Journal of Engineering and Applied Sciences 12 (19): 4865-4869, 2017
ISSN: 1816-949X

Economic-Mathematical Modeling of the Total Costs of Innovative Chemical Enterprise Methods of Fuzzy Set Theory

I.L. Beilin
Institute of Management, Economics and Finance, Kazan Federal University, 420008 Kazan, Russia Federation

Abstract: Three approaches is suggested to determining the total cost of innovative chemical enterprise in the face of uncertainty. This approach is based on the discrete fuzzy numbers gives more accurate results than approaches based on continuous fuzzy numbers solutions through a-sections and the number of L-R-type. Uncertainty intervals for all three solutions are the same.

Key words: Economic-Mathematical modeling, fuzzy sets, total costs, fixed costs, variable costs, continuous and discrete fuzzy numbers, a-section

INTRODUCTION

The development of fuzzy set theory, created as extension of traditional set theory, caused by the need to simulate such phenomena and concepts in which the central role is given to the person making the decision.

It is known that in a particular economic context, the risk premium can be determined on the basis of expert estimations. For example, the expert can make the following statements: to a variety of projects with average risk belong those whose risk premiums are of <8% and projects with a risk premium of more than 16% can be attributed to high-risk set. In the interim from 8-16%, the project with varying degrees of certainty, may be assigned by expert to the set with average risk and to high-risk set. But such a situation is an irreconcilable contradiction to the classical perception of the concept of “multitude” according to which it is required to give a definite answer about belonging to a plurality of elements either “yes” or “no”.

Thus, the blurring of the boundaries of concepts of human language has caused the need to explore new Mathematical object “fuzzy set”. From the foregoing, the elements of fuzzy sets have some common property in varying degrees (Castro *et al*., 2007; Zhou *et al*., 2007; Bojadziev, 1997). Therefore, a fuzzy set characterized by this situation: it’s necessary to specify the elements that belong to the set precisely to identify the elements which do not clearly belong to it to identify the elements that are included in this set with varying degrees of affiliation.

Yet until recently, the use of fuzzy formalism in solving economic problems was seen by economists as a traditional newfangled extravagance. But today we can talk about the turn in the evaluation of the scientific situation (Beilin, 2016; Buckley, 1992, 1987).

It can be imagined with what kind of difficulty found a place in the economic analysis for subjective probabilities, founded by de Finetti, Savage, Kayburg and others. And there is no doubt that the first attempts to introduce a subjective probability to an economic usage have come under fire of criticism for failing and weak Mathematical validity. In the same way ignoram uses of science tried to mix with the mud at the time the research of Lotfi Zadeh. And only the practice of using subjective probabilities in the military and energy applications and the practice of constructing fuzzy controllers (Mamdani, Sugeno, etc.) and other studies (Couturier and Fioleau, 2000; Dimitras *et al*., 1999; Buckley, 1992, 1987; Dimitras *et al*., 1996) with an equally impressive output forced silence on evil tongues.

Thus, to completely remove all doubts about the effectiveness and correctness of the usage of fuzzy economic models need to deploy a full-scale program of “fuzzy” research aimed at achieving by economy of a qualitatively new level.

MATERIALS AND METHODS

Fuzzy number is called normalized and convex fuzzy set A, defined on the set of real numbers for the membership function of which $\mu_A(x)$:

- $\max_{x \in R} \mu_A(x) = 1$, fuzzy number is normalized
- $\mu_A(\lambda x_1+(1-\lambda)x_2) \geq \min(\mu_A(x_1); \mu_A(x_2))$, the number is convex

Example 1: We consider the linguistic variable β = “ profit ability ratio”. We choose as one of its fuzzy variable values A = “high ratio”.

