Journal of Engineering and Applied Sciences 12 (21): 5413-5417, 2017
ISSN: 1816-949X

Development of Mathematical Model for Operating Room Scheduling

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Abstract: One of the problems faced in the process of operating room scheduling is the uncertainty in the surgical duration. Surgical duration uncertainty often causes the hospital to change the initial operating room schedule, even the schedule can not be started earlier. This system leads to prolonged patient waiting time and staff overtime. This study develops a mathematical model for operating room scheduling which aims at minimizing patient waiting time and staff overtime. Clustering analysis is used to determine the priority in scheduling patients while hill climbing is developed to solve the model to find out daily operating room schedule. Computational experiment was performed based on real surgery data of elective patients to examine performance quality of this proposed model. The results indicate that waiting time and overtime experience a decrease of 40.04 and 29.54%, respectively compared to real conditions.

Key words: Clustering analysis, operating room, scheduling, real conditions, surgery, data

INTRODUCTION

Health services are becoming the focuses of many researchers at present due to uncertainty and risk factors involved in it. One of the hospital units which has important roles and needs special handling is operating room. Operating room needs the greatest operating budget and generates the highest income for the hospital (Macario *et al*., 1995; Vancroonenburg *et al*., 2015). Operating room accounts for 40% of the hospital's resource expense due to the use of facilities, equipment, expensive drugs and high-cost medical specialists (Vancroonenburg *et al*., 2015; Essen *et al*., 2012). Nevertheless, operating room makes about 40% of the hospital's total revenues which is obtained from surgery cost, drug charge and hospitalization expenses paid by the patients (Guerriero and Guido, 2011). Operating room as a critical unit in hospitals so that the operating room scheduling and management system will affect the hospital's service quality (Roland *et al*., 2006; Tancrez *et al*., 2013).

The problem found in a surgical procedure at operating room is uncertainty in surgical duration. This uncertainty might be caused by patient's health condition during the surgery or error in predicting surgical duration done by the scheduling department. The fact often forces the hospital to change the operation schedule which has been arranged, particularly when the hospital resources capacity is limited. The main problem is that the hospital only delays all the remaining operation schedules after there is an interruption on the needed time or right shift scheduling (Batun *et al*., 2011).

The delay causes the presence of patient waiting time and staff overtime. Patient waiting time is one of the problems that hospitals need to encounter. Prolonged patient waiting time might influence patient's disease severity and lead to patient dissatisfaction. Another problem hospital management may encounter is high overtime. Overtime expense is often higher than the regular one at some hospitals (Sufahani *et al*., 2012). Cost reduction related to surgical process and overtime is highly important for the hospital since when overtime occurs, the expenditure increases more than that in normal condition. A reduction of the overtime will be more useful for the hospital management than an increase in surgical cases which should be completed during regular working hours.

Santoso *et al*. (2007) formulates an operating room scheduling model for elective patients and groups types of surgical based on the characteristics of surgery using hierarchical and non hierarchical clustering. A scheduling sequence in each cluster formed was conducted using shortest processing time while operating rooms were scheduled using genetic algorithm. In contrast to previous research the scheduling sequence in each cluster formed in this research was randomly determined. The sequence which will be used to determine operating room scheduling priority was clusters sequence resulting

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in minimum waiting time and overtime. In order to speed up and ease the scheduling process, hill climbing was utilized to seek out operating room schedules resulting in minimum values of waiting time and overtime. This strategy is expected to minimize the influence of uncertainty in surgical duration towards patient waiting time and staff overtime.

MATERIALS AND METHODS

Model formulation

Problem definition: Problem arising in the operating room scheduling involves determining activities of operations which will be performed for the elective patients according to the availability of doctors, recovery room and operation staff team during the certain period of time. Operation staff team refers to anesthesiologists and nurses. The purpose of this operating room scheduling model is to minimize patient waiting time and staff overtime. Clustering analysis results are used to determine priority in the stage of operation sequencing and hill climbing was applied to find out daily surgical schedule.

