Journal of Engineering and Applied Sciences 12 (3): 494-500, 2017
ISSN: 1816-949X

Magnetoconvection in a Soret Driven Binary Fluid Mixture Induced by Temperature Dependent Viscosity

[1,4]Nurul Hafizah Zainal Abidin, [1,2]Nor Fadzillah Mohd Mokhtar,
[1]Muhammad Syahid Zuhri Suardi, [1]Zanariah Abdul Majid and [3]Siti Salwa Abd Ghani
[1]Institute for Mathematical Research, Universiti Putra Malaysia,
43400 Serdang, Selangor, Malaysia
[2]Department of Mathematics, Faculty of Science, Universiti Putra Malaysia,
43400 Serdang, Selangor, Malaysia
[3]Department of Chemistry, Faculty of Science, Universiti Putra Malaysia,
43400 Serdang, Selangor, Malaysia
[4]Faculty Science Computer and Mathematial Sciences,
Universiti Teknologi MARA, Tapah Campus, Tapah Road, 35400 Perak, Malaysia

Abstract: Thermosolutal Rayleigh-Benard convection in a binary fluid heated from the below is studied numerically. Soret effects are imposed to analyze the thermo-diffusion effects on the flow. We study the onset of convection in a horizontal binary fluid layer with the effect of temperature dependent viscosity together with vertical magnetic field. The confined boundaries of the binary fluid layer are considered to be free-free, rigid-free and rigid-rigid which described the lower and upper surfaces respectively. The Galerkin method is implemented through the numerical computations in order to observe the stability of the binary liquid mixture and the effect of magnetic field, temperature dependent viscosity and Soret effect to the critical Rayleigh number is also reported.

Key word: Binary fluid, vertical magnetic field, temperature dependent viscosity, soret effect, confined boundaries, critical Rayleigh

INTRODUCTION

A chemical fluid that is produced from two or more fluid will have their own feature and reaction compared to their original fluid. This fluid renamed as binary fluid is used to solve many of problem arising in aeronautical, chemical, mechanical and civil engineering. It also enables many natural phenomena such as in oceanography and astrophysics (Teamah *et al*., 2011). The study have been done experimentally and numerically and also receive considerable attention by many researchers.

Nield (1967) as one of the pioneer in this research analyzed that thermal effect can act as a stabilizing or destabilizing factor in both stationary and oscillatory mode for a thermosolutal convective binary fluid layer induced by thermal and solutal gradients. Soret and Dufour are two effects that may considered in a double-diffusive binary fluid. These effects arises from the variation in temperature gradient or from the cross diffusive coefficients that significantly affect the buoyancy forces. The thermocapillary instability in binary fluid on the onset of convection by considering the Soret effect with applying other physical influences has been studied by Bergeon *et al*. (1998) and Slavtchev *et al*. (1999) followed by Saravanan and Sivakumar (2009).

Chandrasekhar (1961) has presented his research by integrating magnetic field on the dynamical stability of a convective system and showed that the critical value of Rayleigh number, Ra_c can be increased when a higher magnetic file is considered. Hence, it shows that magnetic effect will stabilize the system. Rao (1980) state that an increase value of magnetic field is possible to delay the onset of convection when he studied the effect on a micropolar fluid. Later, Rudraiah *et al*. (1986) also indicate the same result when he integrate the system with magnetic effect together with non-uniform temperature temperature gradient. Maekawa and Tanasawa (1988) explained clearly that the increase of the intensity of the magnetic field increase the critical Marangoni number in a pure fluid. Shivakumara *et al*. (2011) also integrate the magnetic field dependent viscosity parameter in a porous medium where their results shows that an increase of magnetic field dependent viscosity will delayed the onset of ferromagnetic convection but it shows no influence on the critical wave number.

