Journal of Engineering and Applied Sciences 12 (3): 679-683, 2017
ISSN: 1816-949X

Decomposition of Bipolar Fuzzy Finite State Machines and Transformation Semigroups

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Abstract: Bipolar fuzzy sets are the generalization of fuzzy sets. In many domains dealing with bipolar information is an essential. In this study, we deal with bipolar information and introduce the concept of decomposition of fuzzy finite state machines and transformation semigroups and investigated some of their algebraic properties.

Key words: Bipolar fuzzy sets, finite state machines, decomposition, transformation semigroups, algebraic properties

INTRODUCTION

Fuzzy set theory introduced by Zadeh (1965) is an alternative to the probability theory. It has many applications in the field of science and technology. In 1967 Wee (1967) introduced the mathematical formulation of fuzzy automata. Since, then many researchers proposed and studied the various types of fuzzy automata in a different context. Bipolar fuzzy sets introduced by Zhang are the generalization of the concept of fuzzy sets (Zadeh, 1965) which is an extension of fuzzy sets whose membership degree range is (-1, 1). The bipolar fuzzy set is now growing and expanding in its applications including graph theory, automata theory and pattern recognition.

Malik *et al.* (1994) introduced the notions of submachine of a fuzzy finite state machine. They also initiated a decomposition theorem for fuzzy finite state machines considering primary submachines (Mordeson and Malik, 2002). Further, they have introduced and studied fuzzy transformation semigroups in Mordeson and Malik (2002). Inspired by the notion of bipolar fuzzy valued sets, Jun and Kavikumar (2011) introduced the algebraic properties of bipolar fuzzy finite state machines and investigated some related results.

Kavikumar introduced the notion of bipolar fuzzy finite switchboard state machines and investigated several properties. In this study, we generalize the concept of decomposition and transformation semigroups of fuzzy finite state machine into the bipolar fuzzy finite state machine environment and investigate some of its algebraic properties.

Definition (Jun and Kavikumar, 2011): A bipolar fuzzy finite state machine (bffsm, for short) is a triple $M = (Q, X, \phi)$ where Q and X are finite non-empty sets called the set of states and the set of input symbols, respectively and $\varphi = \langle \varphi^-, \varphi^+ \rangle$ is a bipolar fuzzy set in $Q\times X\times Q$. Here, the set of all words of element of X of finite length is denoted by $X^*.\lambda$ and $|x|$ denoted by empty word and length of $x \forall x \in X^*$.

Definition 2 (Jun and Kavikumar, 2011): Let $M = (Q, X, \varphi)$ be a bffsm. Define a bipolar fuzzy set $\varphi_* = \langle \varphi_*^-, \varphi_*^+ \rangle$ in $Q\times X^*\times Q$ by:

$$\varphi_*^-(q,\lambda,p)=\begin{cases}-1 & \text{if } q=p\\ 0 & \text{if } q\neq p\end{cases} \quad \varphi_*^+(q,\lambda,p)=\begin{cases}-1 & \text{if } q=p\\ 0 & \text{if } q\neq p\end{cases}$$

$$\varphi_*^-(q,xa,p) = \inf_{r\in Q}[\varphi_*^-(q,x,r) \vee \varphi^-(r,a,p)]$$

$$\varphi_*^+(q,xa,p) = \sup_{r\in Q}[\varphi_*^+(q,x,r) \vee \varphi^+(r,a,p)]$$

$\forall p, q \in Q$ and $x, y \in X^*$.

Definition 3 (Jun and Kavikumar, 2011): Let $M = (Q, X, \varphi)$ be a bffsm. Let $(\forall T \subseteq Q)$. Let $\varphi_Q = \langle \varphi_Q^-, \varphi_Q^+ \rangle$ be a bipolar fuzzy set in $T\times X\times T$ and $N = (T, X, \varphi_Q)$ be a bffsm. Then N is called a bipolar submachine of M, if 1.$\varphi|_{T\times X\times T = \varphi Q}$ and 2.$S(T)\subseteq T$.

