCS in the form of FCM involved a current hypothesis that the factors (concepts) are interdependent. Let us conventionally imagine the process of fuzzy cognitive analysis of the mechanism of developing a learner's PCS in the form of several stages.

Stage 1: Creating a cognitive map of PCS development mechanism based on heuristic data obtained during implementation of special procedures for extracting and processing information according to the survey of several experts.

Suppose the experts reveal 13 essential factors (concepts) at the first stage that define the efficiency of training and are conditionally divided into 4 groups (Table 1). At the second stage, the experts define cause-and-effect connections between the factors and reveal the character of each connection (enhancing or weakening). These actions result in creating a cognitive map of educational process in university (Fig. 1) which characterizes the mechanism of cause-and-effect relationships without including the intensity of the mutual influence. The cognitive map is represented in Fig. 1. As the third step, the experts evaluate the power of

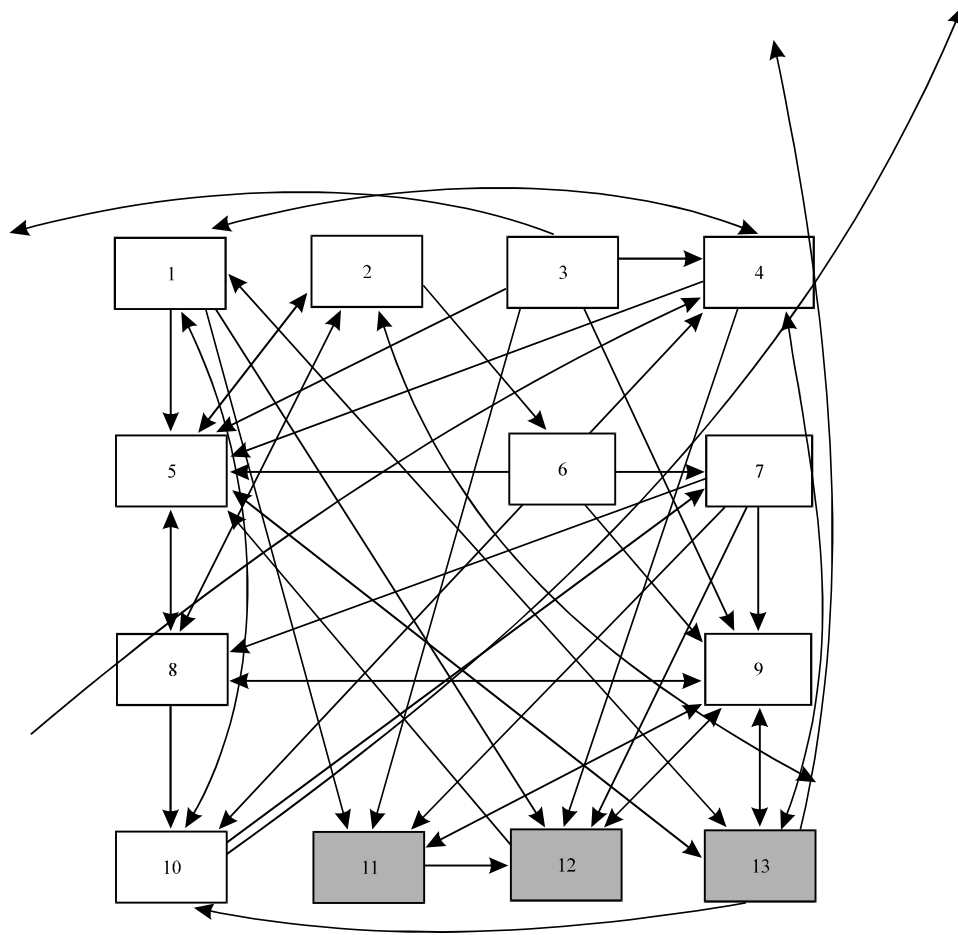


Fig. 1: Cognitive model for the mechanism of PCS development (resulting concepts on the scheme are highlighted in gray)

Table 1: Summary table of cognitive model concepts

Concept name	Concept identifier
Individual qualities of learners	
Basic mathematical knowledge and skills	e_1
Motivation	e_2
Elementary level of personal cognitive style	e_3
Components of training system methodology	
Active training methods	e_4
Research work	e_5
Illustrative training and modeling	e_6
Innovation (scalability, flexibility) of funding mechanism	e_7
Informational and educational environment in the context of interaction of mathematical, economic and information cultures	e_8
Results of training	
Styles of information coding	e_{11}
Cognitive style	e_{12}
Thinking styles and epistemological styles	e_{13}
Additional factors	
Academic load of professors and instructors	e_{10}
Conservatism of education system (system of training methods)	e_9

Table 2: Scale to formalize influence power between concepts

Scale elements	Interpretation in terms of influence power
0.0	No influence
0.1	Minimal possible (almost no) influence
0.3	Weak
0.5	Average
0.7	Significant
0.9	Strong
1.0	Maximal possible

expert. Finally, the first stage results in creation of a cognitive matrix (Table 2), the elements of which are defined as average evaluations of the impact intensity according to the number of experts.