Corresponding Author: Nurul Hafizah Zainal Abidin, Institute for Mathematical Research, Universiti Putra Malaysia, 43400 Serdang, Selangor, Malaysia

Franchi and Straughan (1992) has done a study the effect of temperature dependent viscosity in a micropolar fluid and presented that an increase of temperature will make the critical Rayleigh number decrease. Ramirez and Saez (1990) studied the effect in a porous medium. He stressed out the effect should be taken into account, even for relatively low temperature gradients. Another research to show that stability of convection can be enhanced by decreasing the temperature is done by Lu and Chen (1995).

Currently, there is lack of study on the effect of temperature dependent viscosity on controlling the onset of convection in a binary fluid mixture. In this research, we are interested to study the effect of temperature dependent viscosity in the presence of vertical magnetic field and Soret effects on the onset of binary fluid layer. Three types of bounding surfaces (lower boundary-upper boundary) are considered in this investigation: free-free, rigid-free and rigid-rigid. We assume that the upper surface to be non-deformable and employed the stability analysis theory. The resulting eigenvalue problem is solved using the Galerkin method.

MATERIALS AND METHODS

Mathematical formulation: An infinite horizontal layer of quiescent binary fluid of depth d where the fluid between two horizontal plates is heated from below is considered. The stability of a horizontal layer of quiescent binary fluid in the presence of temperature dependent viscosity is examined. ΔT is be the temperature difference between the lower and upper surfaces. Note that the lower boundary at a higher temperature than the upper boundary and these boundaries maintained at constant temperature.

We select a Cartesian coordinate system with z pointing upward, opposite to the gravity vector and (x, y) in the horizontal direction at the bottom rigid boundary. The onset of double-diffusive convection is studied under the Boussinesq approximation where the density, p is assumed to be linearly dependent upon the temperature, T and the solute concentration, C and is given by:

$$\rho = \rho_0\left[1-\alpha\left(T-T_0\right)+\alpha_c\left(C-C_0\right)\right] \quad (1)$$

Here, $\alpha = -\rho_0^{-1}\partial\rho/\partial T$ and $\alpha_c = -\rho_0^{-1}\partial\rho/\partial C$. Let the solute concentrations to be taken as $C_0+\Delta C$ and C_0. The governing equations used for the Rayleigh-Benard convection under the Oberbeck-Boussinesq approximation are given by following the analysis by Hurle and Jakeman for Soret parameter:

$$\nabla \cdot \nu = 0 \quad (2)$$

$$\rho_0\left[\frac{\partial T}{\partial t}+\left(\nu\cdot\nabla\right)\nu\right] = -\nabla p+\mu\nabla^2\nu-\rho g e_z \quad (3)$$

$$\left[\frac{\partial T}{\partial t}+\left(\nu\cdot\nabla\right)T\right] = \kappa\nabla^2 T \quad (4)$$

$$\left[\frac{\partial T}{\partial t}+\left(\nu\cdot\nabla\right)C\right] = \kappa\nabla^2 C \quad (5)$$

The basic state of the fluid is quiescent and is given by:

$$(u, v, w) = (0, 0, 0),\ T = T_b(z),\quad p = p_b(z),\ \rho = \rho_b(z) \text{ and } C = C_b(z) \quad (6)$$

On the basic state we superpose perturbations in the form:

$$(u, v, w, T, p, \rho, C) = [0, 0, 0, T_b(z), p_b(z), \rho_b(z), C_b(z)]\ [u', v', w', T', p', \rho', C'] \quad (7)$$

where the primes quantities indicate the perturbed variables. Equation 2-5 are non-dimensionalized using the following definitions:

$$\left(x',y',z'\right) = \frac{\left(x,y,z\right)}{d},\ t' = \frac{t\kappa}{d^2}$$
$$\left(u',v',w'\right) = \frac{d\left(u,v,w\right)}{\kappa}$$
$$p = \frac{pd^2}{v\kappa\rho},\ T' = \Delta T$$
$$C' = \Delta C,\ \bar{f}\left(z\right) = \frac{\eta\left(z\right)}{\eta_0} \quad (8)$$

and using Eq. 6 and 7, we obtain the non-dimensional variables:

$$\nabla \cdot \nu' = 0 \quad (9)$$

$$\frac{1}{Pr}\frac{\partial v'}{\partial t} = -\nabla p'+\nabla^2 v'+RaT'\hat{e}_z+LeRsC'\hat{e}_z \quad (10)$$

$$\frac{\partial T'}{\partial t}-w' = \nabla^2 T' \quad (11)$$

$$\frac{\partial C'}{\partial t}-w' = Le\nabla^2 C'+Sr\nabla^2 T' \quad (12)$$

Where:

$$Ra = \frac{\alpha g d^3\nabla T}{v\kappa},\ Rs = \frac{\alpha_c g d^3\nabla C}{v\kappa_c}$$

$$Le = \frac{\kappa}{\kappa_c} \quad (13)$$

$$Sr = \frac{D_{CT}\Delta T}{\kappa \Delta C},\ Pr = \frac{v}{\kappa}$$

Operating on Eq. 10 by eliminating the pressure term by using curl identity together with Eq. 9 and 10 can be written as:

$$\left[\frac{1}{Pr}\frac{\partial}{\partial t}\nabla^2 - \nabla^4\right] w' = RaT'\nabla^2 + RsLeC'\nabla^2 \qquad (14)$$

A normal mode representation is introduced in the form:

$$(w', T', C') = [W(z), \theta(z), \eta(z)] \exp[i(a_x x + a_y y)] \qquad (15)$$

Substituting Eq. 15 into 14 and Eq. 11-12 we obtain:

$$\bar{f}(D^2 - a^2)^2 W + \bar{f}\,2B(D^2 - a^2)DW + \bar{f}B^2(D^2 + a^2)W - \bar{f}HD^2W - a^2Ra\theta - Lea^2Rs\xi = 0 \qquad (16)$$

$$W + (D^2 - a^2)\theta = 0 \qquad (17)$$

$$W + Sr(D^2 - a^2)\theta + Le(D^2 - a^2)\xi = 0 \qquad (18)$$

Where:

$$a = \sqrt{a_x^2 + a_y^2}$$

and D = d/dz $\bar{f}(z)$ is given by:

$$\bar{f}(z) = \exp\left[B\left(z - \frac{1}{2}\right) + \frac{(T_r - T_a)}{\beta d}\right]$$

where, $B = \gamma\beta d$ is the dimensionless viscosity parameter. If the reference temperature $T_r = T_a$, then $\bar{f}(z) = \exp[B(z-1/2)]$ Equation 16-18 are solved subject to the appropriate boundary conditions that are:

$$W = DW = \theta = \xi = 0 \text{ at } z = 0 \qquad (19)$$

For upper free boundary which is at z = 1, we have:

$$W = D\theta = \xi = D^2W = 0 \text{ at } z = 1 \qquad (20)$$

For upper rigid boundary which is at z = 1, we have L:

$$W = D\theta = \xi = DW = 0 \text{ at } z = 1 \qquad (21)$$

We applied the Galerkin-type weighted residuals method to find an approximate solution to the system. The variables are written in a series of basis function as:

$$W = \sum_{p=1}^{N} A_p W_p,\ \theta = \sum_{p=1}^{N} B_p \Theta_p,\ \eta = \sum_{p=1}^{N} C_p \Lambda_p \qquad (22)$$

Substitute Eq. 22 into Eq. 16-18 and make the expressions on the left-hand sides of those equations (the residuals) orthogonal to the trial functions, thereby obtaining a system of 3N linear algebraic equations in the 3N unknowns. The vanishing of the determinant of coefficients produces the eigenvalue equation for the system. One can regard Ra as the eigenvalue and thus Ra is found in terms of the other parameters.