Let X^* is a semigroup with identity λ with respect to the binary operation concatenation of two words. Let $x, y \in X^*$. Define a relation~on X^* by x~y if and only if $\varphi_*^-(q, x, p) = \varphi_*^-(q, y, p)$ and $\varphi_*^+(q, x, p) = \varphi_*^+(q, y, p)$ for

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all q, p∈Q Then ~ is a congruence relation on X^* (Kavikumar *et al*., 2012). For any x∈X^* we denote X[x] = y∈X^*| x~y and S(M) = {[x] |x∈X^*.

Theorem 3: Let M = (Q, X, φ) be a bffsm. Define a binary operation⊙on S(M) by [x]⊙[y] = [xy] for all [x], [y]∈S(M). Thenm (S(M), ⊙) is a finite semigroup with identity.

MATERIALS AND METHODS

Decomposition of bipolar fuzzy finite state machine: Every fuzzy finite state machine can be decomposed to primary submachines (Malik *et al*., 1994). This decomposition property can be extended to bipolar submachines as follows.

Definition 4: Let N = (T, X, φ_Q) and P = (P, X, φ_Q) be bipolar submachine of M = (Q, X, φ). Then P is called a primary bipolar submachine of M if (∃q∈Q) such that T = P = <q>; ∀s∈Q if P⊆<s>then P = <s>.

Theorem 1: (Decomposition theorem) let N = (T, X, φ_Q) be a bipolar submachine of M = (Q, X, φ). Let P = {P_1, P_2, ..., P_n} be the set of all distinct primary bipolar submachines of M. Then:

$$N = U_{i=1}^{n} P_i$$

$$N \neq U_{i=1, i\neq j}^{n} P_i \text{ for any } j \in \{1, 2, ..., n\}$$

Proof 1: Let q_0∈Q if ∀q∈Q then either ⟨q⟩∈P $_i$or ∃q_{i+1}∈Q∖S(q_i) such that ⟨q_i⟩⊂⟨q_{i+1}⟩ = P for some i which implies that ⟨q_i⟩⊂$U_n^{n=1}$ P_i. Now, we have either ⟨q_0⟩∈P or there exists a positive integer k such that ⟨q_0⟩⊂⟨q_k⟩∈p since Q is finite. Thus, Q = $U^n_{n=1}$ S(p_i) where P_i = ⟨p_i⟩, i = 1, 2, ..., n. Hence, N = $U^n_{n=1}$ P_i. Let N = $U^n_{i=1, i\neq j}$ P_i and P_i = ⟨p_i⟩. It suffices to show that P_i≠P_j. If we assume that P_i = P_j for some i, then:

$$P_j = \begin{cases} P_i & \text{if } p_j \in U_{i=1, i\neq j}^{n} S(p_i) \\ 0 & \text{otherwise} \end{cases}$$

If P_j = 0 then P_j≠P_i since P_i≠0 which proved the theorem. Suppose if p_j∈$U^n_{i=1, i\neq j}$ S(p_i) then p_j∈S(p_i) for thensome i≠j. Hence, P_j = ⟨p_j⟩⊂P_i. Since, P_j = P_i, this contradicts that the maximality of P_j. Thus, p_j∉$U^n_{i=1, i\neq j}$ S(p_i). Hence, M≠N.

Corollary 1: Let M = (Q, X, φ) be a bffsm. Then every singly generated bipolar submachines of M≠∅ is a bipolar submachines of a primary bipolar submachine of M.

Corollary 2: Let M = (Q, X, φ) be a bffsm. Then P_1, P_2, ..., P_n in theorem 1 are unique.

Bipolar transformation semigroups: In this study, we introduce the notion of transformation semigroups of a bffsm in order to consider state membership as bipolar fuzzy sets.