The obtained FCM represents the most important direct relationships between the concepts. We need information about the fuzzy manifestations of the concept's influence on each other and the result of the educational process to analyze the cause-and-effect structure of the PCS development mechanism in detail.

the concept's influence on each other. A scale is applied to formally represent the qualitative evaluations of each

Table 3: Cognitive matrix of PCS development mechanism $W = \{w_{i,j}, i, j = \overline{1,13}\}$

Concept No.	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0.00	0.00	0	0.0	0.80	0.00	0.0	0.00	0.00	0.0	0.40	0.5	0.62
2	0.00	0.00	0	0.1	0.70	-0.35	0.0	0.00	0.00	0.0	0.00	0.0	0.00
3	0.25	0.00	0	0.7	0.40	0.00	0.0	0.00	0.85	0.0	0.75	0.0	0.00
4	0.00	0.82	0	0.0	0.65	0.00	0.0	0.00	0.00	0.0	0.00	0.6	0.00
5	0.00	0.45	0	0.0	0.00	0.00	0.0	0.60	0.00	0.0	0.00	0.0	0.71
6	0.00	0.00	0	-0.5	-0.30	0.00	-0.8	0.00	-0.40	0.7	0.00	0.0	0.00
7	0.00	0.00	0	0.0	0.00	0.00	0.0	0.45	0.90	0.0	0.50	0.6	0.70
8	0.00	0.75	0	0.8	0.50	0.00	0.0	0.00	0.75	-0.3	0.00	0.0	0.00
9	0.00	0.55	0	0.0	0.00	0.00	0.0	0.70	0.00	0.0	0.90	0.8	0.82
10	0.00	0.00	0	-0.6	-0.50	0.00	-0.8	0.00	0.00	0.0	0.00	0.0	0.00
11	0.00	0.00	0	0.0	0.00	0.00	0.6	0.00	0.80	0.0	0.00	0.7	0.00
12	0.00	0.00	0	0.0	0.65	0.00	0.0	0.00	0.70	0.6	0.00	0.0	0.00
13	0.20	0.00	0	0.5	0.70	0.00	0.8	0.00	0.90	0.0	0.00	0.0	0.00

Stage 2: Quantitative evaluation of indirect mutual influence of concepts. For this purpose, the operation of cognitive matrix transitive closing is carried out as the first step. Out of many known ways of adjacency matrix transitive closing, let us use the algorithm recommended by Silov (1995). The algorithm involves the following acts.

We move from original cognitive matrix (Table 3) to cognitive matrix of positive relationships R with dimension $(2 \cdot n \times 2 \cdot n)$ based on substitution procedure:

$$\text{if } w_{i,j} > 0, \text{ mo } r_{2i-1, 2j-1} = w_{i,j}, r_{2i, 2j} = w_{i,j}$$

$$\text{if } -w_{i,j} < 0, \text{ mo } r_{2i-1, 2j} = -w_{i,j}, r_{2i, 2j-1} = -w_{i,j}$$

Other elements of matrix R take zero meaning. We define transitive closing of fuzzy relationship R in accordance with the following equation (ibid., 29):

$$\tilde{R} = \bigcup_{i=1}^n R^i = R \cup R^2 \cup \dots \cup R^n \text{ where } R^2 = R \times R$$

The product of fuzzy relationships is calculated according to the procedure:

$$\text{if } D = A \times B, \text{ then } d_{i,j} = \max_{k=1, \dots, n} a_{i,k} \cdot b_{k,j}, i, j = 1, \dots, n$$

We move from auxiliary matrix \tilde{R} to transitively closed cognitive matrix V , the elements of which will be pairs $(v_{i,j}, \tilde{v}_{i,j})$ where $v_{i,j}$ and $\tilde{v}_{i,j}$ characterize the power of positive and negative impact of i concept on j concept, respectively:

$$v_{i,j} = \max(r_{2i-1, 2j-1}, r_{2i, 2j}), \tilde{v}_{i,j} = -\max(r_{2i-1, 2j}, r_{2i, 2j-1})$$

Calculation of two main private indices of a cognitive map; impact of the i concept on the j concept:

$$h_{i,j} = \text{sgn}(v_{i,j} + \tilde{v}_{i,j}) \cdot \max(|v_{i,j}|, |\tilde{v}_{i,j}|), |v_{i,j}| \neq |\tilde{v}_{i,j}| \quad (1)$$

Consonance of the influence of i concept on the j concept:

$$c_{i,j} = \frac{|v_{i,j} + \tilde{v}_{i,j}|}{|v_{i,j}| + |\tilde{v}_{i,j}|} \quad (2)$$

which expresses the degree of trust to the sign of impact. The obtained data may also help to define the integral indices of consonance, dissonance and impact (ibid., 102). The calculation formulas of these indices are summarized in Table 4.

