

## Linear Distribution Algorithm Concerning the Weights of Alternatives During Decision Making Support

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**Abstract:** The study studied the decision-making algorithm at a pairwise comparison of alternatives. The algorithm of alternative linear weight distribution during decision making is proposed for important cases of practical application. An experimental comparison of decision-making results is performed by the method of paired comparison and linear distribution of alternative weights for different amounts of criteria and alternatives. The advantages of the algorithm use concerning alternative linear weight distribution are determined during decision making.

**Key words:** Decision-making methods, decision-making algorithm, pairwise comparison matrix, linear distribution of weights, practical application

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### INTRODUCTION

Non-standard situations are an important area decision making support system application. Such situations are characterized by uncertainty which makes the selection of the best alternative solution difficult. In such situations, the decision-making process involves the use of special means of a Decision Maker (DM) preference system formalizing and a structured comparative analysis of alternatives.

Hierarchy analysis method allows to present a complex decision-making problem in a hierarchical structure, compare and perform a quantitative evaluation of alternative solutions. Using the method of hierarchy analysis one can solve simple problem successfully. This method shows its effectiveness during the solution of complex problems that require a systematic approach and attract a large number of experts (Saaty and Tran 2007; Ozdemir and Saaty 2006; Peng *et al.*, 2011).

However, in order to solve the problems in a number of cases when the amount of time for a problem solution is limited, there is no need for a thorough accuracy of a decision-making result or a number of criteria/alternatives is too large (10 or more). The use of traditional means for decision making, including the ones based on the method of alternative pair comparison which is the basis of hierarchy analysis method, does not provide the required parameters of a solution. In particular, it is extremely difficult to obtain a reliable logical solution with recommended values of coherence index and coherence attitude (Mitihin, 2012; Podinovsky and Podinovskaya, 2011; Podinovskiy and Podinovskaya, 2012).

### MATERIALS AND METHODS

**Decision making solution at pair comparison of alternatives:** The analysis of a decision making problem in the method of hierarchy analysis begins with the construction of the hierarchy which includes a target, alternatives and selection criteria which influence a decision making (Lomakin and Lifirenko, 2014; Lomakin *et al.*, 2015). This structure helps to reflect the understanding of a DM problem.

Priorities are defined using a paired comparison procedure (Lomakin *et al.*, 2014). These priorities represent a relative importance or the preference of elements concerning a developed hierarchical structure.

The synthesis of priorities in a hierarchy is performed during a final stage of analysis. The result of it is the calculation of alternative solution priorities concerning a main goal (Lifirenko and Lomakin, 2013). At that, comparing each pair of alternatives, it is necessary to seek for a consistency value limitation between 0 and 0.1. The best alternative is considered to be the option with the maximum priority value.

In order to take a decision on the basis of a direct assessment of alternatives the following algorithm is used (Fig. 1). However, even when the number of alternatives is <5-6 it is difficult to obtain solutions with desired consistency relationship value. A traditional method of hierarchy analysis method solves this issue by re-filling or correction of paired comparison matrices. However, this is not always possible this raises the issue of a DM judgment reliability.