Definition 5: A bipolar fuzzy transformation semigroup (bfts, for short) is a triple T = (Q, S, ρ) where Q is a finite nonempty set, S is a finite semigroup and ρ is a bipolar fuzzy subset of Q×S×Q such that for all u, v∈S and p, q∈Q we have:

$$\rho^-(q, uv, p) = \inf_{r\in Q}\{\rho^-(q, u, r) \vee \rho^-(r, v, p)\}$$

$$\rho^+(q, uv, p) = \sup_{r\in Q}\{\rho^+(q, u, r) \wedge \rho^+(r, v, p)\}$$

If S contains the identity e then:

$$\rho^-(q, e, p) = \begin{cases} -1 & \text{if } q = p \\ 0 & \text{if } q \neq p \end{cases}$$

$$\rho^+(q, e, p) = \begin{cases} 1 & \text{if } q = p \\ 0 & \text{if } q \neq p \end{cases}$$

For all p, q∈Q. If in addition for all u, v∈S and p, q∈Q satisfies the following property then we can say that T = (Q, S, ρ) is a faithful bipolar fuzzy transformation semigroup (faithful bfts for short):

$$\begin{pmatrix} \rho^-(q, u, p) = \rho^-(q, v, p) \\ \rho^+(q, u, p) = \rho^+(q, v, p) \end{pmatrix} \Rightarrow u = v$$

Theorem 2: Let T = (Q, S, ρ) be a bfts. If we define an equivalence relation "≈" on S by:

$$u \approx v \Leftrightarrow \begin{pmatrix} \rho^-(q, u, p) = \rho^-(q, v, p) \\ \rho^+(q, u, p) = \rho^+(q, v, p) \end{pmatrix}$$

For every p, q∈Q and let [[u]] denote the equivalence class ≈ of each u∈Q then the equivalence relation ≈ is a congruence relation on S and hence the quotient set [[S]]: = {[[u]]|u∈S} is a finite semigroup. The triple $T_{≈}$ = (Q, [[S]], $\dot{\rho}$) is a faithful bfts where $\bar{\rho} = \langle \bar{\rho}^-, \bar{\rho}^+ \rangle$ be a bipolar fuzzy set in Q×[[S]]×Q is defined by for all p, q∈Q and [[x]]∈[[S]]:

$$\bar{\rho}^-(q, [[x]], p) = \rho^-(q, x, p)$$

$$\bar{\rho}^+(q, [[x]], p) = \rho^+(q, x, p)$$

Proof i: Let we can assume that u, v∈S are such that u≈v. Suppose if w∈S, then:

$$\rho^-(q,uw,p) = \inf_{r\in Q}\{\rho^-(q,u,r)\vee\rho^-(r,w,p)\}$$
$$\inf_{r\in Q}\{\rho^-(q,v,r)\vee\rho^-(r,w,p)\} = \rho^-(q,vw,p)$$

$$\rho^+(q,uw,p) = \sup_{r\in Q}\{\rho^+(q,u,r)\vee\rho^+(r,w,p)\}$$
$$= \sup_{r\in Q}\{\rho^+(q,v,r)\vee\rho^+(r,w,p)\} = \rho^+(q,vw,p)$$

For all p, q∈Q. So, uv≈vw. Similarly, v≈w⇒uv≈uw for all u∈S. Hence, the quotient set [[S]] is a finite semigroup. Let $T_{\approx} = (Q, [[S]], \bar{\rho})$ be a bfts. Let e is the identity element of the semigroup S and also ē is the identity element of the semigroup [[S]]. Since:

$$\bar{\rho}^-(q,\bar{e},p) = \rho^-(q,e,p) = \begin{cases} -1 & \text{if } p = q \\ 0 & \text{if } p \neq q \end{cases}$$

$$\bar{\rho}^+(q,\bar{e},p) = \rho^+(q,e,p) = \begin{cases} 1 & \text{if } p = q \\ 0 & \text{if } p \neq q \end{cases}$$

