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Impact Study of Non-Sinusoidal Load Parameters on Real-Power Losses in 3-Phase Double-Wound Power Transformers: Method of Constant Coefficients

Sergey Kostinskiy and Anatoly Troitskiy
Department of Electricity and Electric, Platov South-Russian State Polytechnic University, Novocherkassk, Russia

Abstract: The increase in unbalanced and non-sinusoidal loads is one of the main factors of power quality deterioration. In this regard, new methods of rational use of power resources are required. This study is devoted to the impact study of non-sinusoidal load parameters on real-power losses in three-phase two-wound power transformers by means of constant coefficients. This study provides an algorithm to calculate transformer total power losses using a developed method of constant coefficients. This method allows going back from the classical method of equivalent circuits for each harmonic component. We have used the electrical circuit theory, modern methods of electric circuit analysis and synthesis, the equivalent resistance method and the method of equivalent sine to achieve this purpose. Branch currents are calculated in one phase of equivalent circuit of TWTL system based on this equivalent circuit (magnetic coupling of HV and LV windings, replaced by their electrical equivalent and parameters of LV windings are under HV). In connecting capacity load to LV terminals, signs "plus" of the last real and imaginary numerator terms and of the last denominator term should be changed to negative in the right-hand side of the last equation. Thus, we have determined the total power losses in single-phase transformer under non-sinusoidal load (in one phase of a 3-phase transformer) by summing up the power losses of each harmonic component in the load. The proposed calculation algorithm for transformer real-power losses by developed method of constant coefficients allows going back from the classical method of equivalent circuits for each harmonic component.

Key words: Method of constant coefficients, non-sinusoidal load, technical losses, t-equivalent circuit, three-phase two-wound power transformer, transformer total power losses

INTRODUCTION

In recent years an increasing number of human resources is not a significant factor for increase in production. The issue of worker productivity improvement by more efficient resource usage including power resources, their savings and rational use is now highlighted (Dryden, 2013; Blaabjerg *et al*., 2015; Uraev *et al*., 2016).

Modern technologies make possible to improve productivity which characterizes the level of scientific and technical progress. Modern technologies in power industry are (Masters, 2004; Gonen, 2016; 2014).

Outdated power grid replacement by intelligent grids, mandatory and simultaneous modification of all elements in power industry including digital substations;creation of a single information-technological space-power cluster.

Energy efficiency is a range of organizational, economic and technological measures to make power resource rational use more valuable in industrial, consumer, scientific and technical fields (Exposito *et al*., 2016; Hall and Tyler, 2014; Kreith and Goswami, 2016).

Power quality deterioration is mainly due to increased unbalanced and non-sinusoidal loads. In recent years, unbalanced and non-sinusoidal operation requires close attention as municipal energy consumption has exceeded the industrial power consumption in certain electric power systems. This led to unbalances of harmonicity in voltage and current systems. Therefore, the issue of electric energy rational use and as a consequence, energy loss reduction in distribution networks with unbalanced and non-sinusoidal loads is an actual scientific and technical problem of energy physics.

Electric power supply is impossible without transformers that transmit the electricity over a distance and efficiently distribute it among consumers. Transformers as an integral part of power network are at all voltage levels as a matching element between the

Corresponding Author: Sergey Kostinskiy, Department of Electricity and Electric,
Platov South-Russian State Polytechnic University, Novocherkassk, Russia

network and the consumer. Transformer no-load loss consideration as well as real loss in short circuit situation with due account for additional losses under unbalanced loads is appropriate in calculating technical losses and in substantiating the economic benefit for transformer replacement.

Since, transformer losses are the main problem due to unbalanced and non-sinusoidal transformer loads, the correct choice of connection diagram for transformers and unbalanced loads based on real-power loss calculations under unbalanced and non-sinusoidal load is very important not only in relation to consumers but also to energy physics.

The problem is that while assessing unbalanced and non-sinusoidal voltages and currents, only the special cases of unbalanced and non-sinusoidal operation are considered: continuous unbalance and periodic distortion of current and voltage curves (Agunov, 2004, 2000; Bessonov, 2006). Current electric installations have random disturbances in addition to continuous and periodic ones. Therefore, their impact assessment on installations is requirable. This will eliminate limitations related to different types of disturbances (continuous, periodic and random) (Barutskov *et al.*, 2011). Where in, dynamic programming is the most advanced method (Agunov, 2000; Barutskov *et al.*, 2011) as it provides Electro Magnetic Compatibility (EMC) measurements in physical meanings.

Energy-dispersive analysis is requirable to assess the impact of non-sinusoidal load on power supply and to develop EMC recommendations. Real-power component valuation could be used as a criterion. Since, it is an absolute figure, energy efficiency of the system "source-non-sinusoidal load" is more favorable to assess using relative criteria, expressed in power and current components.

The maximum efficiency of the system "source-non-sinusoidal load" is achieved under zero reactive and distortion power or under non-active current components that are at the point zero.

Balanced active and passive parameters of 3-phase AC networks are necessary for their normal operation (Idelchik, 1989; Melnikov, 1975; Nikitin, 1958) as well as sinusoidal voltage and current (lack of current and voltage harmonics). Operation conditions that do not meet these requirements are named "special".

Power electronics universal application leads to voltage and current diagram distortion (Harlov, 2005; Borovikov *et al.*, 2013). In this regard, EMC issue is urgent as well as reliable assessment of energy processes in electrical network with non-sinusoidal operation. Sine, distortion possibility is being calculated based on the current sum of all harmonic components (or a number of components) matching the specified limit value as well as the possibility of current and voltage resonance and capacitor overload.

In special operation conditions of electrical power supply system, pulsating power has to be considered along with active, reactive and apparent power as it is a kind of measure function for unbalances and distortion degree that characterizes unsinusoidality of operation.

Transformer winding inductive reactance is much greater than the active resistance. Phasor power factor of transformer as a separate network element exceeds the standard; the number of transformers is higher than the number of generators. Considering the above, the study of transformer loss dependence on HV winding load is understandable. The study is important also in terms of non-sinusoidal loads; higher harmonic components in load-carrying winding increase inductive reactance which is equivalent to significant transformer losses.

Thus, the purpose of this study is to study unbalanced and non-sinusoidal operation conditions of power transformers modes with respect to real-power loss.