Mathematical model: Mathematical model formulation for the operating room scheduling model is given as follows. Objective function:

$$\text{Min} \sum_{m=1}^{M}\sum_{r=1}^{R}\sum_{t=1}^{T}(t\text{-}S_m)Y_{mrt} + \sum_{p=1}^{P}\sum_{m=1}^{M}(ET_m\text{-}FT_p)X_{mp} \quad (1)$$

Subject to:

$$ET_m = \sum_{r=1}^{R}\sum_{t=1}^{T}(t+d_m)Y_{mri} \quad \forall m \in O \quad (2)$$

$$\sum_{r=1}^{R}\sum_{t=1}^{T}Y_{mri} = 1 \quad \forall m \in O \quad (3)$$

$$\sum_{r=1}^{R}\sum_{m\in\Omega n}\sum_{\tau=1}^{t+d_m}Y_{mri} \le 1 \quad \forall t \in \{1...T\}, n \in \{1...N\} \quad (4)$$

$$\sum_{m=1}^{M}\sum_{\tau=1}^{t+d_m}Y_{mri} \le 1 \quad \forall r \in \{1...R\}, t \in \{1...T\} \quad (5)$$

$$\sum_{r=1}^{R}\sum_{t=1}^{T}\sum_{m\in\Omega} t \cdot Y_{mri} \ge S_m \quad \forall m \in \Omega \quad (6)$$

$$\sum_{r=1}^{R}\sum_{\tau=1}^{t+d_m}\sum_{m\in\Omega}Y_{mrt} \le G(p,t) \quad \forall t \in \{1...T\} \quad (7)$$

$$\sum_{r=1}^{R}\sum_{m=1}^{M}\sum_{\tau=1}^{t+d_m}X_{mp} \cdot Y_{mrt} \le 1 \quad \forall t \in \{1..T\}, p \in \{1...P\} \quad (8)$$

$$\sum_{p=1}^{P}X_{mp} = 1 \quad \forall m \in \Omega \quad (9)$$

$$\sum_{m=1}^{M}\sum_{\tau=t}^{t+dv_m}Z_{mvt} \le 1 \quad \forall v \in \{1...V\}, t \in \{1...T\} \quad (10)$$

$$ET_m = \begin{cases} ST_m + dv_m, & \text{if } F(v,t) \le 0 \\ ET_m, & \text{otherwise} \end{cases} \quad (11)$$

This operating room scheduling model used three binary variable. Binary variable Y_{mrt} which having value of 1 if surgery m was started at operating room r and was initiated at time slot t, 0 otherwise. Binary variable X_{mp} which having value of 1 if surgery m was assigned by staff team p, 0 otherwise. Another binary variable Z_{mvt} which having value of 1 if surgery m was used recovery room v at time slot t, 0 otherwise. Each operation has a duration estimation (d_m) in initial surgical schedule. Time value is represented by time slot (t) (Dexter, 2002) in which a time slot ranges for 15 min. The number of operations which will be handled by doctor q is symbolized by Ωq. Each doctor is assigned to handle patients and therefore it is impossible for them to change the doctor. The staff team involving in the surgery is symbolized by p while operating room which is used is symbolized by r. Booking time (S_m) is defined as the time which has been booked for a surgical case done by the assigned doctor.

The objective function seeks to minimize patient waiting time and staff overtime. Constraint determines the finish time of each surgical case. Constraint ensures that each treatment was only scheduled once and restricts that a doctor could only carry out one surgery at available time. Constraint ensures that at available time in one operating room there was only one surgical case activity. Constraint guarantees that surgery starting time was bigger than or equals to the initial booking time. Constraint ensures that a surgery was started at operating room at available time by available staff team, indicates that each surgical case was only handled by one staff team at certain time slot and implies that each staff team could only handle a surgical case at available time slot. Constraint restricts that each recovery bed could only be occupied by one patient at certain time slot and ensures that patient would perform a recovery in operating room if a recovery bed was not available.

Computational experiment: Data include historical records of surgical requests and operating room uses for elective patients, provided by central surgery unit in one of Indonesian hospitals. This unit is staffed from 08:00 a.m. to 08:30 p.m. from Monday to Friday. This hospital uses first come first served operating room scheduling system which allows the patients to obtain surgery schedule in accordance to sequence of surgery requests. Surgery requests should be made at least one day before the surgery performed. Lists of the requests are later accessed by scheduling department to be scheduled manually.