For all p, q∈Q and we conclude that the Definition 5 holds. For all u, w∈S and Q. Now:

$$\bar{\rho}^-(q,[[v]][[w]],p) = \bar{\rho}^-(q,[[vw]],p) = \rho^-(q,vw,p)$$
$$= \inf_{r\in Q}\{\rho^-(q,v,r)\vee\rho^-(r,w,p)\}$$
$$= \inf_{r\in Q}\{\bar{\rho}^-(q,[[v]],r)\vee\bar{\rho}^-(r,[[w]],p)\}$$

$$\bar{\rho}^+(q,[[v]][[w]],p) = \bar{\rho}^+(q,[[vw]],p) = \rho^+(q,vw,p)$$
$$= \sup_{r\in Q}\{\rho^+(q,v,r)\wedge\rho^+(r,w,p)\}$$
$$= \sup_{r\in Q}\{\bar{\rho}^+(q,[[v]],r)\wedge\bar{\rho}^+(r,[[w]],p)\}$$

The condition from Definition 5 also holds. Hence, it is clear that bipolar fuzzy transformation $T_{\approx} = (Q, [[S]], \bar{\rho})$ is faithful.

RESULTS AND DISCUSSION

Theorem 4: The triple (Q, S(M), ρ) is a faithful bfts Where:

$$\rho^-(q,[x],p) = \varphi_*^-(q,x,p)$$

$$\rho^+(q,[x],p) = \varphi_*^+(q,x,p)(\forall p,q\in Q, x\in X^*)$$

For any bffsm M = (Q, X, φ).

Proof: By Theorem 3 (S(M), ⊙) is a finite semigroup with identity [λ]. Clearly ρ^- and ρ^+ are single valued. Let p, q∈Q and [x], [y]. Then:

$$\rho^-(q,[x]\odot[y],p) = \rho^-(q,[xy],p) = \varphi_*^-(q,xy,p)$$
$$= \inf_{r\in Q}\{\varphi_*^-(q,x,r)\vee\varphi_*^-(r,y,p)\}$$
$$= \inf_{r\in Q}\{\rho^-(q,[x],r)\vee\rho^-(r,[y],p)\}$$

$$\rho^+(q,[x]\odot[y],p) = \rho^+(q,[xy],p) = \varphi_*^+(q,xy,p)$$
$$= \sup_{r\in Q}\{\varphi_*^+(q,x,r)\vee\varphi_*^+(r,y,p)\}$$
$$= \sup_{r\in Q}\{\rho^+(q,[x],r)\vee\rho^+(r,[y],p)\}$$

Now we have:

$$\rho^-(q,[\lambda],p) = \varphi_*^-(q,\lambda,p) = \begin{cases} -1 & \text{if } q = p \\ 0 & \text{if } q \neq p \end{cases}$$

$$\rho^+(q,[\lambda],p) = \varphi_*^+(q,\lambda,p) = \begin{cases} 1 & \text{if } q = p \\ 0 & \text{if } q \neq p \end{cases}$$

So, φ_* satisfies the definition 5. Suppose:

$$\rho^-(q,[x],p) = \rho^-(q,[y],p), \rho^+(q,[x],p)$$
$$= \rho^+(q,[y],p) \sum \forall q,p\in Q$$

Then:

$$\varphi_*^-(q,x,p) = \varphi_*^-(q,y,p),\ \varphi_*^+(q,x,p),\ \varphi_*^+(q,y,p)\forall p,\ q\in Q$$

Thus, x~y or [x] = [y]. Hence (Q, S(M), ρ) is a faithful bfts.

Definition 6: Let $T_1 = (Q_1, S_1, \rho_1)$ and $T_2 = (Q_2, S_2, \rho_2)$ be a bfftss. A bfts homomorphism from T_1 to T_2 is a pair (f, g) such that:

- f: $Q_1 \rightarrow Q_2$ is a map
- g: $S_1 \rightarrow S_2$ is a semi homomorphism
- If S_1 and S_2 are both semigroups with identity elements $e_1 \in S_1$ and $e_2 \in S_2$, then g $(e_1) = e_2$
- $\rho_1^-(q,u,p) \geq \rho_2^-(f(q),g(u),f(p)\rho_1^+(q,u,p) \leq \rho_2^+(f(q), g(u),f(p))\ (\forall p,q\in Q_1)\text{and}(u\in S_1)$