Stage 3: Analysis of calculation results and justification of recommendations. In accordance with the above mentioned calculation ratios based on obtained transitively closed cognitive matrix $V = [(v_{i,j}, \tilde{v}_{i,j}), i, j = \overline{1, n}]$, we calculate a multitude of partial (Eq. 1 and 2) and systemic (Table 4) indices of fuzzy cognitive model.

Our study defines systemic indices which reflect the influence of cognitive model concepts on the development of information coding style (concept e_{12}), cognitive style (concept e_{11}), thinking styles and epistemological styles (concept e_{13}) of a learner. These data are represented in Table 5.

In accordance with the above mentioned calculation ratios based on obtained transitively closed cognitive matrix $V = [(v_{i,j}, \tilde{v}_{i,j}), i, j = \overline{1, n}]$ we calculate a multitude of partial (Eq. 1 and 2) and systemic (Table 4) indices of fuzzy cognitive model. Table 5 represents the values of system indices of fuzzy cognitive model for PCS DM (further referred to as system). The analysis of the obtained results shows that the positive influence of concepts e_1 , e_2 and e_9 and the negative influence of concept e_{10} dominate in the mechanism of learner's PCS development. Let us comment on the results of the fuzzy cognitive analysis in accordance with Silov (1995)'s recommendations.

Table 4: FCM systemic indices

Index name	Calculation formula
Consonance of influence of concept i on the system	$\bar{H}_i = \sum_{j=1}^n \alpha_{i,j} / n$, $\alpha_{i,j}$ is consonance of influence of concept i on j concept
Consonance of the system's influence on concept j	$\bar{H}_j = \sum_{i=1}^n \alpha_{i,j} / n$, $\alpha_{i,j}$ is consonance of influence of concept i on j concept
Dissonance of influence of concept i on the system	$\bar{D}_i = \sum_{j=1}^n d_{i,j} / n$, $d_{i,j}$ is dissonance of influence of concept i on j concept
Dissonance of influence of the system on concept j	$\bar{D}_j = \sum_{i=1}^n d_{i,j} / n$
Influence of concept i on the system	$\bar{P}_i = \sum_{j=1}^n \omega_{i,j} / n$, $\omega_{i,j}$ is influence of concept i on j concept
Influence of the system on concept j	$\bar{P}_j = \sum_{i=1}^n \omega_{i,j} / n$
Index of influence centralization	$E_i^* = \bar{P}_i - \bar{D}_i$

Table 5: Results of calculation of fuzzy cognitive model systemic indices

Indices of consonance		Indices of dissonance		Indices of influence		Indices of centralization of influence (E_i)
\bar{H}_i	\bar{H}_j	\bar{D}_i	\bar{D}_j	\bar{P}_i	\bar{P}_j	
0.430	0.500	0.570	0.500	0.384	0.104	0.280
0.476	0.442	0.524	0.558	0.260	0.364	-0.104
0.432	0.000	0.568	1.000	0.490	0.000	0.490
0.347	0.360	0.653	0.640	0.351	0.332	0.019
0.451	0.492	0.549	0.508	0.409	0.434	-0.025
0.415	0.466	0.585	0.534	-0.341	-0.146	-0.195
0.392	0.306	0.608	0.694	0.485	0.265	0.220
0.428	0.474	0.572	0.526	0.458	0.386	0.072
0.383	0.432	0.617	0.568	0.519	0.482	0.037
0.442	0.328	0.558	0.672	-0.440	0.324	-0.764
0.371	0.442	0.629	0.558	0.445	0.452	-0.007
0.200	0.467	0.800	0.533	0.345	0.431	-0.086
0.420	0.477	0.580	0.523	0.520	0.457	0.062

RESULTS AND DISCUSSION

Concepts e_1 , e_2 and e_3 in this cognitive model represent the primary factors that characterize individual qualities of learners (students). The strongest influence of these concepts is revealed at the opening stages of training which is linked to the task of choosing optimal educational trajectory. With the targeted impact of the system, concept e_1 can change to some extent ($\bar{P}_1 = 0.104$). In this case, its influence on the development of learner's cognitive style (concept e_{12}) can become more significant. Concept e_2 in turn, is subjected to positive impact of the system ($\bar{P}_2 = 0.364$) too and actively increases the quality of training if there are appropriate educational conditions. Concept e_3 expresses a starting level of student's personal professional style and the influence that the system has on him is hardly identified ($\bar{P}_3 = 0$).