MATERIALS AND METHODS

Since, an electrical network represents the equivalent circuit of our transformer, we have used a chaining method to study operating conditions. General calculation objectives in the context of theoretical foundations of electrical engineering are the following (Bessonov, 2006; Zeveke *et al.*, 1975): the parameters of active and passive network elements are known to determine the current distribution in the network to draw balances of current in independent nodes and of voltage in independent circuits to draw a power balance in the whole chain.

The following challenge is favorable, according to the above. Passive parameters of a single-phase two-wound transformer (either one phase of 3-phase transformer) and its loads are known. The challenge is to define transformer total power losses under load based on equivalent circuit of the system "Two-Wound Transformer-Load" (TWTL) which elements are under High Voltage (HV). Let us consider two options for this challenge:

Obtain an expression for TWTL system's total power by multiplying the modulus of a square current of HV winding by its (TWTL system) total resistance. Find an expression for transformer total power losses $\dot{S}_t$ by matching the load power $\dot{S}_l$ against the total power. Identify transformer real power loss ΔP and reactive power loss ΔQ by determining the active and reactive components of this expression.

Determine the total power loss of all the equivalent circuit elements of TWTL system $\underline{Z}_1, \underline{Z}_0 \underline{Z}_2'$: total resistance of HV winding, shunt and Low-Voltage winding (LV). Then extract active and reactive power losses. The results on Options 1 and 2 must be the same:

$$I_1^2 \times \underline{Z}_Э - \left(I_2'\right)^2 \times \underline{Z}_H' = I_1^2 \times \underline{Z}_1 + I_0^2 \times \underline{Z}_0 + \left(I_2'\right)^2 \times \underline{Z}_2'$$

Where:

$\underline{Z}_Э$ = Total resistance of TWTL system which includes HV winding resistance and transformer's equivalent circuit under HV

$\underline{Z}_2', \underline{Z}_H'$ = Resistance of LV winding and load

$\underline{Z}_0$ = Resistance of a shunt

I_1, I_0 = Modulus of current in HV winding (1st Branch) and shunt (2nd Branch)

I_2' = Modulus of current in HV winding, transmitted from LV winding (3rd Branch)

RESULTS

Branch currents are calculated in one phase of equivalent circuit of TWTL system based on this equivalent circuit (magnetic coupling of HV and LV windings, replaced by their electrical equivalent and parameters of LV windings are under HV). This equivalent circuit is shown in (Fig. 1).

Figure 1 shows branch current directions and directions of Traversals 11 and 22. We have used branch current method for calculations.

The equivalent circuit (Fig. 1) has 2 nodes (Node 2 is a backbone node) and 3 Branches. According to Kirchhoff's 1st Law and specified current directions of the Node 1, we have:

$$I_1 - I_0 + I_2' = 0 \quad (1)$$

Equation 1 here is a equation of Magneto Motive Force (MMF). It is the fundamental equation in choosing mathematical models to study all operation conditions pf power transformers (Kostenko and Piotrovskiy, 1964). This is because MMF is a product of ampere-winding at 4π and the parameters of LV winding under HV. Thus, MMF equation can be reduced by $4\pi w$ where: w-number of HV winding turns.

In terms of independent Circuits 11 and 22 (directions of traversals is clockwise), Kirchhoff's 2nd Law determines the voltage balance Eq. 2:

$$I_1 \times \underline{Z}_1 + I_0 \times \underline{Z}_0 = \dot{U}_1 \quad (2)$$

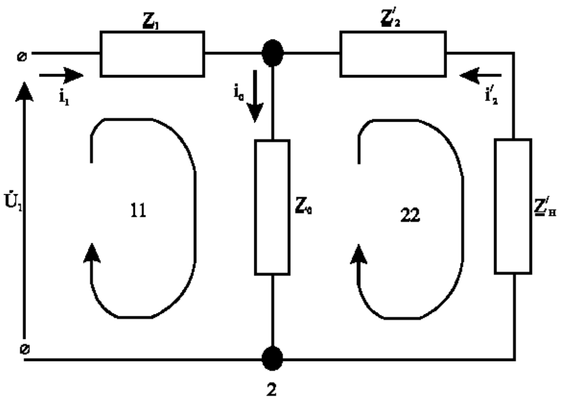

Fig. 1: Equivalent circuit for one phase of TWTL system

$$I_0 \times \underline{Z}_0 + I_2' \times \underline{Z}_{2H}' = 0 \quad (3)$$

where, $\dot{U}_1$ complex voltage across the terminals in double-wound transformer.

Equation 3 includes summarized resistance of LV windings under HV and load:

$$\underline{Z}_{2H}' = \underline{Z}_2' + \underline{Z}_H' = r_2' + r_H' + j\left(x_2' + x_H'\right)$$

and complex total resistance values for LV windings and for load:

$$\underline{Z}_2' = r_2' + jx_2' = k^2 \times r_2 + jk^2 \times x_2,$$

$$\underline{Z}_H' = r_H' + jx_H' = k^2 \times r_H + jk^2 \times x_H$$

where, k-transformation ratio, we have replaced the complete system of Eq. 1-3 for equivalent circuit (Fig. 1) with extended matrix:

$$\left(\begin{array}{ccc|c|c} 1 & -1 & 1 & \dot{I}_1 & 0 \\ \underline{Z}_1 & \underline{Z}_0 & 0 & \dot{I}_0 & \dot{U}_1 \\ 0 & \underline{Z}_0 & \underline{Z}_{2H}' & \dot{I}_2' & 0 \end{array}\right)$$

We have found the solution by Cramer's Rule. The main determinant of extended matrix:

$$\Delta = \begin{pmatrix} 1 & -1 & 1 \\ \underline{Z}_1 & \underline{Z}_0 & 0 \\ 0 & \underline{Z}_0 & \underline{Z}_{2H}' \end{pmatrix} = \underline{Z}_0 \underline{Z}_{2H}' + \underline{Z}_1\left(\underline{Z}_0 + \underline{Z}_{2H}'\right)$$

is not equal to zero and consequently, the system of equations has a single solution. We have calculated additional determinants:

$$\begin{pmatrix} 0 & -1 & 1 \\ \dot{U}_1 & \underline{Z}_0 & 0 \\ 0 & \underline{Z}_0 & \underline{Z}'_{2H} \end{pmatrix} = \Delta_1 = \dot{U}_1\left(\underline{Z}_0 + \underline{Z}'_{2H}\right)$$

$$\begin{pmatrix} 1 & 0 & 1 \\ \underline{Z}_1 & \dot{U}_1 & 0 \\ 0 & 0 & \underline{Z}'_{2H} \end{pmatrix} = \Delta_2 = \dot{U}_1 \underline{Z}'_{2H}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ \underline{Z}_1 & \underline{Z}_0 & \dot{U}_1 \\ 0 & \underline{Z}_0 & 0 \end{pmatrix} = \Delta_3 = -\dot{U}_1 \underline{Z}_0$$