Basic variable for this fuzzy number given by the ratio x = profit/gross proceeds. Obviously in this case, we have

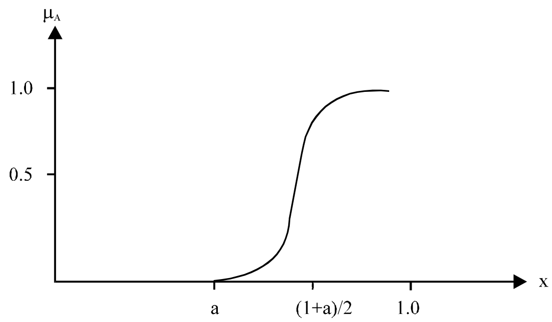

Fig. 1: High coefficient of profitability

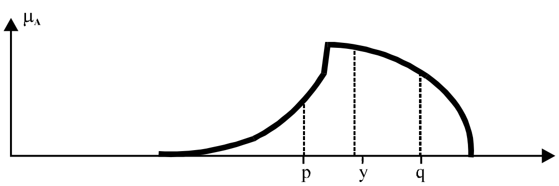

Fig. 2: Membership function of fuzzy number

a segment U = [0; 1], x∈U. Membership function can be represented graphically (Fig. 1) or by the equation:

$$\mu_A(x) = \begin{cases} 0,\ 0 \le x \le a \\ 2\left(\dfrac{x-a}{1-a}\right)^2,\ a \le x \le \dfrac{1+a}{2} \\ 1-2\left(\dfrac{1-x}{1-a}\right)^2,\ \dfrac{1+a}{2} \le x \le 1 \end{cases}$$

Fuzzy number A are convex, if its α sections are segments, i.e., for every $p \le y \le q$ performed inequality $\mu_A(y) \ge \mu_A(p) \wedge \mu_A(q)$. Membership function of convex fuzzy number is shown in Fig. 2. It should be noted that if numbers A and B are convex, their intersection A∩B is convex too.

Example 2: Below are some of the most frequently used membership functions of fuzzy numbers. Membership function of continuous triangular fuzzy number A (when S_A is a segment) shown in Fig. 3 (right) is equal to $\mu_A(x) = \max(0, 1-|x-a|/e)$ a the average value of fuzzy number, e the magnitude of variation with respect to a.

Such symmetrical triangular fuzzy numbers A is convenient to denote A = (a; e, e) = (a; e). If the values of average dispersion values a are different then the following notation is introduced as A = (a; e_1, e_2).

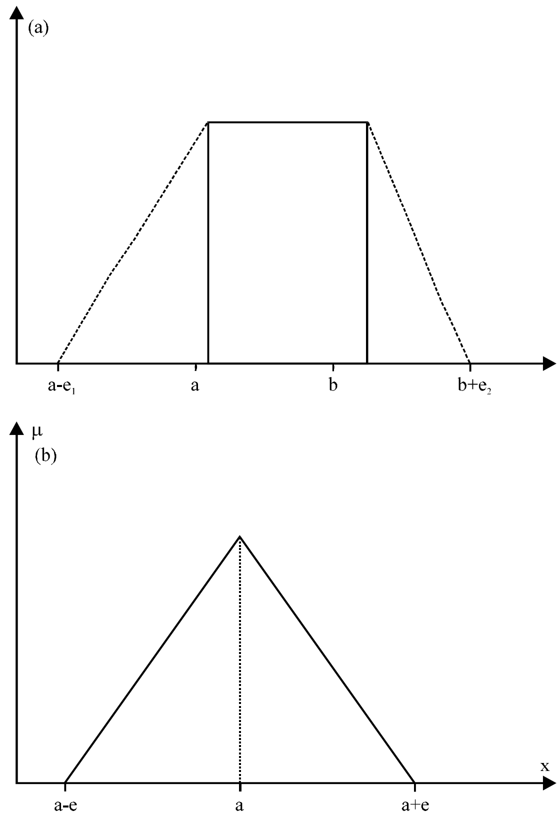

Fig. 3: Membership functions

Similarly, in Fig. 3a designated trapezoidal fuzzy numbers A = (a, b; e_1, e_2). A fuzzy number A is called: a fuzzy zero if $\mu_A(0) = \max_{x \in R} \mu_A(x)$; positive number if $\mu_A(x) = 0$ for x<0; negative number if $\mu_A(x) = 0$ for x>0; $x \in S_A$.