Concept e_4 "active training methods" makes a significant impact on the system ($\bar{P}_1 = 0.351$) while the system in its turn, positively influences the development of this concept with the same power ($\bar{P}_1 = 0.332$). This shows (with account for the value of concept dissonance)

that educational system can initiate the development and implementation of new forms of active training in the educational process at particular stages of training.

Concept e_5 "Research work" as our calculations have confirmed has a great didactic potential. The influence of concept₅ on the system is quite significant ($\bar{P}_5 = 0.409$), the system in its turn, makes a strong positive impact on this concept ($\bar{P}_5 = 0.434$).

Concept e_6 "conservatism of education system (system of training methods)" acts as a damping factor during the implementation of innovations in educational process. The influence of the concept e_6 on the system is quite significant ($\bar{P}_6 = -0.341$), the system in its turn, poses strong positive impact on this concept ($\bar{P}_6 = 0.195$). Concept e_7 "Innovation of funding mechanism" is considered as the main generating factor which influences the system ($\bar{P}_7 = 0.485$) most significantly while the system enhances the influence of this concept ($\bar{P}_7 = 0.265$). According to the calculation results, this regularity is stable (as shown by rather high consonance coefficients).

Concept e_8 "informational and educational environment in the context of interaction of mathematical,

economic and information cultures”, the content of which characterizes methodological and intellectual potential of university, makes the strongest influence on the system ($\bar{p}_8 = 0.458$). The influence of the system on concept e_8 is manifested a little weaker ($\bar{p}_8 = 0.386$). The proximity of the system’s consonances and concept ($\bar{u}_8 = 0.428$ and $\bar{u}_8 = 0.474$) shows that there are good perspectives of system enhancement based on further development of services of informational and educational environment towards individualized education.

The influence of concept e_9 “illustrative training and modeling” (group B) on the system is evaluated as the most significant ($\bar{p}_9 = 0.519$). Moreover, the data of cognitive modeling show that the system actively enhances the weight influence of this concept ($\bar{p}_9 = 0.482$) on the results of study.

Concept e_{10} “academic load of professors and instructors” characterizes the number of classes on professor’s and instructor’s schedule. In case of overloaded schedule, excessive normative values of individual workload significantly restrict a professor’s opportunities for creative research, development and implementation of innovative educational technologies. The negative influence of concept e_{10} on the system is quite significant ($\bar{p}_{10} = -0.440$). Besides, the system contributes to further enhancement of this concept ($\bar{p}_{10} = 0.324$) in strictly regulated educational activity. The results of the cognitive analysis demonstrate that the greatest positive effect should be expected from an agreed change of a group of controlled FCM concepts which are located in the chain of cause-and-effect relationships and together provide a stable impact on the system-PCSDM. Three chains of concepts can be typical examples of this:

$$e_1 \rightarrow e_5 \rightarrow e_8 \rightarrow e_9 \rightarrow e_{11}, e_2 \rightarrow e_5 \rightarrow e_{13}$$

$$\text{and } e_2 \rightarrow e_9 \rightarrow e_{11} \rightarrow e_{12}$$

The results of the critical analysis and weighted evaluation of the potential of this resource can give a new impulse to innovative transformations in the methodological system of training future economists with an emphasis on individualized education.

CONCLUSION

The automated fuzzy cognitive analysis enabled us to reveal significant FCM concepts: e_7 -innovation of funding mechanism (personal experience) e_8 -informational

and educational environment in context of mathematical, economic and informational cultures; e_9 -illustrative training and modeling. In case of agreed adjustment of these concept’s characteristics with account for a university’s specifics, one can create conditions for better implementation of didactic potential of innovation technology of training based on the method of illustrative modeling. It is necessary to develop specific mechanisms for developing key components of personal cognitive style of learners based on illustrative training and modeling. Implementing the principles of funding personal experience, it is also possible to develop a method of quantitative evaluation of the achieved synergetic effect in order to cope with the traditional conservatism of the current training methodology system. A significant reduction of the negative impact of the factor “Academic load of professors and instructors” (concept e_{10}) on student’s PCS DM can be achieved through implementing the policy of differentiated distribution of academic load in the educational process with account for personal contribution of each professor to the methodology of future economist’s individual training.

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