Equation for branch current calculation will the following: 1st Branch:

$$\dot{I}_1 = \frac{\Delta_1}{\Delta} = \frac{\dot{U}_1\left(\underline{Z}_0 + \underline{Z}'_{2H}\right)}{\underline{Z}_0 \underline{Z}'_{2H} + \underline{Z}_1\left(\underline{Z}_0 + \underline{Z}'_{2H}\right)} = \frac{\dot{U}_1}{\underline{Z}_1 + \underline{Z}_0 \underline{Z}'_{2H} / \left(\underline{Z}_0 + \underline{Z}'_{2H}\right)}$$

Where:

$$\underline{Z}_1 + \underline{Z}_0 \underline{Z}'_{2H} / \left(\underline{Z}_0 + \underline{Z}'_{2H}\right) = \underline{Z}_Э -$$

Equivalent resistance of TWTL system; 2nd Branch:

$$\dot{I}_0 = \frac{\Delta_2}{\Delta} = \frac{\dot{U}_1 \underline{Z}'_{2H}}{\underline{Z}_0 \underline{Z}'_{2H} + \underline{Z}_1\left(\underline{Z}_0 + \underline{Z}'_{2H}\right)}$$

3rd Branch:

$$\dot{I}'_2 = \frac{\Delta_3}{\Delta} = \frac{-\dot{U}_1 \underline{Z}_0}{\underline{Z}_0 \underline{Z}'_{2H} + \underline{Z}_1\left(\underline{Z}_0 + \underline{Z}'_{2H}\right)}$$

In plugging branch current values in Eq. 1, we can see that the current balance in the node 1 is observed. To check our equations we have calculated the power balance for the entire network:

$$\dot{U}_1 \times I_1^* = I_1^2 \times \underline{Z}_Э$$

that can be traced to power balance:

$$\dot{U}_1 = \dot{I}_1 \times \underline{Z}_Э$$

Plugging the vector current of the 1st Branch in to last equality will get an identity $\dot{U}_1 = \dot{U}_1$. Where: I_1^* current, conjugated with the current $\dot{I}_1$.

We have checked the equations for branch currents using mesh current method. Direction of Traversal 11 coincides with direction of vector current $\dot{I}_{11}$ in the 1st Circuit; direction of Traversal 22 coincides with direction of current $\dot{I}_{22}$ in the 2nd circuit. In this case, equation for power balances in these circuits take the form:

$$\begin{cases} \dot{I}_{11}\left(\underline{Z}_1 + \underline{Z}_0\right) - \dot{I}_{22}\underline{Z}_0 = \dot{U}_1 \\ -\dot{I}_{11}\underline{Z}_0 - \dot{I}_{22}\left(\underline{Z}'_{2H} + \underline{Z}_0\right) = 0 \end{cases}$$

Their extended matrix:

$$\left(\begin{array}{cc|c|c} \underline{Z}_1 + \underline{Z}_0 & -\underline{Z}_0 & \dot{I}_{11} & \dot{U}_1 \\ -\underline{Z}_0 & \underline{Z}'_{2H} + \underline{Z}_0 & \dot{I}_{22} & 0 \end{array}\right)$$

Contains main and additional determinants:

$$\begin{pmatrix} \underline{Z}_1 + \underline{Z}_0 & -\underline{Z}_0 \\ -\underline{Z}_0 & \underline{Z}'_{2H} + \underline{Z}_0 \end{pmatrix} = \underline{Z}_1 \underline{Z}'_{2H}; + \underline{Z}_0 \underline{Z}'_{2H} + \underline{Z}_1 \underline{Z}_0$$

$$\begin{pmatrix} \dot{U}_1 & -\underline{Z}_0 \\ 0 & \underline{Z}'_{2H} + \underline{Z}_0 \end{pmatrix} = \dot{U}_1\left(\underline{Z}'_{2H} + \underline{Z}_0\right)$$

$$\begin{pmatrix} \underline{Z}_1 + \underline{Z}_0 & \dot{U}_1 \\ -\underline{Z}_0 & 0 \end{pmatrix} = \dot{U}_1 \underline{Z}_0$$

Since, the main determinant is nonzero, there is only one equation for mesh current:

$$\dot{I}_{11} = \frac{\dot{U}_1\left(\underline{Z}'_{2?} + \underline{Z}_0\right)}{\underline{Z}_1 \underline{Z}'_{2H} + \underline{Z}_0 \underline{Z}'_{2H} + \underline{Z}_1 \underline{Z}_0}$$

$$\dot{I}_{22} = \frac{\dot{U}_1 \underline{Z}_0}{\underline{Z}_1 \underline{Z}'_{2H} + \underline{Z}_0 \underline{Z}'_{2H} + \underline{Z}_1 \underline{Z}_0}$$

Their modules are equal to current modules of Branches 1 and 3. Where in:

$$\dot{I}_{11} = \dot{I}_1, \dot{I}_{22} = -\dot{I}'_2$$

Magnetizing branch (shunt) current value is equal to their difference:

$$\dot{I}_0 = \dot{I}_{11} - \dot{I}_{22} = \dot{I}_1 + \dot{I}'_2 = \frac{\dot{U}_1 \underline{Z}'_{2H}}{\underline{Z}_1 \underline{Z}'_{2H} + \underline{Z}_0 \underline{Z}'_{2H} + \underline{Z}_1 \underline{Z}_0}$$