The integration by parts with respect to z between z = 0 and 1 is performed. By using the boundary conditions (Eq. 21-23), we obtain the system of linear homogeneous algebraic equations:

$$CA_i + DG_i + ED_i = 0 \qquad (23)$$

$$FX_i + GA_i + 0 = 0 \qquad (24)$$

$$ID_i + JX_i + KA_i = 0 \qquad (25$$

Where:

$$C = \langle (D^2W)^2 \rangle + 2a^2 \langle (DW)^2 \rangle + a^4 \langle W^2 \rangle + H \langle (DW)^2 \rangle - 2B \langle D^3W^2 \rangle + 2Ba^2 \langle DW^2 \rangle + B^2 \langle DW^2 \rangle - B^2a^2W^2$$

$$D = a^2 \langle \Theta W \rangle$$

$$E = a^2 LeRs \langle \Lambda W \rangle$$

$$F = \langle W\Theta \rangle$$

$$G = \langle D\Theta \rangle^2 - a^2 \langle \Theta^2 \rangle$$

$$I = \langle W\Lambda \rangle$$

$$J = Sr \langle D^2 \Lambda \Theta \rangle - a^2 Sr \langle \xi\Theta \rangle,$$

$$K = Le \langle (D\Lambda)^2 \rangle - a^2 Le \langle \Lambda^2 \rangle$$

where the angle bracket $\langle ... \rangle$ denotes the integration with respect to z from 0 to 1. The vanishing of coefficients produces the eigenvalue equation for the system. Then, the equations is restructure in a matrix form:

$$\begin{vmatrix} C_{ji} & D_{ji} & E_{ji} \\ F_{ji} & Gji & 0 \\ I_{ji} & J_{ji} & K_{ji} \end{vmatrix} = 0 \quad (26)$$

Various boundary conditions have been used for instant free-free, rigid-free and rigid-rigid referring to upper-lower boundaries. The sensitiveness of the critical Rayleigh number, Ra_c to the changes of the difference physical parameters K, Rs, Df and Sr are also studied.

RESULTS AND DISCUSSION

The criterion for the onset of Rayleigh-Benard convection in a binary fluid in the presence of vertical magnetic field, temperature dependent viscosity and Soret effects are investigated theoretically. The sensitiveness of the critical Rayleigh number, Ra_c to the changes in the binary fluid parameters; Rs and Le is also discussed.

To validate the results obtained, we neglect the feedback control and obtained $Ra_c = 657.5$ for free-free, $Ra_c = 1140$ for free-rigid and $Ra_c = 1750$ for rigid-rigid boundaries which complied with Nield and Kutzenotsv (2011) findings.

The variation of Rayleigh number, Ra with wave number, a and various values of vertical magnetic field, H is illustrate in Fig. 1. According to Sassi *et al*. (2016), the most efficient stabilizing effect is obtained when the magnetic field is perfectly vertical compared to horizontal magnetic field or with any azimuthal angle and a polar angle. Hence, we are interested to study the effect of vertical magnetic in this research. From the graph it clearly shows that when H values increased, it elevate the critical Rayleigh numbers in all type of boundaries considered. This indicates that vertical magnetic field can help to stabilize the system. The illustration also shown that the free-free system has a lower critical values making this type boundary is the most unstable boundary of one system. Since the particles of the liquids are more freely to carry the heat without too many obstacles, resulting to an easier heat transfer. It is interesting to also take note that the critical value of rigid-rigid boundary is highest compared to the other type of boundary. This could suggest the use of rigid-rigid boundary can hold stability in the system.

Figure 2 shows the variation of Ra versus a with different temperature viscosity values, B. We chose B = 2, 4, 6 and we found that as B increases, the marginal stability curves shift downwards and thus destabilize the no-motion state for all wave numbers. The pattern appear consistently in all type of boundaries as seen in the graph.

