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Effect of the Central Rise on the Behavior of Dome-Like Structures

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Abstract: Construction and design of domes have been of great engineering challenge for thousands of years. Great domes are still fascinating structures that reflect power and unity. Domes have been used extensively in long span public structures. The dome structures reflect power and project sense of community. Building domes require experience, time and great effort. Dome like structures can be easily built at relatively lower cost. Pyramid and conical shape roofs are some examples of the dome like structures considered in this study. The pyramids are of square base or rectangular base while the conical shape roofs are of circular base. Domes are very efficient in supporting loads, taking advantage of the strength of the building material in compression. Flat slabs can shift its behavior to dome like behavior by increasing the central rise. This process converts the two dimensional structural element into three dimensional structural element. The slab will shift its behavior from plate element to shell element, depending on the slab thickness and the value of the central rise. The increase in the central rise up to certain values, converts the flexural stresses into compressive stresses. Most of the building materials are stronger in compression compared to tension or flexure which makes the domes and the dome like structures efficient structures. Square and circular slabs are used in this study where the central rise is increased gradually to study the effect of the central rise on the central deflection and the maximum stress. Finite Element Model is developed to study the effect of the slab thickness on the maximum Von Mises stress at a given value of the central rise. The slabs are subjected to uniformly distributed dead loads and live loads.

Key words: Central rise, pyramid, shell element, plate element, circular conical slab, compressive

INTRODUCTION

Dome structures have been proven to be long lasting efficient structures due to their capability to carry the applied loads. The vertical loads will be resisted mainly by internal compressive stresses. The base of the dome must be reinforced for outward forces. The shape of the domes is advantages for wind forces especially for the domes which are built at high structures. The main advantage of domes is that it covers large areas without the need for columns. Domes usually give an impressive and attractive view from the inside and from the outside which difficult to forget. Unfortunately, constructing domes could be costly and requires special skills. Several shapes similar to domes were constructed all over the world.

Building structures similar to domes but not as elaborate as a real dome could be a reasonable compromise. Hollow conical shape roofs and hollow pyramid roofs are two examples of dome like structures that can be easily built. These two types of structures take some of the advantages of the domes as structural elements especially when it comes to deflection and stresses. The main difference between hollow conical shape roof and hollow pyramid roof is the shape of the base which can be either circular or rectangular. Flab circular slabs or square slabs can be subjected to vertical rise forming cones and pyramids of variable apex out of flat slabs. This study focuses on the central deflection of these newly formed shapes as well as the maximum Von Mises stress for a given value of the slab thickness.

Long span flat slabs are subjected to high punching shear values. This requires an increase in the slab thickness as well as the increase in the steel reinforcing bars. Al-Nasra *et al*. (2013), studied the flat slabs subjected to high punching shear. They provided additional steel reinforcement to reduce the cost of the flat slabs of long span. Swimmer bars were introduced to resist the applied shear stresses (Al-Nasra *et al*., 2013,1926; Al-Nasra and Wang, 1994; Duweib *et al*., 2013).

Finite Element Model is developed to study, the stresses and deflection of slabs and plates of increasing central rise. The Von Mises concept is applied to calculate the maximum stresses. Several mesh generating techniques were explored to generate a mesh that suitable for this current study. The purpose of the mesh is to give an acceptable and consistent accuracy.

The research in this area focused on circular plates subjected to symmetrical loadings. Numerical techniques were used to solve for stresses and deflections. The maximum deflection at the center of the plate was the focus of many researchers. The plates used were mostly simply supported. The maximum deflection at the center of the plates is expressed in Eq. 1 (Ventsel and Krauthammer, 2001; Szilad, 1974; Timoshinko and Woinowsky-Krieger, 1959):

$$D_{cen,max} = \frac{U_o R^4 (5+\upsilon)}{64\lambda(1+\upsilon)} \quad (1)$$

Where:
υ = The poison's ratio
R = The plate radius
U_o = The applied uniformly distributed external load
λ = The plate flexural rigidity

The value of the plate flexural rigidity depends of the modulus of elasticity of the material used and can be expressed as follows (Bucalem and Bathe, 1997; Ugural, 1999; El-Sobky *et al*., 2001):

$$\lambda = \frac{1}{12} \times \frac{E t^3}{1-\upsilon^2} \quad (2)$$

Where:
E = The material modulus of elasticity
t = The thickness of plate

The maximum deflection at the center of the plate can be used to solve for the applied load:

$$U_o = \frac{D_{cen,max}}{R} \times \frac{64\lambda(1+\upsilon)}{R^3(5+\upsilon)} \quad (3)$$

Equation 3 is known as the plate small deflection theory. It relates the applied uniformly distributed load to the maximum deflection at the center of the plate. This equation solves for the applied load that generate a maximum deflection at the center of the plate (Al-Nasra and Daoud, 2015; 2016; Sadowski and Rotter, 2013; Reese, 2007).