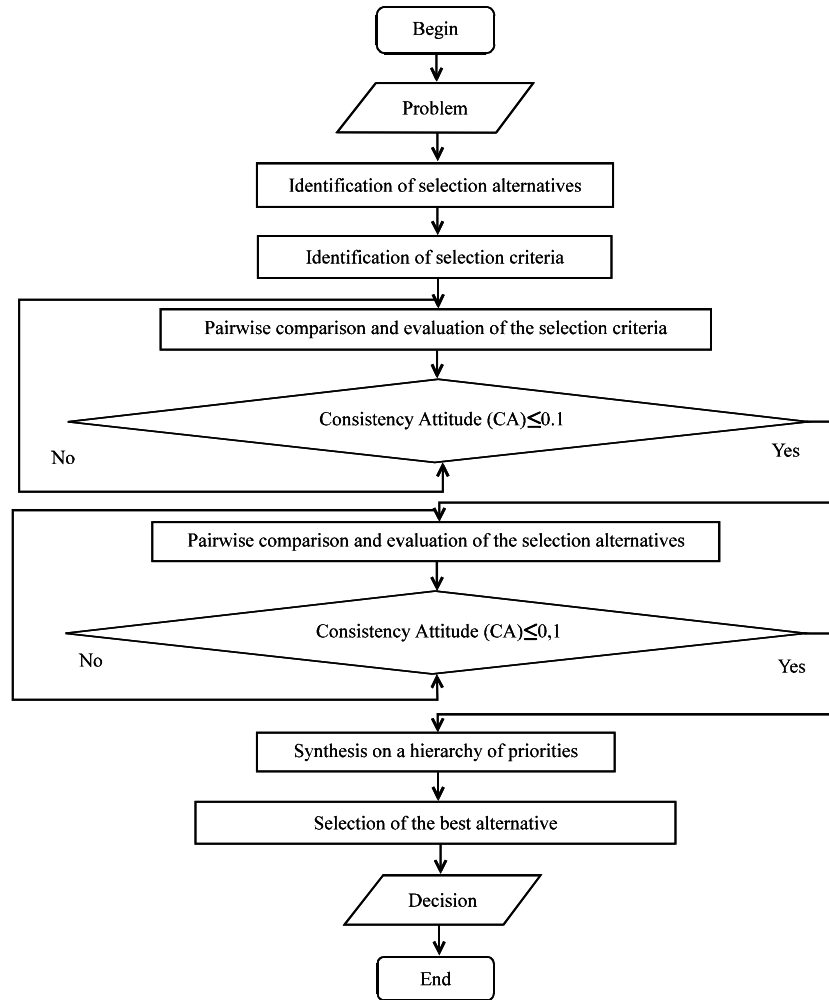


Fig. 1: Decision-making algorithm at a direct assessment of alternatives

#### Development of alternative weight formation algorithm:

As we noted previously, in some cases a paired comparison does not provide the necessary parameters of a solution. For example, if the number of criteria for a decision making is 15 and the number of alternatives is 10, then during the calculation of the total number of alternative paired comparisons according to the formula  $m \times (n \times (n-1))$  where  $m$  is the number of criteria and  $n$  is the number of alternatives, the total number of alternative paired comparisons will be equal to 1350. At that, comparing each pair of alternatives, it is necessary to seek for the limitation of coherence index of the coherence relationship within the range of 0-0.1.

Considering that the operation of a simple ordering of options according to their importance without the assigning of weights is simple, i.e., a DM does not use

simplified strategies here and does not make mistakes, its use during a decision-making seems reasonable instead of paired comparisons, especially in the cases of preliminary selection from a large number of alternatives. Thus, the development of means for a direct evaluation of alternatives without a pairwise comparison procedure seems to be reasonable. The approach proposed by us is the use of step sequence for alternative ordering on the basis of alternative weight linear distribution. The process of alternative ordering consists of six phases shown in Table 1. When criteria or alternatives have the same weight, it is proposed to distribute the weight according to the Eq. 1:

$$MK_j = \frac{\sum_{j=c}^d K_j}{m} \quad (1)$$

Table 1: Linear distribution of alternative weight stages

Stage No.	Stage name	Formulae
1	The linear ordering of criteria by importance without an explicit specification of weight	$K_j = n-j+1/n$
2	The linear ordering of alternatives for each criterion, without the specification of weight explicitly	$A_j = k-i+1/n$
3	The calculation of each criterion on the basis of a unit of weight	$RK_j = \frac{n-j+1/n}{\sum_{j=1}^n K_j}; \sum_{j=1}^n K_j = \frac{K_1 + K_n}{2} \times n$
4	The calculation of an alternative for each criterion on the basis of a unit of weight	$RA_i = \frac{k-i+1/k}{\sum_{i=1}^k A_i}; \sum_{i=1}^k A_i = \frac{A_1 + A_k}{2} \times k$
5	The calculation of total weight for each alternative	$TA_j = \sum_{j=1}^n A_{ij}; A_{ij} = a \times b$
6	The best alternative selection	Decision = max (TA <sub>i</sub> )

j-criterion serial number, i-alternative serial number, n-total number of criteria, k-total number of alternatives n-total number of criteria, a-criterion j weight, b-alternative i weight, estimated by criterion j