In addition for all q, p∈Q_1 and u∈S_1, the following properties:

$$\rho_2^-(f(q),g(u),f(p)) = \inf_{r\in Q}\{\rho_1^-(q,u,t)\,|\,f(r) = f(p)\}$$

$$\rho_2^+(f(q),g(u),f(p)) = \sup_{r\in Q_1}\{\rho_1^+(q,u,t) \mid f(r)=f(p)\}$$

and satisfies both (f, g) is said to be a strong homomorphism. A bipolar (strong) homomorphism (f, g): $T_1 \rightarrow T_1$ is called a bipolar (strong) isomorphism if f and g are both bijective. Let T = (Q, S, δ) be a faithful bfts where S is a semigroup with identity. Define the bffsm M = (Q, X, φ) by taking Q = δ. Consider BFTS (M) = (Q, S(M), ρ) where S(M) = S^*/~ and ρ^- (q, [u], p) = φ_*^- (q, u, p) and p^+ (q, [u], p) = φ^+_* (q, u, p). Let e be the identity element of S and λ the empty word in S^*. Then:

$$\rho^-(q,[e],p) = \varphi_*^-(q,e,p) = \delta^-(q,e,p) = \begin{cases} -1 & \text{if } q=p \\ 0 & \text{if } q\neq p \end{cases}$$

$$\rho^+(q,[e],p) = \varphi_*^+(q,e,p) = \delta^+(q,e,p) = \begin{cases} 1 & \text{if } q=p \\ 0 & \text{if } q\neq p \end{cases}$$

Hence, ρ^- (q, [e], p) = ρ^- (q, [λ], p), ρ^+ (q, [e], p) = ρ^+ (q, [λ], p) for all [e] = [λ].

Theorem 5: Let $M_1 = (Q_1, X_1, \varphi_1)$ and $M_2 = (Q_2, X_2, \varphi_2)$ be bffsms. If for each bffsm strong homomorphism (f, g): $M_1 \rightarrow M_2$ with f bijective we define g^*: $X_1^* \rightarrow X_2^*$ by means of:

$$g^*(\lambda) = \lambda$$

$$g^*(ux) = g^*(u)g^*(x)$$

whenever $u \in X_1^*$ and $x \in X$ and the map $\overline{g^*}: S(M_1) \rightarrow S(M_2)$ by means of $\overline{g^*}: ([u]) = \overline{g^*[u]}$ for any $[u] \in S(M_1)$ then the pair $(f, \overline{g^*}): BFTS(M_1) \rightarrow BFTS(M_2)$ is a strong bfts homomorphism.

Proof: Consider the faithful bftss BFTS (M_1) and BFTS (M_2) We have to show that g^*: $X_1^* \rightarrow X_2^*$ is a semigroup homomorphism that maps the identity element of X_1^* to that of X_2^*. Now we have to prove that $g^*(uv) = g^*(u)\, g^*(v)$ ($\forall u, v \in X_1^*$) since. We can approach the prove by induction method on the length of v: If |v| = 0, then v = λ and $g^*(uv) = g^*(u\lambda) = g^*(u) = g^*(u)\lambda = g^*(u)g^*(\lambda)$. Let |v| = n>0 and write v = wx with $w \in X_1^*$ and $x \in X_1$. Then $h^*(uv) = h^*(uwx) = h^*(uw)h(x) = h^*(u)h^*(w)h(x) = h^*(u)h(v)$ by using the induction hypothesis in the last but one equality. By using induction on |v| we show that for any q, $p \in Q_1$ and $v \in X_1^*$:

$$\varphi_2^-(f(q),g^*(u),f(p)) = \varphi_1^-(q,v,p)$$

$$\varphi_2^+(f(q),g^*(v),f(p)) = \varphi_1^+(q,v,p)$$

If |v| = 0, then v = λ and we have:

$$\varphi_2^-(f(q),g^*(\lambda),f(p)) = \varphi_2^-(f(q),\lambda,f(p)) = \varphi_1^-(q,\lambda,p)$$

$$\varphi_2^+(f(q),g^*(\lambda),f(p)) = \varphi_2^+(f(q),\lambda,f(p)) = \varphi_1^+(q,\lambda,p)$$