Expressions for branch current calculations, found by branch current method and mesh current method are equal. In the future, we accept the following equations for branch currents:

$$\dot{I}_1 = \frac{\dot{U}_1\left(\underline{Z}_0 + \underline{Z}'_{2H}\right)}{\underline{Z}_0 \underline{Z}'_{2H} + \underline{Z}_1\left(\underline{Z}_0 + \underline{Z}'_{2H}\right)}$$
$$\dot{I}_0 = \frac{\dot{U}_1 \underline{Z}'_{2H}}{\underline{Z}_0 \underline{Z}'_{2H} + \underline{Z}_1\left(\underline{Z}_0 + \underline{Z}'_{2H}\right)} \quad (4)$$
$$\dot{I}_2 = \frac{-\dot{U}_1 \underline{Z}_0}{\underline{Z}_0 \underline{Z}'_{2H} + \underline{Z}_1\left(\underline{Z}_0 + \underline{Z}'_{2H}\right)}$$

We introduce additional notation for the equivalent complex total resistance of TWTL system:

$$\underline{Z}_э = \underline{Z}_1 + \underline{Z}_B \tag{5}$$

Where:

$$\underline{Z}_B = \underline{Z}_0 \underline{Z}'_{2H} / \left(\underline{Z}_0 + \underline{Z}'_{2H}\right) = \frac{a+jb}{e+jf} = \frac{ae+bf+j(be-af)}{e^2+f^2} \tag{6}$$

Is complex total resistance, inserted in HV winding circuit that determines the current distribution in transformer branches (equivalent shunt resistance and summarized resistance of LV winding under HV and load, connected in parallel).

$\underline{Z}_0 \underline{Z}'_{2H} = a+jb$ product of complex total shunt resistances and resistance for LV windings under HV and load. The right side of Eq. 6 and the last equation:

$$a = r_0 r'_2 + r_0 r'_H - x_0 x'_2 - x_0 x'_H$$
$$b = x_0 r'_2 + x_0 r'_H + r_0 x'_2 + r_0 x'_H$$

The sum of complex total shunt resistances and total resistance of LV winding under HV is represented in the following form:

$$\underline{Z}_0 + \underline{Z}'_{2H} = e+jf = \underline{Z}$$

Where:

$$e = r_0 + r'_2 + r'_H,\ f = x_0 + x'_2 + x'_H$$

The physical meaning of $\underline{Z}$ is a complex resistance that is equal to summarized resistance of circuit elements connected in series: shunt, LV winding under HV and load.

If, we divide the numerator and denominator in right side of Eq. 4 in the sum of resistance $(\underline{Z}_0 + \underline{Z}'_{2H})$ with due account for Eq. 5 and 6, we could make a certain conclusion. According to Ohm's law, the current in HV winding is proportional to the voltage across the terminals of the transformer and is inversely proportional to the equivalent resistance of the module:

$$\dot{I}_1 = \frac{\dot{U}_1}{\underline{Z}_э}$$

The product of the complex resistance of the 1st branch multiplied by summarized complex resistances of the shunt, LV winding and load:

$$\underline{Z}_1 \left(\underline{Z}_0 + \underline{Z}'_{2H}\right) = \underline{Z}_1 \underline{Z} = c+jd$$

Its square module is:

$$\left|\underline{Z}_1 \left(\underline{Z}_0 + \underline{Z}'_{2i}\right)\right|^2 = Z_1^2 Z^2 = c^2 + d^2$$

The last equation shows that:

$$Z_1^2 = \frac{c^2+d^2}{e^2+f^2}$$

Where:

$$c = r_0 r_1 + r_1 r'_2 + r_1 r'_H - x_0 x_1 - x_1 x'_2 - x_1 x'_H = r_1 \times e - x_1 \times f$$
$$d = r_0 x_1 + x_1 r'_2 + x_1 r'_H + x_0 r_1 + r_1 x'_2 + r_1 x'_H = r_1 \times f + x_1 \times e$$

Here:
e = r
f = x

The complex equivalent resistance of TWTL system based on accepted notations of Eq. 5 is:

$$\underline{Z}_e = \frac{c+a+j(b+d)}{e+jf} \tag{7}$$

Its square module is:

$$\underline{Z}_e^2 = \frac{(c+a)^2 + (b+d)^2}{e^2+f^2}$$

We have multiplied the numerator and denominator of the right side of Eq. 7 by the conjugate complex (e-jf) of the complex resistance $\underline{Z}$:

$$\underline{Z}_e = \frac{(c+a)\times e + (b+d)\times f + j\left[(b+d)\times e - (a+c)\times f\right]}{e^2+f^2} \tag{8}$$

Where:

$$\operatorname{Re}(\underline{Z}_e) = \frac{(c+a)\times e + (b+d)\times f}{e^2+f^2} = Z_1^2 \frac{(c+a)\times e + (b+d)\times f}{c^2+d^2}$$
$$\operatorname{Im}(\underline{Z}_e) = \frac{-(c+a)\times f + (b+d)\times e}{e^2+f^2} = Z_1^2 \frac{-(c+a)\times f + (b+d)\times e}{c^2+d^2}$$

Let it be: $\dot{U}_1 = U_1 = \text{const.}$ equation for branch currents Eq. 1-3 based on previous notations, we be the following:

$$\dot{I}_1 = \frac{U_1 \underline{Z}}{a+c+j(b+d)}$$
$$\dot{I}_2 = -\frac{U_1 \underline{Z}_0}{a+c+j(b+d)}$$
$$\dot{I}_0 = \frac{U_1 \underline{Z}'_{2H}}{a+c+j(b+d)}$$

In this case, square module are equal:

$$I_1^2 = \frac{U_1^2 \times Z^2}{(a+c)^2 + (b+d)^2} \tag{9}$$

$$\left(I_2'\right)^2 = \frac{U_1^2 \times Z_0^2}{(a+c)^2 + (b+d)^2} \tag{10}$$

$$I_0^2 = \frac{U_1^2 \times \left(Z_{2H}'\right)^2}{(a+c)^2 + (b+d)^2} \tag{11}$$

Transformer real-power loss in Case 1:

$$\Delta\dot{S}_{O1} = \frac{U_1^2}{Z_{y'}^2} \times \underline{Z}_{y'} - \left(\dot{I}_2'\right)^2 \underline{Z}_1' \tag{12}$$