MATERIALS AND METHODS

Plate-shell theory: The finite element model used in this study solves for stresses and deflections of the slabs subjected to induced central rise at the center and subjected to uniformly distributed applied load. The Von Mises theory is used to calculate the maximum stresses which is based on the concept of the distortion energy theory. The distortion energy Ω can be expressed of the principal stresses as shown in Eq. 4:

$$\Omega = \frac{(\sigma 1-\sigma 2)^2 + (\sigma 2-\sigma 3)^2 + (\sigma 3-\sigma 1)^2}{2}\left(\frac{1+\nu}{3E}\right) \quad (4)$$

The distortion energy can also be rewritten as:

$$\Omega = \frac{1+\nu}{3E}(\sigma_v)^2 \quad (5)$$

Where:
σ_1-σ_3 = The principal stresses
σ_v = Von Mises stress

Where in this case the Von Mises stress is defined as a function of the principal stresses as shown in Eq. 6:

$$\sigma_v = \sqrt{\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)}{2}} \quad (6)$$

Equation 6 can also be expressed in terms of the three dimensional stresses as shown in Eq. 7:

$$\sigma_v = \sqrt{\frac{(\sigma_{xx}-\sigma_{yy})^2 + (\sigma_{yy}-\sigma_{zz})^2 + (\sigma_{zz}-\sigma_{xx}) + 6(\tau^2{}_{xy} + \tau^2{}_{yz} + \tau^2{}_{zx})}{2}} \quad (7)$$

Where:
τ_{xy}, τ_{yz}, τ_{zx} = The shear in the planes xy, yz and zx, respectively
σ_{xx}, σ_{yy}, σ_{zz} = The axial stresses in x, y and z axes, respectively

In case of two dimensional stresses, the Von Mises equation can be expressed as:

$$\sigma_v = \sqrt{\sigma^2{}_{xx} + \sigma^2{}_{yy} - \sigma_{xx}\sigma_{yy} + 3\tau^2{}_{xy}} \quad (8)$$

RESULTS AND DISCUSSION

Two different types of slabs were used; circular and square. The center of each slab is subjected to a gradual central rise. The circular slab will be forming a conical shape slab while the square slab will be forming a pyramid shape slab. Both are subjected to the same loading case. The behavior of each slab in terms of stresses and central deflection is the focus of this study. Comparison between the two slabs is presented in terms of equations and graphs. Three slab thicknesses (0.10, 0.15 and 0.20 m) are also considered to study the effect of the slab thickness on the maximum Von Mises stress and the central deflection. The slabs are analyzed as simply supported slabs loaded with a dead load (5 kN/m^2) in addition to the self-weight and a live load (3 kN/m^2). For the ultimate load condition, the load combination of 1.4 by the dead load and 1.6 by the live load is used. For the service load condition, the load combination of 1.0 by the dead load and 1.0 for the load live is used.