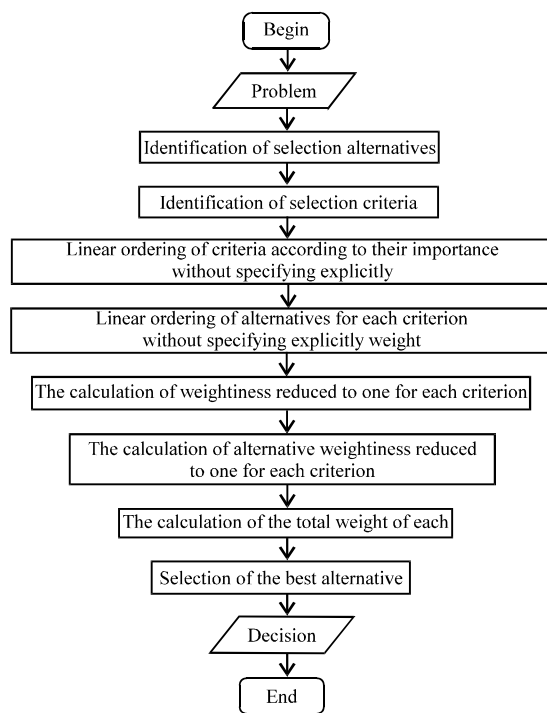


Fig. 2: The algorithm of a linear sequence of alternative weight development

$$MA_i = \frac{\sum_{i=p}^q A_i}{r} \quad (2)$$

Where:

- c = The number of the first order criterion with the same weight
- d = The number of the last criterion by order with an equal weight
- m = The number of criteria with equal weight
- p = The number of the first alternative by order with the same weight

q = The number of the last alternative by order with the same weight

r = The number of alternatives with the same weight

Let's formalize previously presented steps in the algorithm of alternative weight development presented on Fig. 2. Thus, we determined the priorities of alternatives and selection criteria. The global priorities are calculated using a convolution. In order to assess the possibility of the resulting algorithm use, let's perform a comparative assessment of the classical method of hierarchy analysis based on a linear distribution of alternative weights.

## RESULTS AND DISCUSSION

**Comparative analysis of pairwise comparison methods and linear distribution of alternative weights:** Within a DM study, the problems are offered with a number of criteria and alternatives from 6-11. An experimental research of pairwise comparison and linear distribution of alternative weight methods during the solution of problems with the number of criteria and alternatives from Eq. 6-11 are shown on Fig. 3-8. For the clarity of a management solution improvement for each set of alternatives and criteria the number of paired comparisons and the number of alternative evaluation operations was calculated at a linear ordering of alternative weights (Fig. 9). According to the performed study it can be noted that the results of the best alternative selection using the linear ordering of alternative weights are consistent with the choice results using the method of alternative pair comparison.

At that the total number of pairwise alternative comparisons makes 1210 when the number of alternatives and selection criteria equal to eleven. At that the number of ordering operations is equal to 132 in a linear ordering of alternative weights.

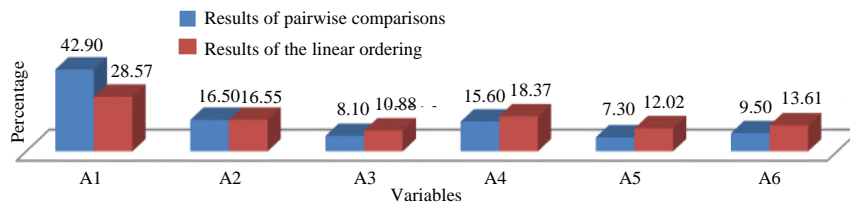


Fig. 3: The results of a problem solution when the number of alternatives and criteria is equal to six

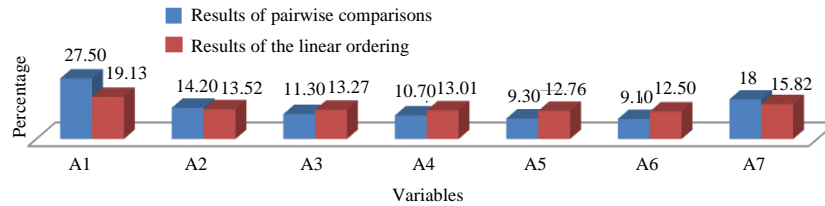


Fig. 4: The results of a problem solution when the number of alternatives and criteria is equal to seven

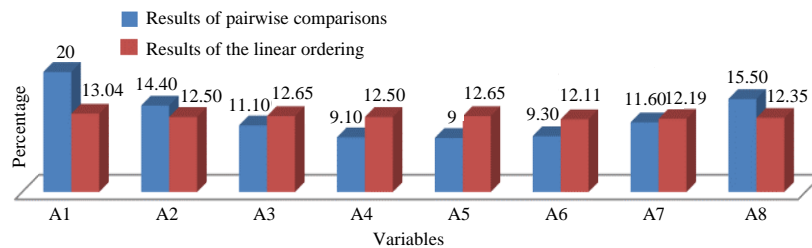


Fig. 5: The results of a problem solution when the number of alternatives and criteria is equal to eight