According to Eq. 7, 8, 10 and 12 shall be:

$$\Delta\dot{S}_{t1} = \frac{U_1^2}{(a+c)^2 + (b+d)^2} \times \begin{bmatrix} (c+a)\times e + (b+d)\times f + \\ j\left[(b+d)\times e - (a+c)\times f\right] - Z_0^2 \underline{Z}_H' \end{bmatrix} \tag{13}$$

Transformer real-power loss in Case 1:

$$\Delta P_{t1} = \frac{U_1^2}{(a+c)^2 + (b+d)^2} \times \left[(c+a)\times e + (b+d)\times f - Z_0^2 r_H'\right] \tag{14}$$

Transformer reactive power loss in Case 1:

$$\Delta Q_{t1} = \frac{U_1^2}{(a+c)^2 + (b+d)^2} \times \left[(b+d)\times e - (a+c)\times f - Z_0^2 x_H'\right] \tag{15}$$

Transformer real-power loss in Case 2:

$$\Delta\dot{S}_{t2} = I_1^2 \times \underline{Z}_1 + I_0^2 \times \underline{Z}_0 + \left(I_2'\right)^2 \times \underline{Z}_2' \tag{16}$$

According to Eq. 9-11 and 16 shall be:

$$\Delta\dot{S}_{t2} = \frac{U_1^2}{(a+c)^2 + (b+d)^2} \times \begin{bmatrix} Z^2 r_1 + Z_0^2 \times r_2' + \left(Z_{2H}'\right)^2 r_0 + \\ j\left(Z^2 x_1 + Z_0^2 \times x_2' + \left(Z_{2H}'\right)^2 x_0\right) \end{bmatrix} \tag{17}$$

Transformer real-power loss in Case 2:

$$\Delta P_{T2} = \frac{U_1^2}{(a+c)^2 + (b+d)^2} \left[Z^2 r_1 + Z_0^2 \times r_2' + \left(Z_{2H}'\right)^2 r_0\right] \tag{18}$$

Transformer reactive power loss in Case 2:

$$\Delta Q_{T2} = \frac{U_1^2}{(a+c)^2 + (b+d)^2} \left[Z^2 x_1 + Z_0^2 \times x_2' + \left(Z_{2H}'\right)^2 x_0\right] \tag{19}$$

Right-hand sides of Eq. 13-15 and 17-19 are identical. We have reduced left and right-hand sides of paired equations by an algebraic expression:

$$\frac{U_1^2}{(a+c)^2 + (b+d)^2}$$

Thus, we will have new and less lengthy equations with identically equal left-hand and right-hand sides.

The certainty of these expressions can be confirmed by verifying identical equation:

$$(c+a)\times e + (b+d)\times f - Z_0^2 r_1' = Z^2 r_1 + Z_0^2 \times r_2' + \left(Z_{21}'\right)^2 r_0$$
$$(b+d)\times e - (a+c)\times f - Z_0^2 x_1' = Z^2 x_1 + Z_0^2 \times x_2' + \left(Z_{21}'\right)^2 x_0$$

Total power of TWTL system:

$$\dot{S} = I_1^2 \times \underline{Z}_e \tag{20}$$

where, square current module I_1^2 of Branch 1 is taken according to Eq. 9:

$$\underline{Z}_e = \underline{Z}_1 + \frac{a+jb}{e+jf} \tag{21}$$

In plugging expressions Eq. 9, 21 and 20, we get:

$$\dot{S} = U_1^2 \times \frac{r_1 Z^2 + ae + bf + j\left(r_1 Z^2 + be - af\right)}{(a+c)^2 + (b+d)^2} \tag{22}$$

Transformer total power loss is determined by taking the right-hand side of expression for real load power $\dot{S}_1$ from the right-hand side of expression Eq. 22 for real power of TWTL system:

$$\Delta\dot{S}_t = \dot{S} - \dot{S}_1$$

Total load power is:

$$\dot{S}_1 = \left(I_2'\right)^2 \times \underline{Z}_1'$$

where, $(I_2')^2$ square current module in "LV winding-load" system is determined by Eq. 10; $\underline{Z}_1' = r_1' + jx_1'$ total load resistance under HV. With due account for previous notations:

$$ae + bf = Z_0^2 \times r_{21}' + \left(Z_1\right)^2 r_0 \tag{23}$$

$$\text{be-af} = Z_0^2 \times x'_{21} + \left(Z'_{21}\right)^2 x_0 \quad (24)$$

Equation 13 with Eq. 22-24 in Case 1 will be:

$$\Delta\dot{S}_{tl} = \frac{U_1^2}{(a+c)^2+(b+d)^2} \times \left[\begin{array}{l} Z^2 r_1 + Z_0^2 \times r_2' + \left(Z'_{2?}\right)^2 r_0 + \\ j\left(Z^2 x_1 + Z_0^2 \times x_2' + \left(Z'_{2H}\right)^2 x_0\right) \end{array}\right] \quad (25)$$

Thus, the right-hand side of Eq. 25 is identically equal to the right-hand side of Eq. 17 which was to be proved.

The fractional expression in the right-hand side of Eq. 25 should be presented in such a way so that the load resistances were separated from equivalent circuit resistances under HV to assess the impact of load parameters on transformer losses.

In this case, the real part of the numerator $Z^2 r_1 + Z_0^2 \times r_2' + (Z'_{21})^2 r_0$ can be represented as the sum of two terms: term, independent from load resistance:

$$A_1 = r_1\left[Z_0^2 + Z_1^2 + 2\left(r_0 r_2' + x_0 x_2'\right)\right] + Z_0^2 \times r_2' + \left(Z_2'\right)^2 r_0 \quad (26)$$

term, dependent on load resistance:

$$B = \left(Z'_H\right)^2 \left(r_1 + r_0\right) + 2\left(r_0 r_2' + r_1 r_2' + r_0 r_1\right) \times r'_H + 2\left(x_0 x_2' + r_1 x_2' + x_0 r_1\right) \times x'_H \quad (27)$$