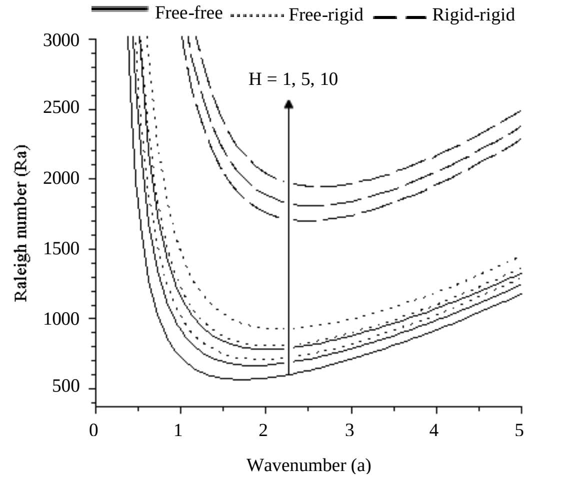

Fig. 1: Marginal stability curve Ra vs. a for different values of H

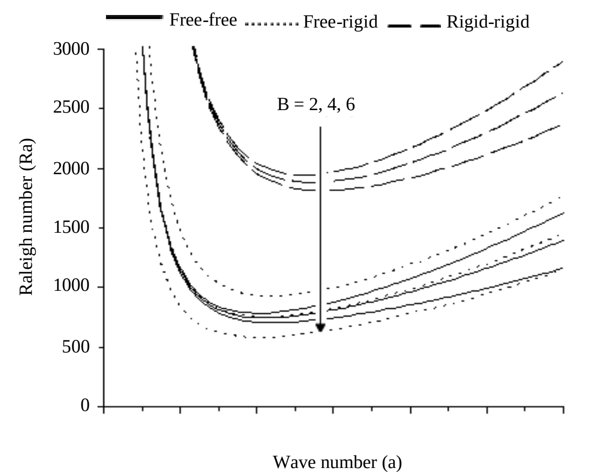

Fig. 2: Marginal stability curve Ra vs. a for different values of B

In this research, we also consider the Soret effect as these effect is important to analyze the thermo diffusion in a binary fluid. The effect exists when small light molecules and large heavy molecules separate under a temperature gradient. Figure 3 shows the plot of Ra versus different values of Soret number, Sr, namely Sr = 0.2, 0.4 and 0.6. In each of these plots, the critical number decreases with increasing of the Soret number. Since, the system is heated from below the increase in the temperature flux contributes to the initiation of natural convection in binary fluid mixtures.

The trends of stability curves in a binary fluid within the system have been plotted in Fig. 4 and 5. Figure 4 shows the effects when we consider the thermo-solutal

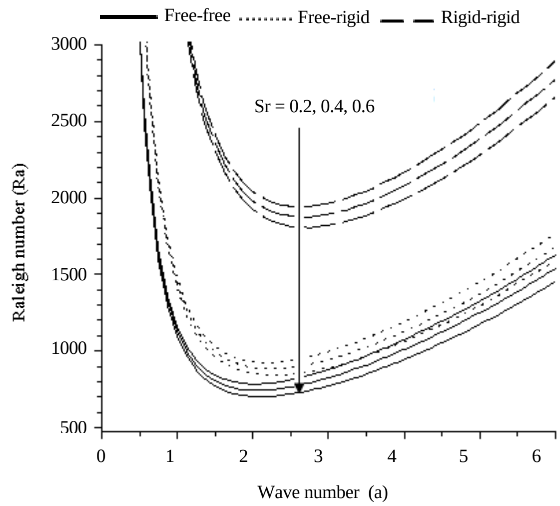

Fig. 3: Marginal stability curve Ra vs. a for different values of Sr

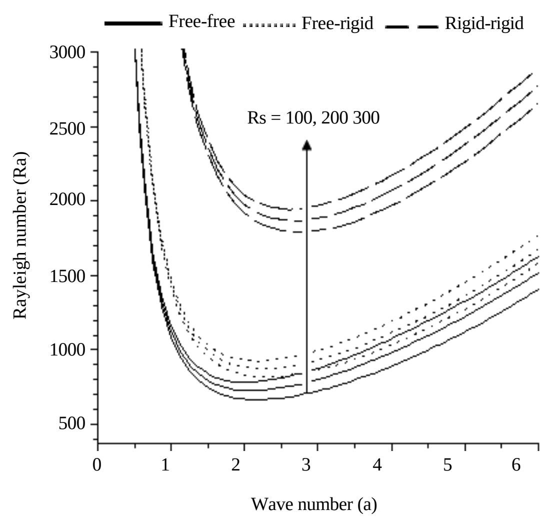

Fig. 4: Marginal stability curve Ra vs. a for different values of Rs

Rayleigh number, Rs and Fig. 5 shows the effect of the Lewis number, Le. As can be seen clearly from the graph, an increase of Rs number will increase the Rayleigh number. Since, the effective thermal conductivity is reduce through the influences of thermo-solutal Rayleigh number (Basu and Layek, 2013). Contrast to the Lewis effect, an increase of Le number will decrease the Rayleigh number.