For any operations ∘ on fuzzy numbers, set discretely in accordance with the principle of generalization, we have the general equation:

$$C = A \circ B \Leftrightarrow \mu_C(z) = \max_{\substack{x,\,y \\ z = x \circ y}} \left(\mu_A(x) \wedge \mu_B(y)\right)$$

It follows a special case:

$$C = \min(A;B) \Leftrightarrow \mu_C(z) = \max_{z=\min(x;y)} \left(\mu_A(x) \wedge \mu_B(y)\right)$$

And in the case where the continuous fuzzy numbers are convex you can build the binary operation using a sections of fuzzy sets. To demonstrate this approach to the operation of addition of two triangular fuzzy numbers:

$$\mu_A(x) = \max\left(0; 1 - \frac{|x-a|}{d}\right);\ \mu_A(x) = \max\left(0; 1 - \frac{|x-b|}{c}\right)$$

It is established that:

$$\mu_{A+B}(x) = \max\left(0; 1 - \frac{|x-(a+b)|}{(d+c)}\right)$$

It is clear that a-section of convex fuzzy number is a segment and is determined from the Eq. $\mu_A(x) = \alpha$, i.e., if $x \in S_A$, it is sufficient to solve Eq. $1-|x-a|/d = \alpha$ and thereby define the boundary of the segment $A_\alpha = [a-(1-\alpha)d; a+(1+\alpha)d]$. In the same way can be found α-sections of fuzzy number B:

$$B_\alpha = \left[b-(1-\alpha)c; b+(1-\alpha)c\right]$$

and fuzzy number A+B:

$$(A+B)_\alpha = \begin{bmatrix}(a+b)-(1-\alpha)(d+c);\\ (a+b)+(1-\alpha)(d+c)\end{bmatrix}$$

It is obvious that α-sections of numbers A_α, B_α, $(A+B)_\alpha$ are segments for which the equality $A_\alpha + B_\alpha = (A+B)_\alpha$ is set. This equation combined with a formula of decomposition of fuzzy number $(A+B) = \bigcup_\alpha \alpha(A+B)_\alpha$, allows you to define the desired expression for $\mu_{A+B}(x)$. Established that operations of addition and multiplication:

- Commutative: A+B = B+A, A×B = B×A
- Associative: (A+B)+C = A+(B+C), (A×B)×C =A×(B×C)
- Generally not distributive: A×(B+C)⊇A×B+A×C

However, if the fuzzy numbers A,B, C only positive or only negative, the distributive property holds. Additionally, fuzzy numbers have opposite and inverse numbers $A+(-A) \neq \tilde{0}$ $A \times (1/A) \neq \tilde{1}$. Here, the symbols $\tilde{0}$ and $\tilde{1}$ are marked fuzzy zero and fuzzy one. This fact makes it impossible to use the elimination method for solving equations in which fuzzy numbers exist.

Operation on fuzzy numbers may lead to the loss of properties of convexity and then the result can not be a fuzzy number (Zopounidis *et al*., 2001; Kaufmann and Gupta, 1991; Dourra and Siy, 2002; Zadeh *et al*., 1996; Zadeh, 1978, 1973; Dubois and Prade, 1979; Buckley, 1992, 1987). This problem is recoverable if the fuzzy numbers are presented by continuous membership functions.

Further considered single (unary) operations f on fuzzy sets A, defined by principle of generalization.

If f: R→R, A⊂R, y = f (x) then fuzzy set B = f (A) has membership function $\mu_B(y) = \max_{x, y=f(x)} \mu_A(x)$. Here are some examples of unary operations:

- Changing the sign of a fuzzy number A, $\mu_{(-A)}(x) = \mu_A(-x)$

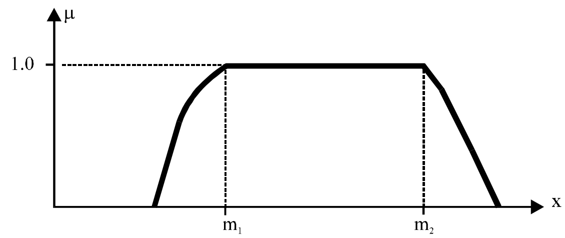

Fig. 4: Fuzzy interval

- Turning of fuzzy number $\mu_A^{-1}(x) = \mu_A(x^{-1})$ is possible if it is negative or positive. Otherwise A^{-1} is not convex
- Absolute value $\mu_{|A|}(x) = \begin{cases}\max(\mu_A(x), \mu_A(-x)), \text{если } x > 0\\ 0, \text{если } x < 0\end{cases}$
- Exponent $\mu_{\exp(A)}(y) = \max_{x:y=\exp(x)} \mu_A(x) = \mu_A(\ln y), B = \exp(A)$
- Multiplied by the number $\mu_{\eta \times A}(y) = \max_{x:y=\eta \times x} \mu_A(x) = \mu_A(y/\eta)\ \eta > 0$