We have replaced the Eq. 27 by a trinomial:

$$B = B_1\left(Z'_H\right)^2 + B_2 r'_H + B_3 x'_H \quad (28)$$

with load resistance coefficients:

$$B_1 = \left(r_1 + r_0\right) \quad (29)$$

$$B_2 = 2\left(r_0 r_2' + r_1 r_2' + r_0 r_1\right) \quad (30)$$

$$B_3 = 2\left(r_0 x_2' + r_1 x_2' + x_0 r_1\right) \quad (31)$$

Equation 27 corresponds to the situation of a transformer load being active-inductive. In the case of active-capacitive load, the last term in the right-hand side of Eq. 27 changes in sign. Equation for active load Eq. 27 with notations Eq. 29-31 will be:

$$B = B_1 (r_1')^2 + B_2 r_1'$$

In connecting capacitors to LV terminals:

$$B = B_1 (x_1')^2 + B_3 x_1'$$

According to Eq. 25, B = 0, if capacitor resistance under HV is:

$$x_i' = \frac{2(r_0 x_2' + r_1 x_2' + x_0 r_1)}{(r_1 + r_0)}$$

If equivalent circuit elements (their parameters) are used under LV, the capacitors resistance will be:

$$x_i = \frac{2(r_0 x_2 + r_1 x_2 + x_0 r_1 / k^2)}{(r_1 + r_0)}$$

Now, lengthy expression for the real part of the numerator in Eq. 25 with due account for Eq. 26 and 29-31 is shorter:

$$A_1 + B_1 (Z_1')^2 + B_2 r_1' + B_3 x_1'$$

The imaginary part $Z^2 x_1 + Z_0^2 \times x_2' + (Z'_{2H})^2 x_0$ as well as the real part of the numerator of fractional expression in right-hand side of Eq. 25 can also be represented as the sum of two terms: term, independent from load resistance:

$$A_2 = x_1\left[Z_0^2 + Z_1^2 + 2\left(r_0 r_2' + x_0 x_2'\right)\right] + Z_0^2 \times r_2' + \left(Z_2'\right)^2 x_0 \quad (32)$$

Term, dependent on load resistance:

$$D = \left(Z_1'\right)^2 \left(x_1 + x_0\right) + 2\left(x_0 r_2' + x_1 r_2' + r_0 x_1\right) \times r_1' + 2\left(x_0 x_2' + x_1 x_2' + x_0 x_1\right) \times x_1' \quad (33)$$

We have replaced Eq. 33 by a trinomial:

$$D = D_1\left(Z_1'\right)^2 + D_2 r_1' + D_3 x_1' \quad (34)$$

with load resistance coefficients:

$$D_1 = \left(x_1 + x_0\right) \quad (35)$$

$$D_2 = 2\left(x_0 r_2' + x_1 r_2' + r_0 x_1\right) \quad (36)$$

$$D_3 = 2\left(x_0 x_2' + x_1 x_2' + x_0 x_1\right) \quad (37)$$

The denominator of the fraction in right-hand side of Eq. 25 is:

$$(a+c)^2+(b+d)^2 = Z_1^2Z^2+Z_0^2\times(Z_{21}^{/})^2+2\times(r_0r_1+x_0x_1)\times(rr_{21}^{/}+xx_{21}^{/})+2\times(x_0r_1-r_0x_1)\times(xr_{21}^{/}+rx_{21}^{/})$$

Where: $r_0r_1+x_0x_1) = [\underline{Z}_0\times\underline{Z}_1]$ scalar product of total resistance in the 1st and 2nd Branches of equivalent circuit of TWTL system.

$(rr_{21}^{/}+xx_{21}^{/}) = [\underline{Z}\times\underline{Z}_{21}^{/}]$ scalar product of total resistance and z total resistance in the 3rd Branch of equivalent circuit of TWTL system under HV.

$(x_0r_1-r_0x_1) = \|[\underline{Z}_0\times\underline{Z}_1]\|$ module value of vector product of total resistance in the 1st and 2nd Branches of equivalent circuit of TWTL system under HV.

$xr_{21}^{/}+rx_{21}^{/}) = \|[\underline{Z}\times\underline{Z}_{21}^{/}]\|$ module value of vector product of total resistance Z and total resistance in the 3rd Branch of equivalent circuit of TWTL system under HV. The physical meaning of total resistance $\underline{Z}$ have been introduced earlier.

The physical meaning of Eq. 25 is as follows. The denominator of the fractional expression in the right-hand side of the equation for transformer real-power losses is the sum of 4 terms.

Product of square module of total resistance in Branch 1 (Fig. 1) multiplied by square module of total resistance in Branche 2 and 3, if they are connected in series.

Product of square module of total resistance in Branch 2 (Fig. 1) multiplied by square module of total resistance in Branch 3.

Twice the product of the scalar product value of total resistance in the 1st and 2nd Branches of equivalent circuit of TWTL system, multiplied by scalar product of total resistance $\underline{Z}$ and total resistance in the 2nd Branch of equivalent circuit of TWTL system.

Twice the product of the module value of vector product of total resistance in the 1st and 2nd Branches of equivalent circuit of TWTL system, multiplied by the module value of vector product of total resistance $\underline{Z}$ and total resistance in the 3rd Branch of equivalent circuit of TWTL system.

Load resistance and other types of resistance were divided in the common denominator in right-hand side of Eq. 25. In this case, algebraic expression for denominator can be represented as the sum of 2 terms: term without load resistance:

$$A_3 = Z_1^2r_0^2+Z_1^2\times\left[Z_0^2+(Z_2^{/})^2\right]+Z_0^2(Z_2^{/})^2+2\times Z_1^2\times(r_0r_2^{/}+x_0x_2^{/})+2\times Z_0^2\times(r_1r_2^{/}+x_1x_2^{/})+2\times(Z_2^{/})^2\times(r_0r_1+x_0x_1) \quad (38)$$

term with load resistance:

$$C = C_1(Z_1^{/})^2+C_2r_1^{/}+C_3x_1^{/} \quad (39)$$

with load resistance coefficients:

$$C_1 = Z_0^2+Z_1^2+2\times(r_0r_1+x_0x_1) \quad (40)$$

$$C_2 = 2\times\left[Z_1^2\times(r_0+r_2^{/})+Z_0^2(x_1+x_2^{/})+2\times(r_0r_1r_2^{/}+x_0x_1r_2^{/})\right] \quad (41)$$

$$C_3 = 2\times\left[Z_1^2\times(x_0+x_2^{/})+Z_0^2(x_1+x_2^{/})+2\times(r_0r_1x_2^{/}+x_0x_1x_2^{/})\right] \quad (42)$$