The effect of temperature dependent viscosity, B, Soret parameter and Lewis number on the onset of convection in binary mixtures is plotted clearly in Fig. 6-11. It is known that the onset of convection occurs beyond a critical value of the Rayleigh number. Figure 6-8

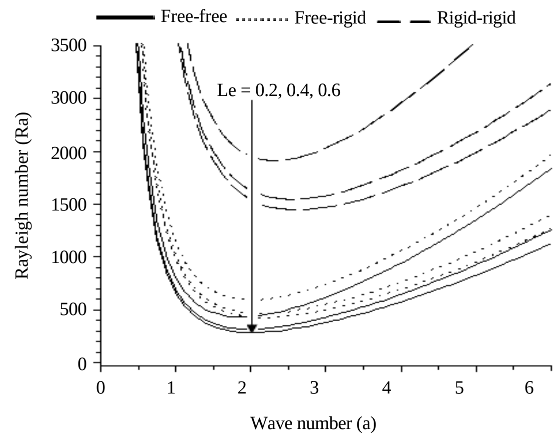

Fig. 5: Marginal stability curve Ra vs. a for different values of Le

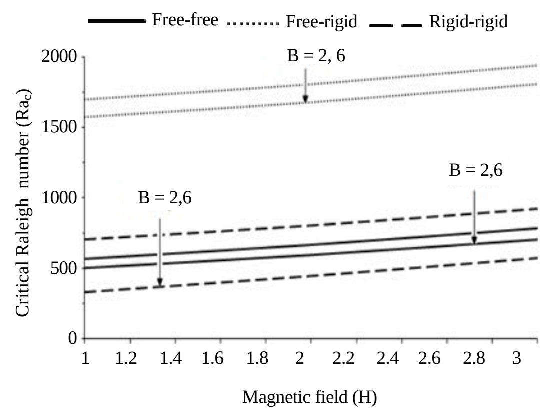

Fig. 6: Marginal stability curve Ra_c vs. H for different values of B

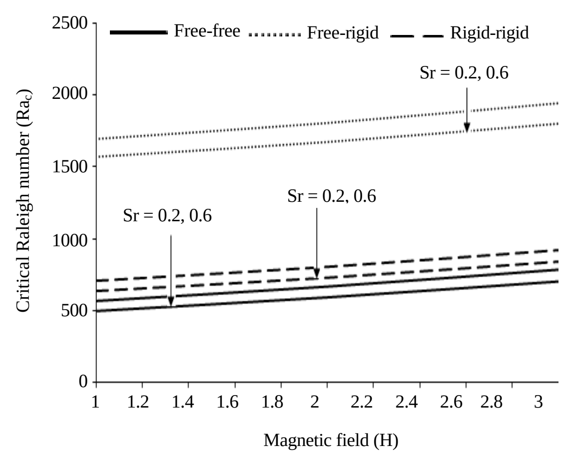

Fig. 7: Marginal stability curve Ra_c vs. H for different values of Sr

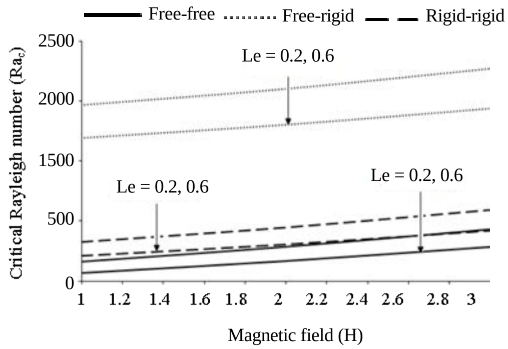

Fig. 8: Marginal stability curve Ra_c vs. H for different values of Le

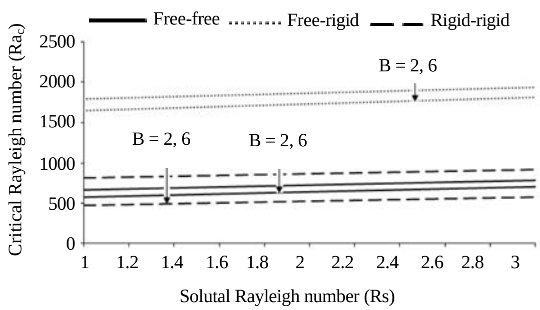

Fig. 9: Marginal stability curve Ra_c vs. Rs for different values of B

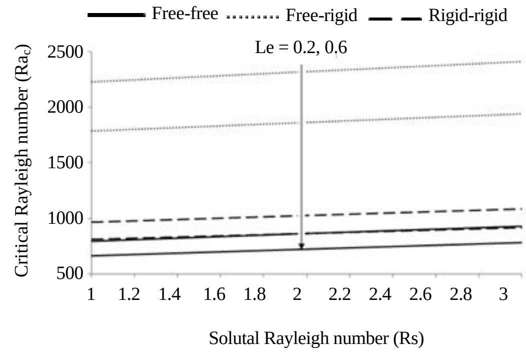

Fig. 10: Marginal stability curve Ra_c vs. Rs for different values of Sr

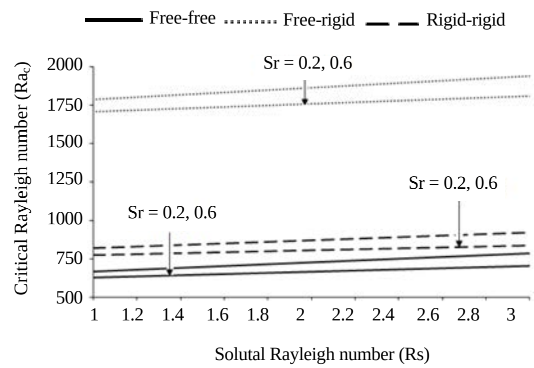

Fig. 11: Marginal stability curve Ra_c vs. Rs for different values of Le

shows the plot of magnetic field, H for variation values of B, Sr and Le number on the onset of natural convection in binary mixture.

To analyze the stability behavior of the system, we increase the values of the magnetic field, H for all bounded boundary considered. As expected, large value of magnetic field will increase the critical Rayleigh number, Ra_c. This implies that H yields a reduction in the rate of convection and stabilized the system.

Figure 6 also shows that when the values B increase, the system destabilize. Rigid-rigid boundary also show a higher critical Rayleigh number compared to free-rigid and free-free boundary. Figure 7 shows the plot of Ra_c versus different values of Soret number, Sr. When Sr increase, the critical Rayleigh number decrease. In other word, the Soret number destabilize the convection.

The effect of temperature dependent viscosity, Soret effects and Lewis number on the variation of Ra_c versus Rs are depicted in Fig. 9-11, respectively. The results are similar with previous results presented. An increase of Solutal Rayleigh number, Rs will increase the critical Rayleigh number, Ra. This means that the system are more stable. As for others parameter, an increase of temperature dependent viscosity, Soret effects and Lewis number shows that it will destabilizes the system. As the effect of lower-upper boundary, the onset of convection happen rapidly in free-free type of system, followed by rigid-free and rigid-rigid system. This observation could point out that the use of rigid-rigid boundary can sustain stability in one system.