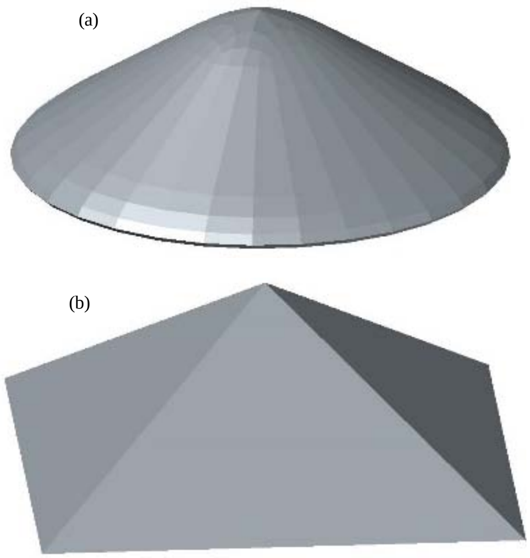

Fig. 1: Slabs with central rise: a) Circular slab and b) Square slab

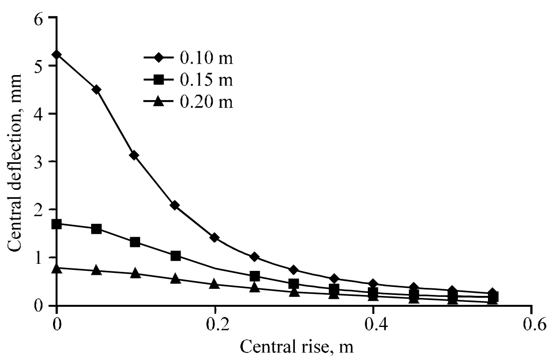

Fig. 2: Effect of the central rise on the central deflection of circular slabs

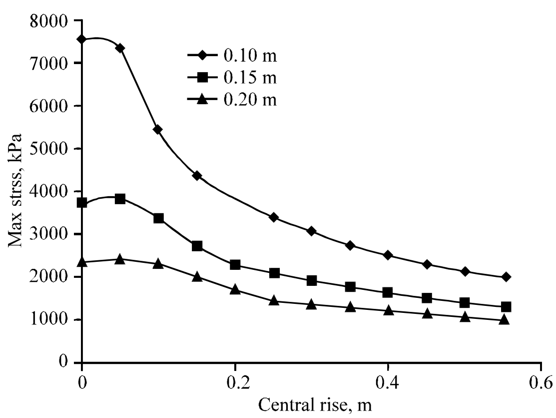

Fig. 3: Effect of the central rise on the maximum Von Mises stress of circular slabs

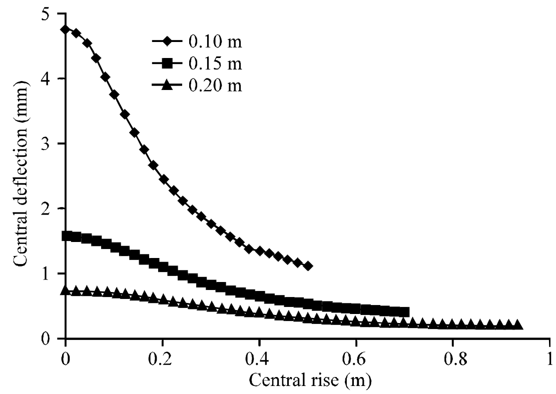

Fig. 4: Effect of the central rise on the square slab central deflection

The incremental increase in the rise of 2 cm is applied in this study. The slab transforms from two-dimensional structural element to three-dimensional structural element. Consequently, the stresses and deflections will change for a given load combination. The amount of material used will be increased in this process but the stresses decrease in return. Figure 1 shows a typical transforming circular slab as well as transforming square slab.

Figure 2 shows the effect of the central rise on the central deflection for a given circular slab thickness. The diameter of the slab is 4 m. It can be observed that the central deflection is relatively high for the slabs of small thickness at small value of central rise. Also it can be noticed that the slab thickness becomes non-sensitive to the central rise at large values of central rise. Figure 3 shows the effect of the central rise on the max Von Mises stress. In general the stress reduces at lower rates after the central rise of 0.20 m. The stress decreases with the increase in the slab thickness. For high values of central rise the effect of the slab thickness diminishes.

Figure 4 shows the effect of the central rise on the central deflection of a square slab. The slab dimensions are 4 m by 4 m. The central deflection decreases substantially with the increase in the central rise. The increase in the slab thickness decreases the central deflection especially at low values of central rise. The effect of the slab thickness on the central deflection decreases with the increase in the central rise. Figure 5 shows the effect of the central rise on the max value on Von Mises stress. The max Von Mises stress decreases with the increase in the central rise to a certain value of central rise, then the stress shift its direction. It can be observed from Fig. 5 that the increase in the slab thickness decreases the maximum Von Mises stress for the same case of loading. The effect of the slab thickness decreases with the increase in the central rise.

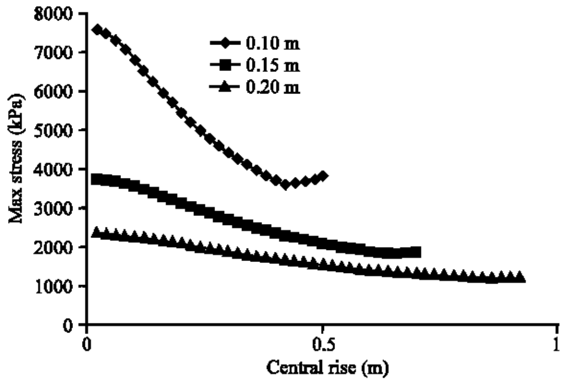

Fig. 5: Effect of the central rise on the maximum Von Mises Stress of square slabs