Clustering analysis: Clustering was carried out to group types of surgical case with the same characteristics. Variables used in making clusters comprise the average value and the standard deviation value of surgery duration. The clustering method applied in this research is hierarchical clustering. This type of clustering is more advantageous to determine the number of clusters which is going to be used. The distance between clusters was measured using centroid linkage method. This method was chosen due to its better performance compared to other methods when the number of targeted clusters was different (Ferreira and Hitchcock, 2009).

Clustering analysis with centroid linkage method was applied in the research. Algorithm of the clustering analysis is given as follows: variable data input of each type of operations which will be categorized. Calculation of each type of operations based on Euclidean distance theory which was formulated as:

$$d(i, j) = \sqrt{(x_{1i}-x_{1j})^2+(x_{2i}-x_{2j})^2+\cdots+(x_{ki}-x_{kj})^2}$$

Where:

$d(i, j)$ = Distance from data of i to data source of j

x_{ki} = The ith data on the kth data variable

x_{kj} = The jth data on the kth data variable

Categorization of types of operations in which certain type with the closest distance would be located in the same cluster. New clusters would have new centroid value by taking the mean value of all new objects in the clusters. Repetition of Step 2 and 3 until one cluster was left.

Determination of the number of optimal clusters. The number of optimal clusters was obtained when there existed significant change in similarity level and when the value of similarity level was considered high (>80) computed as:

$$S(ij) = 100 \cdot \left(1-\frac{d(ij)}{d(max)}\right)$$

Where:

$S(ij)$ = Similarity level of cluster i and j

$d(ij)$ = Distance between cluster i and j

$d(max)$ = Maximum distance on original distance

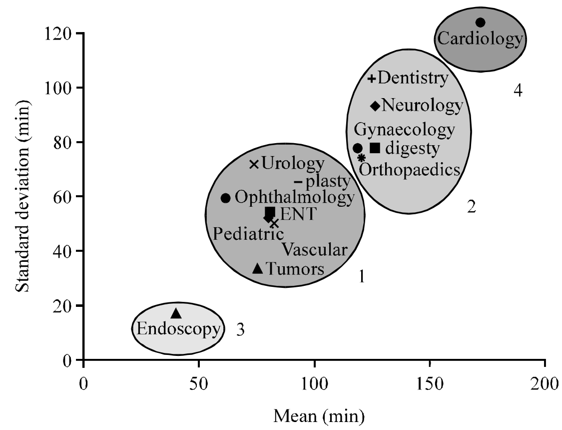

Fig. 1: Results of surgery activities clustering

Determination of optimum number of clusters was conducted by taking into account the changes of similarity level in each number of clusters used. On the basis of the research results, significant changes occur when the number of cluster is 3 and 4. Therefore, four clusters were used in this research. It is considered optimum due to good results of statistical test on homogenity within each cluster and heterogeneity between clusters. Those four clusters are presented in Fig. 1.

To ensure that the clusters produced are good enough, homogenity test within cluster (intra-cluster homogenity test) and heterogenity test between clusters were carried out. Intra-cluster homogenity test was performed to indicate the similarity level in a cluster. This type of test was conducted by calculating Sum of Square Error Within Cluster (SSW) and Sum of Square Between Cluster (SSB) ratio. The ratio is quite low that is 0.0532 and hence, it is interpreted that the relationship among members in the cluster is homogenous. Heterogenity test was performed to reveal the difference level between clusters. This test was carried out by measuring R^2 value. This study results in the R^2 value of 0.95 which means that the algorithm obtained can divide the data into four clusters which are significantly different.

Hill climbing: Hill climbing is an optimization method which belongs to the family of local search. It makes use of the best candidate solution to find out new points in search space. The method carries out iterations in which current solution is used to produce new candidate solution. If the new candidate solution is found better than the previous best solution, it will serve as the best solution and the process repeats. The steps of Hill Climbing algorithm are illustrated in Fig. 2.