Fuzzy number L-R-type is defined according to certain rules in order to reduce the amount of computation during the operations on them. The memebership functions of fuzzy numbers L-R-type defined by the functions L (x) and R (x) (left and right branches of the fuzzy number). These functions do not increase on [0; ∞] to meet properties:

$$L(x) = L(-x)R(x) = R(-x)R(-x),$$
$$(o) = R(0) = 1; L(\infty) = R(\infty) = 0$$

Example 5: As examples of L (x) and R (x) functions:

$$L(x) = R(x) = \max\left(0; 1 - \frac{|x-m|}{\alpha}\right), L(x) = R(x) = \frac{1}{1+x^2}$$
$$L(x) = R(x) = \max\left(0; 1 - \frac{(x-m)^2}{\alpha}\right); L(x) = R(x) = e^{-|x|^p}, p>0$$

In general, the fuzzy numbers L-R-type are determined by the membership function:

$$\mu_A(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), x \le m, \alpha > 0\\ R\left(\frac{x-m}{\beta}\right), x \ge m, \beta > 0\end{cases}$$

L-R fuzzy number is determined by three parameters and abbreviated written as $A = m_A; \alpha_A, \beta_A)_{L\text{-}R}$. If a fuzzy number is equal to 1, not only at the point x = m but on points of segment $[m_1, m_2]$ we get the fuzzy number used for the simulation of a fuzzy interval (Fig. 4):

$$\mu_A(x)=\begin{cases} L\left(\frac{m_1-x}{\alpha}\right), & x\le m_1, \alpha>0 \\ 1, & m_1\le x\le m_2 \\ R\left(\frac{x-m_2}{\beta}\right), & x\ge m_2, \beta>0 \end{cases}$$

In general, when comparing two fuzzy numbers A and B two situations can occur. The first one when the intersection of the medias of fuzzy numbers is blank, i.e., $S_A \cap S_B = \varnothing$. Obviously as most of them acts the number of which is located to the right on the abscissa, i.e., A<B.

The second one when the intersection of the medias of fuzzy numbers is not blank, i.e., $S_A \cap S_B \neq \varnothing$. In this case, the procedure comparing fuzzy numbers A and B is not as simple as in the first case.

It becomes necessary to calculate the index of ranking I(A, B) = F(A)-F(B), F-called fuzzy number ordering function.

There is a sufficiently large number of ranking indices (Peray, 1999; Dubois and Prade, 1980; Bart, 1993, 1991; Maria and Kandel, 1984; Zimmerman, 2001; Trippi and Lee, 1995). We consider one of them, it is constructed as follows:

$$I(A,B)=\int_0^{\alpha_{max}} M(A_\alpha)d\alpha-\int_0^{\alpha_{max}} M(B_\alpha)d\alpha$$

α_{max} the maximum value of membership function, A_α, B_α-α sections of fuzzy numbers A and B:

$$M(A_\alpha)=\frac{a+b}{2}; a(\alpha)=\inf_{x\in A_\alpha}(x); b(\alpha)=\sup_{x\in A_\alpha}(x)$$

If I (A, B)≥0, then A≥B.

RESULTS AND DISCUSSION

We define Total Costs (TC) of innovative chemical enterprise "Copolymer +" for the production of copolymer product, based on the cyclic carbonate compounds and isocyanate containing compounds (Beilin *et al*., 2006; Beilin and Arkhireev, 2009, 2005, 2011a, b) if Fixed Costs (FC) are predicted to be:

$$FC=\left\{\frac{0}{16},\frac{0.8}{17},\frac{1}{18},\frac{1}{19},\frac{0.7}{20},\frac{0.3}{21},\frac{0}{22}\right\}\text{mln. rub, a Variable Costs (VC)}$$

$$VC=\left\{\frac{0}{8},\frac{0.4}{9},\frac{0.7}{10},\frac{0.9}{11},\frac{1}{12},\frac{0.6}{13},\frac{0}{14}\right\}\text{mln. rub}$$

using "FuziCalc" tool we solve the problem (Approach 1) Table 1:

Table 1: Fuzi calc tool

Factors	0/1	0/2	0/3	0/4	0/5	0/6	0/7
VC\FC	0/16	0.8/17	1/18	1/19	0.7/20	0.3/21	0/22
0/8	0/24	0/25	0/26	0/27	0/28	0/29	0/30
0.4/9	0/25	0.4/26	0.4/27	0.4/28	0.4/29	0.3/30	0/31
0.7/10	0/26	0.7/27	0.7/28	0.7/29	0.7/30	0.3/31	0/32
0.9/11	0/27	0.8/28	0.9/29	0.9/30	0.7/31	0.3/32	0/33
1/12	0/28	0.8/29	1/30	1/31	0.7/32	0.3/33	0/34
0.6/13	0/29	0.6/30	0.6/31	0.6/32	0.6/33	0.3/34	0/35
0/14	0/30	0/31	0/32	0/33	0/34	0/35	0/36

$$TC=\left\{\begin{array}{l}\frac{0}{24},\frac{0}{25},\frac{0.4}{26},\frac{0.7}{27},\frac{0.8}{28},\frac{0.9}{29},\\ \frac{1}{30},\frac{1}{31},\frac{0.7}{32},\frac{0.6}{33},\frac{0.3}{34},\frac{0}{35},\frac{0}{36}\end{array}\right\}$$

Next, we express the analytical expression of fixed costs and variable costs in the form of continuous fuzzy numbers (Approach 2) and represent the solution of this problem through α-sections:

$$\mu_{FC}(x)=\begin{cases}1, x\in[18;19]\\ \frac{x-16}{2}, x\in(16;18)\\ \frac{22\text{-}x}{3}, x\in(19;22)\\ 0, \text{ otherwise}\end{cases};\ \mu_{VC}(x)=\begin{cases}1, & x=12\\ \frac{x\text{-}8}{4}, & x\in(8;12)\\ \frac{14\text{-}x}{2}, & x\in(12;14)\\ 0, & \text{otherwise}\end{cases}$$

We produce the addition operation of two continuous fuzzy numbers:

$$FC_\alpha=[2\alpha+16;\ 22\text{-}3\alpha]; VC_\alpha=[4\alpha+8;\ 14\text{-}2\alpha]$$

As a result, we receive $Tc_\alpha = Fc_\alpha+VC_\alpha$, = [6α+24; 36-5α], analytic expression which has the form:

$$\mu_{TC}(x)=\begin{cases}1, x\in[30,31]\\ \frac{x-24}{6}, x\in(24,30)\\ \frac{36-x}{5}, x\in(31,36)\\ 0, \text{ otherwise}\end{cases}$$

CONCLUSION

In the analysis of the approach of finding the common costs of this innovative chemical enterprise through fuzzy number L-R-type express how fixed costs FC = (16; 18; 19;22) and variables as VC = (8; 12; 14).Then, by adding the uncertainty intervals overall costs will amount to: TC = (24; 30; 31; 36) which confirms the above calculations are made on the example of discrete (Approach 1) and continuous (Approach 2) fuzzy numbers.

REFERENCES

Bart, K., 1991. Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence. Prentice Hall, Englewood Cliffs, New Jersey, ISBN:9780136114437, Pages: 175.

Bart, K., 1993. Fuzzy Thinking: The New Science of Fuzzy Logic. Hyperion Books, New york, USA., ISBN:9781562828394, Pages: 318.

Beilin, I.L. and V.P. Arkhireev, 2005. New copolymers of propylene carbonate with controlled complex of properties. Plast. Weights: Synth. Prop. Recycl. Appl., 7: 12-15.

Beilin, I.L. and V.P. Arkhireev, 2009. New copolymer products from cyclic carbonates and isocyanate-containing compounds. Prot. Met. Phys. Chem. Surf., 45: 450-454.

Beilin, I.L. and V.P. Arkhireev, 2011a. Synthesis and structure of copoly (amide esters) based on cyclic carbonates and monofunctional isocyanates. Prot. Met. Phys. Chem. Surf., 47: 478-483.

Beilin, I.L. and V.P. Arkhireev, 2011b. The supermolecular structure of new copolymer products based on cyclic carbonates. Int. Polym. Sci. Technol., 38: T37-T40.

Beilin, I.L., 2016. Analysis of efficiency of the innovative project in the field of chemistry fuzzy logic. J. Econ. Educ. Res., 17: 177-185.