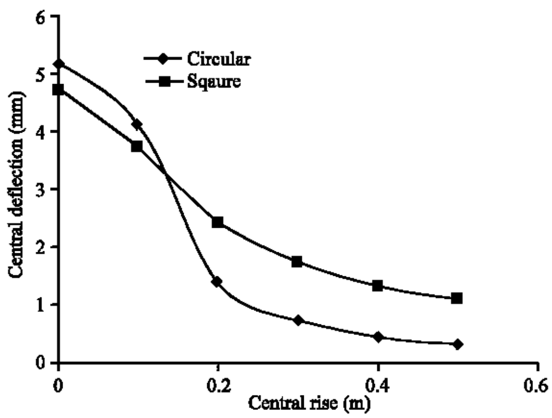

Fig. 6: Effect of the base shape on the central deflection of 0.10 m thick slab

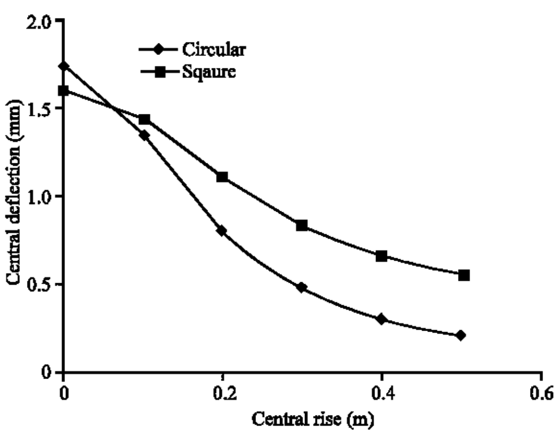

Fig. 7: Effect of the base shape on the central deflection of 0.15 m thick slab

Figure 6-8 shows the comparisons between square slabs and circular slabs as the central rise is related

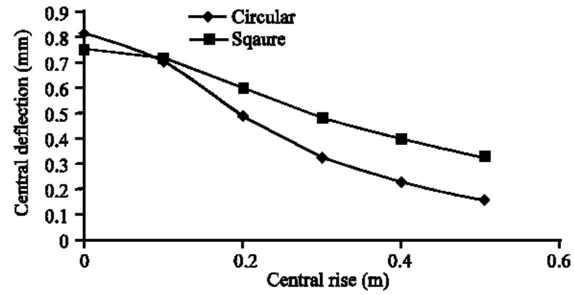

Fig. 8: Effect of the base shape on the central deflection of 0.20 m thick slab

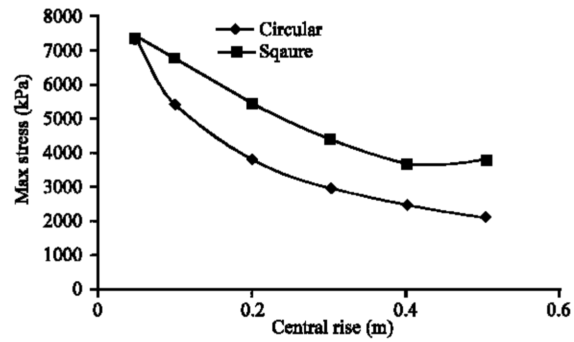

Fig. 9: Effect of the base shape on the maximum Von Mises stress of 0.10 m slab thickness

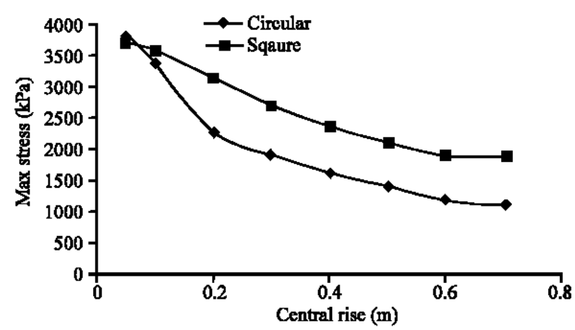

Fig. 10: Effect of the base shape on the maximum Von Mises stress of 0.15 m slab thickness

to the central deflection of the slab. The base shape has substantial effect on the central deflection at larger values of central rise. The diameter of the circular slab is 4 m and the dimensions of the square slab are 4 m by 4 m. The circular slabs exhibited lower values of central deflection beyond the central rise of 0.10 m.