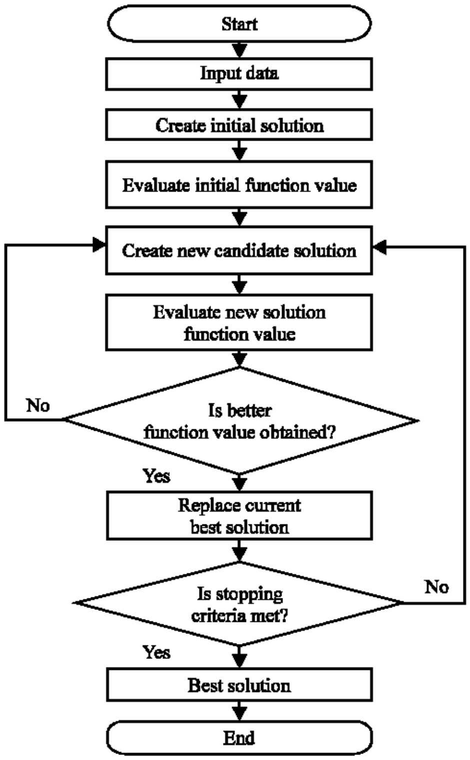

Fig. 2: Steps of hill climbing algorithm

Input data in operating room scheduling using hill climbing include data of surgery requests, duration and types of surgical procedures, data of doctors and operating rooms and sequence from cluster formed. To ease data processing, all data of surgical procedure duration are represented in form of time slot. Hill climbing will perform iterations (repetitions of the process) to find out the best solution. The repetitions will stop when stopping condition has been fulfilled. Stopping criterion includes either minimum objective function value (0) or iteration numbers of 2000.

Scheduling priority in each clusters formed in this study was randomly determined. According to experiment results, the order for scheduling priority is as the following: patients in cluster 3, 1, 2 4, respectively. This strategy, hence was proved to be able to minimize waiting time and overtime due to the impacts of uncertainty in surgical duration.

Summary: This research results indicate that operating room scheduling model using clustering and hill climbing appears to have the better values compared to real condition. The comparison between patient waiting time and staff overtime of scheduling results in June 2016 is displayed in Table 1. In real scheduling system, total patient waiting time is 812.65 hand staff overtime is 122.46 h. Meanwhile, in scheduling system using clustering and hill climbing, patient waiting time decreases by approximately 40.04% from real scheduling (487.25) and staff overtime decreases by approximately 29.54% from real scheduling (86.28 h).

Table 1: Operating room scheduling result comparison

Performance criteria	Actual	Proposed model
Waiting time (h)	812.65	487.25
Overtime (h)	122.46	86.28
Total	935.11	573.53

CONCLUSION

In conclusion, operating room scheduling model using clustering analysis and hill climbing which prioritizes patients shows results in lower patient waiting time and staff overtime than actual scheduling condition. This type of scheduling model is capable of reducing patient waiting time to 40.04% and staff overtime to 29.54%. However, this proposed model is merely limited to considerations in determining scheduling priority which include surgical duration and types of operations. This, therefore, requires further research on such another consideration in determining the scheduling priority as contamination level of surgical activities.

NOMENCLATURE

Γ = Set of operating rooms
R = Number of operating rooms, $R = |\Gamma|$
r = An operating room, $r \in \{1, ..., R\}$
Ω = Set of surgical activity
M = Number of surgical activity, $M = |\Omega|$
m = A surgical case, $m \in \{1, ..., M\}$
τ = Set of time slots
T = Number of time slots, $T = |\tau|$
t = A time slot, $t \in \{1, ..., T\}$
Ω_n = Set of surgical activity allocated to doctor n
N = Number of doctors
n = A doctor, $n \in \{1, ..., N\}$
P = Set of staff teams
p = Number of staff teams, $P = |\rho|$
G = Available staff team in time slot
γ = Set of recovery bed
V = Number of recovery bed, $V = = |\gamma|$
ν = A recovery bed, $\nu \in \{1, ..., V\}$
F = Available recovery bed in time slot
d_m = Duration of surgical case m
d_{vm} = Recovery duration of surgical case m
S_m = Surgical case m booking time
E_{Tm} = End time of surgical case m
F_{Tp} = Finish time of staff team p
S_{Tm} = Start time of surgical case m recovery
$E_{Tm'}$ = End time of operating room r usage

ACKNOWLEDGEMENT

Researchers acknowledge support from Felicia Alverina Monindra and Ayu Norma Affiati for their help during the development of this project.

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