If |v| = 1, then $v \in X_1^*$ and $g^*(v) = g(v)$. So:

$$\varphi_2^-(f(q),g^*(v),f(p)) = \varphi_2^-(f(q),g(v),f(p)) = \varphi_1^-(q,v,p) = \varphi_1^-(q,v,p)$$

$$\varphi_2^+(f(q),g^*(v),f(p)) = \varphi_2^+(f(q),g(v),f(p)) = \varphi_1^+(q,v,p) = \varphi_1^+(q,v,p)$$

Let |v| = n>1 and take v = wx with $w \in X_1^*$ and $x \in X$. Then:

$$\begin{aligned}\varphi_2^-(f(q), g^*(v), f(p)) &= \varphi_2^-(f(q), g^*(w)g(x), f(p)) \\ &= \inf_{s\in Q_2}\{\varphi_2^-(f(q), g^*(w), s) \vee \varphi_2^-(s, g(x), f(p))\} \\ &= \inf_{r\in Q_2}\{\varphi_2^-(f(q), g^*(w), f(r)) \vee \varphi_2^-(f(r), g(x), f(p))\} \\ &= \inf_{r\in Q_2}\{\varphi_1^-(q, w, r) \vee \varphi_1^-(r, x, p)\} = \varphi_1^-(q, wx, p) = \varphi_1^-(q, v, p)\end{aligned}$$

$$\begin{aligned}\varphi_2^+(f(q),g^*(v),f(p)) &= \varphi_2^+(f(q),g^*(w)g(x),f(p)) \\ &= \sup_{S\in Q_1}\{\varphi_2^-(f(q),g^*(w),s) \wedge \varphi_2^-(s,g(x),f(p))\} \\ &= \sup_{r\in Q_1}\{\varphi_2^-(f(q),g^*(w),f(r)) \wedge \varphi_2^-(f(r),g(x),f(p))\} \\ &= \sup_{r\in Q_2}\{\varphi_1^-(q,w,r) \wedge \varphi_1^-(r,x,p)\} = \varphi_1^+(q,wx,p) = \varphi_1^+(q,v,p)\end{aligned}$$

Finally, we have to show that $\overline{g^*}: S(M_1) \rightarrow S(M_2)$ is well defined. Let [x], $[y] \in S(M_1)$ and [x] = [y]. Then φ_*^- (q, x, p) = φ_*^- (q, y, p), φ_*^+ (q, x, p) = φ_*^+ (q, y, p) for all q, $p \in Q_1$. Now:

$$\varphi_2^-(f(q),g^*(x),f(p)) = \varphi_1^-(q,x,p) = \varphi_1^-(q,y,p) = \varphi_2^-(f(q),g^*(y),f(p))$$

$$\varphi_2^+(f(q),g^*(x),f(p)) = \varphi_1^+(q,x,p) = \varphi_1^+(q,y,p) = \varphi_2^+(f(q),g^*(y),f(p))$$

For all q, $p \in Q_1$. Thus, since f is onto $[g^*(x)] = [g^*(y)]$. Hence $\overline{g^*}$ is well defined. We conclude that $\overline{g^*}$ is a semigroup homomorphism from BFTS (M_1) to BFTS (M_2) which maps $\bar{\lambda} - \bar{\lambda}$.

Theorem 6: Let M_1, M_2 and M_3 be bffsms. If (f_1, g_1): $M_1 \rightarrow M_2$ and $f_2, g_2)$: $M_2 \rightarrow M_3$ are bffsm strong homomorphisms with f_1 and f_2 bijective, then the pair $(f_2 \circ f_1, g_2 \circ g_1)$ is also a bffsm strong homomorphism with $f_2 \circ f_1$ bijective and $\overline{(g_2 \circ g_1)}^* = \overline{g_2^*} \circ \overline{g_1^*}$. BFTS $f_2 \circ f_1, g_2 \circ g_1) = \text{BFTS}(f_2, g_2) \circ \text{BFTS}(f_1, g_1)$.