Now, lengthy expression for common denominator in right-hand side of Eq. 25 with due account for Eq. 38-42 is shorter:

$$C = A_3+C_1(Z_1^{/})^2+C_2r_1^{/}+C_3x_1^{/}$$

According to Eq. 25, 26, 28 and 33-36, the expression for transformer real-power loss shall be:

$$\Delta\dot{S}_{t1} = U_1^2\times\frac{A_1+B_1(Z_1^{/})^2+B_2r_1^{/}+B_3x_1^{/}+j\left[A_2+D_1(Z_1^{/})^2+D_2r_1^{/}+D_3x_1^{/}\right]}{A_3+C_1(Z_1)^2+C_2r_1^{/}+C_3x_1^{/}} \quad (43)$$

Its application with transformer constants (A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3, D_1, D_2, D_3) makes it possible to conduct an in-depth analysis of any operation condition depending on the load resistance. Putting the load resistance equal to zero, we have a Short-Circuit condition (SC). In this case, transformer real-power loss is:

$$\Delta\dot{S}_{t1} = U_1^2\times\frac{A_1+jA_2}{A_3}$$

In connecting active load to LV terminals:

$$\Delta\dot{S}_{t1} = U_1^2\times\frac{A_1+B_1(r_1^{/})^2+B_2r_1^{/}+j\left[A_2+D_1(r_1^{/})^2+D_2r_1^{/}\right]}{A_3+C_1(r_1^{/})^2+C_2r_1^{/}}$$

In connecting inductive load to LV terminals:

$$\Delta\dot{S}_{t1} = U_1^2\times\frac{A_1+B_1(x_1^{/})^2+B_3x_1^{/}+j\left[A_2+D_1(x_1^{/})^2+D_3x_1^{/}\right]}{A_3+C_1(x_1^{/})^2+C_3x_1^{/}}$$

In connecting capacity load to LV terminals, signs "plus" of the last real and imaginary numerator terms and of the last denominator term should be changed to negative in the right-hand side of the last equation.

DISCUSSION

In determining additional transformer losses under its non-sinusoidal loads, reactive resistance included in all expressions for transformer constants shall be multiplied by a whole number of harmonic components (for characteristic harmonics) or by a fractional number (for inter-harmonics and sub-harmonics) (Kostinskii and Troitskii, 2015).

Fractional harmonic number is obtained by harmonic frequency divided by 50. For example, sub-harmonics of 20 Hz correspond to n = 2/5, inter-harmonics of 75 Hz to n = 1.5.

The expression for additional transformer losses under harmonic components, previously used to calculate its constants and notions in the equivalent circuit of TWTL system will be the following:

$$A_{1n} = r_1\left(ß+\left(r_2^{/}\right)^2+a\right)+ß\times r_2^{/}+\gamma r_0 \quad (44)$$

$$B_{1n} = B_1 = \left(r_1+r_0\right) \quad (45)$$

$$B_{2n} = B_2 = 2\left(B_1 r_2^{/}+r_0 r_1\right) \quad (46)$$

$$B_{3n} = 2\left(n^2\times r_0 x_2^{/}+n\times r_1 x_2^{/}+n\times x_0 r_1\right) \quad (47)$$

$$A_{2n} = n\times x_1\left(ß+\left(r_2^{/}\right)^2+a\right)+ß\times r_2^{/}+\gamma r_0 \quad (48)$$

$$D_{1n} = n\times D_1 = n\times\left(x_1+x_0\right) \quad (49)$$

$$D_{2n} = 2n\times\left(x_0 r_2^{/}+x_1 r_2^{/}+r_0 x_1\right) \quad (50)$$

$$D_{3n} = 2n^2\times\left(x_0 x_2^{/}+x_1 x_2^{/}+x_0 x_1\right) \quad (51)$$

$$\begin{aligned}A_{3n} = {} & r_0^2\delta+\delta\times\left(ß+\gamma\right)+ß\gamma+a\delta+ \\ & 2ß\times\left(r_1 r_2^{/}+n^2\times x_1 x_2^{/}\right)+\gamma d\end{aligned} \quad (52)$$

$$C_{1n} = ß+Z_1^2+\theta \quad (53)$$

$$C_{2n} = 2\times\left[\delta\left(r_0+r_2^{/}\right)+ß\varepsilon+\theta r_2^{/}\right] \quad (54)$$

$$C_{3n} = 2\times\left(n\times\delta\varepsilon+n\times ß\varepsilon+\theta x_2^{/}\right) \quad (55)$$

Certain algebraic expressions repeated in Eq. 44-55. A number of notations extended them in order to make the programming process related to power loss calculations easier:

$$\alpha = 2\left(r_0 r_2^{/}+n^2\times x_0 x_2^{/}\right) \quad (56)$$

$$ß = \left(r_0^2+n^2\times x_0^2\right) \quad (57)$$

$$\gamma = \left(r_2^{/}\right)^2+n^2\times\left(x_2^{/}\right)^2 \quad (58)$$

$$\delta = \left(r_1^2+n^2\times x_1^2\right) \quad (57)$$

$$\theta = 2\times\left(r_1 r_0+n^2\times x_1 x_0\right) \quad (60)$$

$$\varepsilon = n\times\left(x_1+x_2^{/}\right) \quad (61)$$

If, we accept the restrictions $r_2^{/} = r_1$ and $x_2^{/} = x_1$ that are inherent in all sorts of transformers, than $\alpha = \theta$, $\gamma = \delta$, $\varepsilon = 2x_1 n$, respectively equations for transformer constants should be changed.

The following algorithm is to calculate the total power loss in double-wound transformer under non-sinusoidal load.

Constant transformer coefficients A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3, D_1, D_2, D_3 under sinusoidal load are calculated based on active and reactive resistance of transformer windings and load by Eq. 26, 29-32, 35-38 and 40-42.

Transformer real-power loss under non-sinusoidal load without taking into account additional losses is calculated based on numerical values of these coefficients and load resistance of the 1st harmonic component.

Transformer parameters α, β, γ, δ, ε, θ in terms of characteristic harmonics, sub-harmonics or inter-harmonics v are calculated according to Eq. 56-61.

Transformer coefficients A_{1v}, A_{2v}, A_{3v}, B_{1v}, B_{2v}, B_{3v}, C_{1v}, C_{2v}, C_{3v}, D_{1v}, D_{2v}, D_{3v} in terms of v-type harmonic component are calculated by plugging these numeric values in Eq. 44-55.