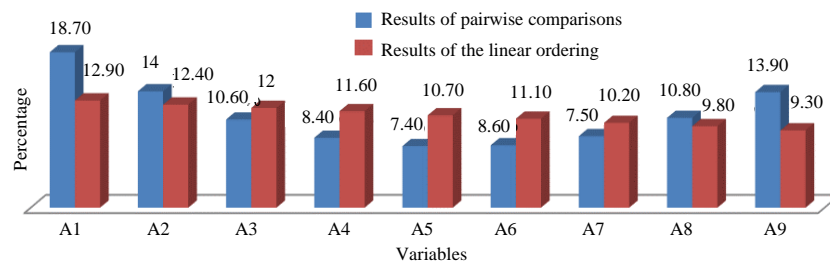


Fig. 6: The results of a problem solution when the number of alternatives and criteria is equal to nine

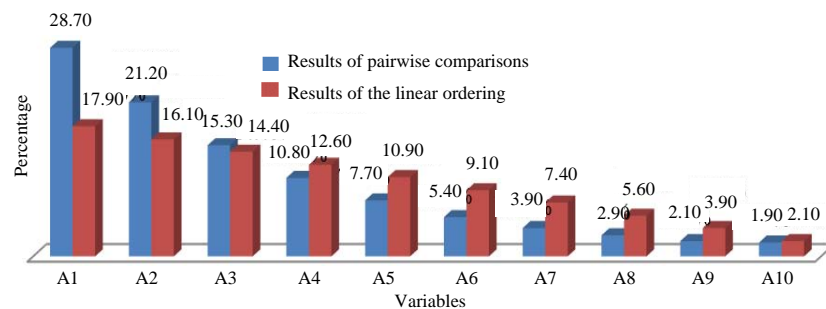


Fig. 7: The results of a problem solution when the number of alternatives and criteria is equal to ten

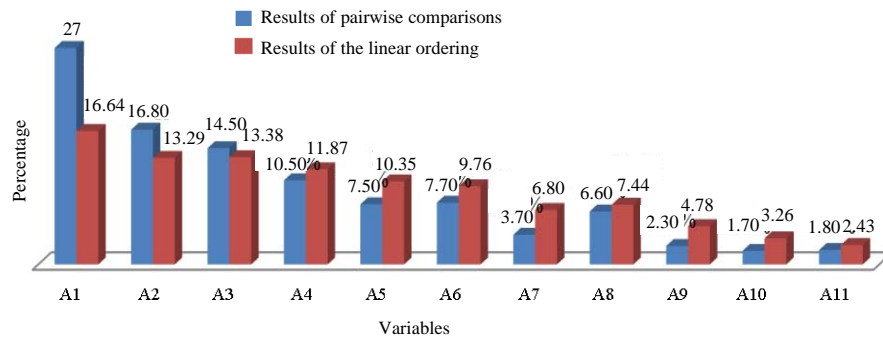


Fig. 8: The results of a problem solution when the number of alternatives and criteria is equal to ten

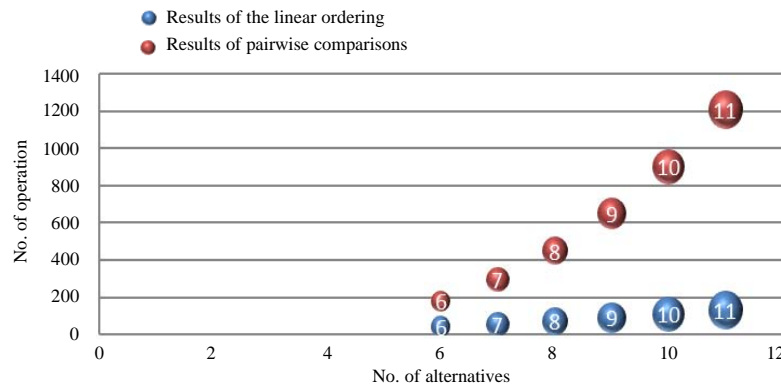


Fig. 9: Operation number comparison results

## CONCLUSION

The study developed the algorithm of alternative linear weight distribution during a decision making. The comparison of hierarchy analysis classical method and the method based on alternative weight linear distribution is performed within the study. The following problems were analyzed with the number of criteria and alternatives from 6-11. According to the survey results the following advantages were determined during a decision-making: a significant reduction of time to make a decision, the possibility of a solution adoption with a large number of criteria and alternatives, a small number of operations and their simplicity.

Thus, the algorithm of alternative weight linear distribution during a decision making was developed. The proposed means of decision-making can be used separately from the hierarchy analysis process and in conjunction with this method. The proposed decision means may be used to solve semi structured problems in various subject areas.

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