Proof: It is straightforward.

Theorem 7: Let S be a semigroup with identity. Then BFTS (M) is isomorphic to $T = (Q, S, \delta)$.

Proof: Define f: $Q \rightarrow Q$ by $f(q) = q \forall q \in Q$ and g: $S \rightarrow S(M)$ by $g(x) = [x] \forall x \in S$ and so $\forall q, p \in Q$, $\varphi_*^-(q, x, p) = \varphi_*^-(q, y, p)$, $\varphi_*^+(q, x, p) = \varphi_*^+(q, y, p)$. Hence, $\varphi^-(q, x, p) = \varphi^-(q, y, p)$, $\varphi^+(q, x, p) = \varphi^+(q, y, p)$ $(\forall q, p \in Q) \Rightarrow \delta^-(q, x, p) = \delta^-(q, y, p)$, $\delta^+(q, x, p) = \delta^+(q, y, p)$ $(\forall q, p \in Q)$. Since, $T = (Q, S, \delta)$ is faithful, it follows that x = y. Thus, g is one-one. Let · denote the binary operation of the semigroup S. Let $a, b \in S$, then $a \cdot b \in S$ and $a, b \in S^*$. For any $q, p \in Q$ we have:

$$\varphi_*^-(q, a \bullet b, p) = \varphi^-(q, a \bullet b, p) = \delta^-(q, a \bullet b, p)$$
$$= \inf_{r \in Q} \{\delta^-(q, a, r) \vee \delta^-(r, b, p)\}$$
$$= \inf_{r \in Q} \{\varphi^-(q, a, r) \vee \varphi^-(r, b, p)\} = \varphi_*^-(q, ab, p)$$

$$\varphi_*^+(q, a \bullet b, p) = \varphi^+(q, a \bullet b, p) = \delta^+(q, a \bullet b, p)$$
$$= \sup_{r \in Q} \{\delta^+(q, a, r) \vee \delta^+(r, b, p)\}$$
$$= \sup_{r \in Q} \{\varphi^+(q, a, r) \vee \varphi^+(r, b, p)\} = \varphi_*^+(q, ab, p)$$

Hence, $g(a \cdot b) = [a \cdot b] = [ab] = [a][b]] = g(a)g(b)$. Using induction, it can be shown that if $y_i \in S$, $1 \leq i \leq n$, then $[y_1 \cdot y_2 \cdot \dots \cdot y_n] = [y_1, y_2, \dots, y_n]$. Let $[u] = \lambda]$then $[\lambda] = [e]$ and $g(e) = [\lambda]$. Suppose $u = a_1, a_2, \dots, a_n$for $a_i \in S$ and $1 \leq i \leq n$. Then, $g(a_1 \cdot a_2 \cdot \dots \cdot a_n) = [a_1 \cdot a_2 \cdot \dots \cdot a_n] = [a_1 a_2 \dots a_n] = [u]$ and thus g is onto. Finally:

$$\rho^-(f(q), g(x), f(p)) = \rho^-(q, [x], p)$$
$$= \varphi_*^-(q, x, p) = \varphi^-(q, x, p) = \delta^-(q, x, p)$$

$$\rho^+(f(q), g(x), f(p)) = \rho^+(q, [x], p)$$
$$= \varphi_*^+(q, x, p) = \varphi^+(q, x, p) = d^+(q, x, p)$$

This completes the proof.

CONCLUSION

In this nutshell in spired from Horry (2016), the concepts of decomposition of a fuzzy finite state machines and fuzzy transformation semigroups have been generalized by substituting the interval [-1, 1] as the truth structure of the transition function in bipolar setting for the study of algebraic automata. It will be of interest to study with other types of products in the bipolar fuzzy finite state machines (Jun and Kavikumar, 2011) which is for future research.

ACKNOWLEDGEMENT

This research was partly supported by the Centre for Graduate Studies, UTHM.

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