Computation on Points 3 and 4 is repeated for all harmonic components in the load. The loss of each harmonic component is calculated by Eq. 43 based on power quality measurements, current harmonics and calculation data on Point 4.

Additional power losses under non-sinusoidal load are determined by summing the power loss of each harmonic component.

Total power losses in single-phase transformer under non-sinusoidal load (in one phase of 3-phase transformer) are found by summing final calculations on Points 2 and 7.

In terms of three-phase transformers, calculation data on Point 8 shall be tripled. In this case, we have to use the method of equivalent sinusoids along with the method of equivalent resistance, assuming that the sum of square harmonic current is equal to square equivalent sinusoidal current in LV windings of a transformer.

CONCLUSION

The algorithm for total loss calculations in double-wound power transformer under non-sinusoidal load is based on constant coefficients A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3, D_1, D_2, D_3, calculated by formulas presented in study based on active and reactive resistance of transformer windings and load. Transformer real-power loss under non-sinusoidal load without taking into account additional losses is calculated based on numerical values of these coefficients and load resistance of the 1st harmonic component. Transformer parameters α, β, γ, δ, ε, θ in terms of characteristic harmonics, sub-harmonics or inter-harmonics ν are calculated according to formulas, provided in the study. These numerical values are used to define the coefficients $A_{1\nu}$, $A_{2\nu}$, $A_{3\nu}$, $B_{1\nu}$, $B_{2\nu}$, $B_{3\nu}$, $C_{1\nu}$, $C_{2\nu}$, $C_{3\nu}$, $D_{1\nu}$, $D_{2\nu}$, $D_{3\nu}$in terms ofν-type harmonic component. Such a calculation is repeated for all harmonic components in the load. The loss of each harmonic component is calculated based on power quality measurements, current harmonics and calculation data. Additional power losses under non-sinusoidal load are determined by summing the power loss of each harmonic component. Peal-power losses in single-phase transformer under non-sinusoidal load (in one phase of three-phase transformer) are found by summing final calculations.

REFERENCES

Agunov, A.V., 2000. Energy processes in electrical networks with non-sinusoidal operation and their effectiveness. Ph.D Thesis, Togliatti State University, Tolyatti, Russia.

Agunov, M.V., 2004. Methodology and principles of electric energy quality control systems in ships electric systems with non-linear elements. Ph.D Thesis, State Marine Technical University, Saint Petersburg, Russia.

Barutskov, I.B., S.A. Vdovenko and E.V. Tsygankov, 2011. Frequency converter harmonic distortions. Russ. J. Chief Power Eng., 6: 5-15.

Bessonov, L.A., 2006. Theoretical Foundations of Electrical Engineering, Electrical Systems: Textbook. 11th Edn., Gardariki Publishing House, Moscow, Russia, Pages: 701.

Blaabjerg, F., H. Wang, P. Davari, X. Qu and F. Zare, 2015. Energy saving and efficient energy use by power electronic systems. Proceedings of the Conference on World Engineering and Convention (WECC), November 29-December 03, 2015, Japan Federation of Engineering Societies Publisher, Kyoto, Japan, pp: 1-6.

Borovikov, V.S., N.N. Kharlov and T.B. Akimzhanov, 2013. On the necessity of including stray-load current harmonics losses into process losses at electric energy transmission. Bull. Tomsk Polytech. Univ., 4: 91-93.

Dryden, I.G.C., 2013. The Efficient use of Energy. 2nd Edn., Elsevier, Amsterdam, Netherlands, ISBN:9781483101576, Pages: 616.

Exposito, A.G., A.G. Exposito, A.J. Conejo and C. Canizares, 2016. Electric Energy Systems: Analysis and Operation. CRC Press, Boca Raton, Florida, ISBN:978-0-8493-7365-7, Pages: 649.

Gonen, T., 2014. Electrical Power Transmission System Engineering: Analysis and Design. CRC Press, Boca Raton, Florida, ISBN:978-1-4822-0700-2, Pages:1053.

Gonen, T., 2016. Electric Power Distribution Engineering. CRC Press, Boca Raton, Florida, ISBN:9781498785846, Pages: 1061.

Hall, E.J. and S.J. Tyler, 2014. Electric power system control with measurement of energy demand and energy efficiency using T-distributions. Dominion Resources Inc, Richmond, Virginia.

Harlov, N.N., 2005. Energy spectrum of voltage and current in load centers. Bull. Tomsk Polytech. Univ., 7: 75-79.

Idelchik, V.I., 1989. Electrical Systems and Networks: Textbook for Universities. Energoatomizdat Publisher, Moscow, Russian, Pages: 592.

Kostenko, M.P. and L.M. Piotrovskiy, 1964. Electric Machinery: Book 1 DC Machines. 2nd Edn., Energiya Publisher, Moscow, Russia, Pages: 548.

Kostinskii, S.S. and A.I. Troitskii, 2015. Method definition of stray load losses of active power in transformers of the branch circuits caused by non-linear loadings. Sci. Tech. J. Russ. Electromechanics, 3: 61-67.

Kreith, F. and D.Y. Goswami, 2016. Energy Management and Conservation Handbook. CRC Press, Boca Raton, Florida, USA., ISBN:9781315356617, Pages: 437.

Masters, G.M., 2004. Renewable and Efficient Electric Power Systems. Wiley-IEEE Press, USA., ISBN-13: 9780471280606, pp: 680.

Melnikov, N.A., 1975. Reactive Power in Electric Networks. Energiya Publisher, Moscow, Russia, Pages: 128.

Nikitin, V.P., 1958. Foundations of Arc-Welding Transformer and Generator Theory: Monograph. Nauka Publisher, Moscow, Russia, Pages: 238.

Uraev, N.N., G.F. Mingaleev, A.T. Kushimov and N.A. Kolesov, 2016. Methodological aspects of strategic development of regional socio-economic system (following the example of radio-electronic industry enterprises in the Republic of Tatarstan). Intl. J. Environ. Sci. Educ., 11: 5094-5108.

Zeveke, G.V., P.A. Ionkin, A.V. Netusil and S.V. Strahov, 1975. Fundamentals of Circuit Theory: Textbook for University. 4th Edn., Energiya Publisher, Moscow, Russia, Pages: 752.