Beilin, I.L., V.P. Arkhireev and S.S. Galibeev, 2006. Copolymerization of cyclic carbonates with isocyanates under anionic initiation conditions and structure of the new copolymers. Russ. J. Appl. Chem., 79: 133-136.

Bojadziev, G., 1997. Fuzzy Logic for Business, Finance and Management. Vol. 12, World Scientific, Singapore, ISBN:9810228945, Pages: 232.

Buckley, J.J., 1987. The fuzzy mathematics of finance. Fuzzy Sets Syst., 21: 257-273.

Buckley, J.J., 1992. Solving fuzzy equations in economics and finance. Fuzzy Sets Syst., 48: 289-296.

Castro, J.R., O. Castillo, P. Melin, L.G. Martínez and S. Escobar *et al*., 2007. Building Fuzzy Inference Systems with the Interval Type-2 Fuzzy Logic Toolbox. In: Analysis and Design of Intelligent Systems Using Soft Computing Techniques, Castro, J.R., O. Castillo, P. Melin, L.G. Martínez and S. Escobar *et al*. (Eds.). Springer, Berlin, Germany, ISBN:978-3-540-72431-5, pp: 53-62.

Couturier, A. and B. Fioleau, 2000. Debt level and company efficiency: Independence or implication? An evaluation of fuzzy implications. Eur. J. Econ. Soc. Syst., 14: 17-25.

Dimitras, A.I., R. Slowinski, R. Susmaga and C. Zopounidis, 1999. Business failure prediction using rough sets. Eur. J. Oper. Res., 114: 263-280.

Dimitras, A.I., S.H. Zanakis and C. Zopounidis, 1996. A survey of business failures with an emphasis on prediction methods and industrial applications. Eur. J. Oper. Res., 90: 487-513.

Dourra, H. and P. Siy, 2002. Investment using technical analysis and fuzzy logic. Fuzzy Sets Syst., 127: 221-240.

Dubois, D. and H. Parade, 1979. Fuzzy real algebra: Some results. Fuzzy Sets Syst., 2: 327-348.

Dubois, D. and H. Prade, 1980. Fuzzy Sets and Systems: Theory and Applications. 4th Edn., Academic Press, Cambridge, Massachusetts, ISBN:9780122227509, Pages: 393.

Kaufmann, A. and M.M. Gupta, 1991. Introduction to Fuzzy Arithmetic: Theory and Applications. Van Nostrand Reinhold Co., New York, USA., ISBN-13: 9780442008994, Pages: 361.

Maria, Z.L. and A. Kandel, 1984. Fuzzy Relational Data Bases: A Key to Expert Systems. TUV Media GmbH TUV Rheinland Group, Cologne, Germany, Pages: 180.

Peray, K., 1999. Investing in Mutual Funds Using Fuzzy Logic. St. Lucie Press, Delray Beach, Florida, ISBN:1-57444-264-3, Pages: 234.

Trippi, R.R. and J.K. Lee, 1995. Artificial Intelligence in Finance and Investing: State of the Art Technologies for Securities Selection and Portfolio Management. Irwin Professional Publishing, Burr Ridge, Illinois,.

Zadeh, L.A., 1973. Outline of a new approach to the analysis of complex systems and decision processes. IEEE Trans. Syst. Man Cybernet., 3: 28-44.

Zadeh, L.A., 1978. Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets Syst., 1: 3-28.

Zadeh, L.A., G.J. Klir and B. Yuan, 1996. Fuzzy Sets, Fuzzy Logic and Fuzzy System. World Scientific, Singapore Pages: 826.

Zhou, S.M., R. John, F. Chiclana and J.M. Garibaldi, 2007. New type-2 rule ranking indices for designing parsimonious interval type-2 fuzzy logic systems. Proceedings of the 2007 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE'07), July 23-26, 2007, IEEE, London, UK., ISBN:1-4244-1209-9, pp: 1-6.

Zimmerman, H.J., 2001. Fuzzy Sets Theory: And its Applications. 4th Edn., Kluwer Academic Publishers, Boston, Massachusetts, ISBN:0-7923-7435-5, Pages: 519.

Zopounidis, C., P.M. Pardalos and G. Baourakis, 2001. Fuzzy Sets in Management, Economy and Marketing. World Scientific, Singapore, ISBN:9810247532, Pages: 270.