CONCLUSION

This present research has focused on a soret driven binary fluid between two horizontal plates which is heated from below by a fix and constant heat flux. The effects of magnetic vertical field, temperature dependent viscosity, Soret effects, thermo-solutal Rayleigh and the Lewis number was investigated and analyzed. The study revealed the following:

- Only two of the parameter tested in this research has a stabilization effect which are the vertical magnetic field, H and thermo-solutal Rayleigh, Rs
- Temperature dependent viscosity, B, Soret parameter, Sr and Lewis number, Le drives a destabilization effect within the system

Three types of boundary conditions on the horizontal boundaries that are free-free, rigid-free and rigid-rigid are considered in this investigation. Based on the results obtained, it is found that the system with rigid-rigid horizontal boundaries is the most stable followed by rigid-free and free-free horizontal boundaries.

ACKNOWLEDGEMENT

The researchers gratefully acknowledge the financial support received from Ministry of Higher Education Malaysia under Fundamental of Research Grant Scheme (02-01-15-1703FR).

REFERENCES

Basu, R. and G.C. Layek, 2013. Influences of cross-diffusive coefficients on the onset of double-diffusive convection in a horizontal fluid layer. Int.J. Appl. Math. Mech., 9: 86-107.

Bergeon, A., D. Henry, H. Benhadid and L.S. Tuckerman, 1998. Marangoni convection in binary mixtures with Soret effect. J. Fluid Mech., 375: 143-177.

Chandrasekhar, S., 1961. The Stability of Superposed Fluids: The Rayleigh-Taylar Instability, in Hydrodynamic and Hydromagnetic Stability. Clarendon Press, Oxford.

Franchi, F. and B. Straughan, 1992. Nonlinear stability for thermal convection in a micropolar fluid with temperature dependent viscosity. Intl. J. Eng. Sci., 30: 1349-1360.

Lu, J.W. and F. Chen, 1995. Onset of double-diffusive convection of unidirectionally solidifying binary solution with variable viscosity. J. Crystal Growth, 149: 131-140.

Maekawa, T. and I. Tanasawa, 1988. Effect of magnetic field on the onset of Marangoni convection. Intl. J. Heat Mass Transfer, 31: 285-293.

Nield, D.A. and A.V. Kuznetsov, 2011. The onset of double-diffusive convection in a nanofluid layer. Intl. J. Heat Fluid Flow, 32: 771-776.

Nield, D.A., 1967. The thermohaline Rayleigh-Jeffreys problem. J. Fluid Mech., 29: 545-558.

Ramirez, N.E. and A.E. Saez, 1990. The effect of variable viscosity on boundary-layer heat transfer in a porous medium. Intl. Commun. Heat Mass Transfer, 17: 477-488.

Rao, K.R., 1980. Thermal instability in a micropolar fluid layer subject to a magnetic field. Intl. J. Eng. Sci., 18: 741-750.

Rudraiah, N., O.P. Chandna and M.R. Garg, 1986. Effect of non-uniform temperature gradient on magnetoconvection driven by surface tension and buoyancy. Indian J. Technol., 24: 279-284.

Saravanan, S. and T. Sivakumar, 2009. Exact solution of Marangoni convection in a binary fluid with throughflow and Soret effect. Appl. Math. Modell., 33: 3674-3681.

Sassi, B.M., S. Kaddeche, A. Abdennadher, D. Henry and B.H. Hadid *et al.*, 2016. Influence of a magnetic field on the stability of a binary fluid with Soret effect. C. R. Mec., 344: 1-11.

Shivakumara, I.S., J. Lee, M. Ravisha and R.G. Reddy, 2011. Effects of MFD viscosity and LTNE on the onset of ferromagnetic convection in a porous medium. Intl. J. Heat Mass Transfer, 54: 2630-2641.

Slavtchev, S., G. Simeonov, V.S. Vaerenbergh and J.C. Legros, 1999. Technical note Marangoni instability of a layer of binary liquid in the presence of nonlinear soret effect. Intl. J. Heat Mass Transfer, 42: 3007-3011.

Teamah, M.A., A.F. Elsafty, M.Z. Elfeky and E.Z. El-Gazzar, 2011. Numerical Simulation of Double-Diffusive Natural Convective Flow in an Inclined Rectangular Enclosure in the Presence of Magnetic Field and Heat Source, Part A: Effect of Rayleigh Number and Inclination Angle. Alexandria Eng. J., 50: 269-282.