Figure 9-11 show the effect of the slab base shape on the maximum Von Mises stress. It can be observed from these figures that the stresses in the circular slabs are consistently lower for the circular slabs compared to the square slabs. Also it can be noted the square slabs

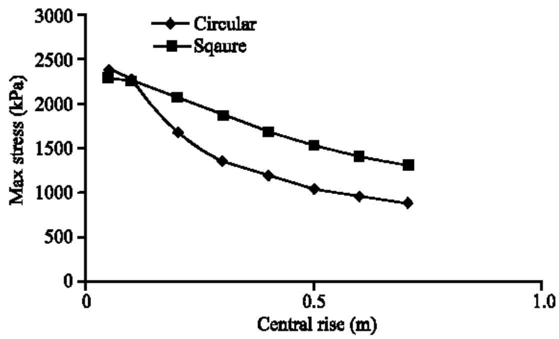

Fig. 11: Effect of the base shape on the max Von Mises stress of 0.20 m slab thickness

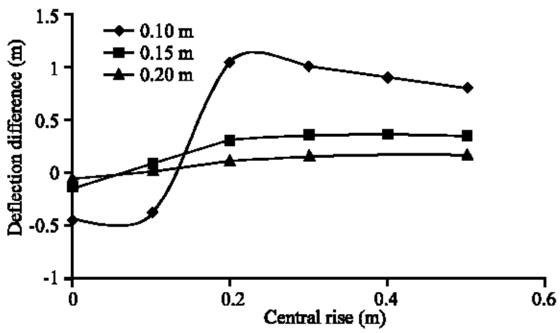

Fig. 12: The effect of the central rise on the deflection difference between circular and square slabs

experience an increase in the maximum Von Mises stress after certain value of central rise. The difference between the maximum Von Mises stresses between the circular slabs and the square slabs is at its lowest values at a very low central rise.

Figure 12 shows the effect of the central rise on the difference in central deflection between circular slabs and square slabs at different values of slab thickness. At small values of central rise the difference seems to be minor. The increase in the central rise decreases the central deflection difference. One can observe from Fig. 12 that the slab thickness has a major factor on the central deflection difference. The central deflection difference is defined in the following equation:

$$\Delta D = Ds\text{-}Dc \tag{9}$$

Where:

ΔD = The central deflection difference between square slab and circular slab of correlated dimensions of the same slab thickness

Ds = The central deflection of a square slab, expressed in millimeters

Dc = The central deflection of circular slab, expressed in millimeters

An approximate formula for circular slab of low central rise has been derived in this study from statistical data analysis at the given load combination. The formula relates the slab thickness to the central deflection of a circular slab provided that the central rise does not exceed 0.25 of the slab diameter as shown in the following equation where Dc is expressed in millimeters:

$$Dc = 14t^2 - 6t + 0.8 \tag{10}$$

where, t = the slab thickness expressed in meters. The central deflection of a square slab can consequently be expressed as follows where Ds is expressed in millimeters:

$$Ds = Dc + \Delta D \tag{11}$$

The central rise has a major effect on the values of the central deflection. Large number of data has been analyzed to study the effect of the central rise on the central deflection at a given value of slab thickness. The following equation is derived for low to moderate central rise that does not exceed 0.5 of the slab diameter. The slab thickness is expressed in meters and the central deflection is expressed in millimeter:

$$Dc = \left(2.25t^2\text{-}14.5t+25.5\right)\psi^2 + \left(0.46t^2\text{-}3.1t+6.1\right)\psi + 2t^2 + 13t\text{-}24 \tag{12}$$

where, ψ is the central rise expressed in meters.

CONCLUSION

Finite element model has been developed successfully to generate relevant data which is used to conduct parametric study. There is a major difference in behavior between circular slabs and square slabs when subjected to central rise. The circular slab in general, shows relatively lower values of stresses when subjected to the same case of loading compared to square slabs. The central deflection decreases with the increase in the central rise. This applies for both circular slab and square slab. The circular slabs exhibit relatively less deflection compared to the square slabs subjected to the same conditions. The maximum Von Mises stress decreases with the increase in the central rise for the circular slabs. This decrease is at a higher rate at lower values of central rise. The square slab behaves similar to the circular slab but at some certain value of the central rise the maximum stress trend shift direction and starts to increase at a low rate. The slab thickness affects both the central deflection and the maximum Von Mises stress. The increase in the slab thickness decreases both the central deflection and the maximum Von Mises stress for both circular slabs and squares slabs at variable rates.

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