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Solving Dissipative Nonlinear Schrodinger Equation with Variable Coefficient using Homotopy Perturbation Method

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Abstract: This study focuses on the solution of the Dissipative Nonlinear Schrodinger equation with Variable coefficient (DNLSV) by using Homotopy Perturbation Method (HPM) to obtain approximate analytical solution. The procedure of the method is systematically illustrated. The result obtained is then compared with the progressive wave solution to verify the accuracy of the HPM solution. The absolute error of the HPM solution of the DNLSV equation with the progressive wave solution will later be carried out using the MATLAB Program.

Key words: Homotopy perturbation method, DNLSV equation, progressive wave, HPM solution, MATLAB Program, Malaysia

INTRODUCTION

Over recent years, there has been an explosive growth of interest in the development of non-linear science among researchers in the analytical techniques for nonlinear problems. Homotopy Perturbation Method (HPM) is one of the techniques proposed to solve these nonlinear problems. HPM was firstly proposed by He (1999). It was further developed and improved by He (2000, 2003, 2004). The HPM is a combination of homotopy technique in topology and classical perturbation technique. It deforms a difficult problem into a simple problem. This method does not require a small parameter in an equation as needed in traditional perturbation method. The solution is obtained as a summation of an infinite series which usually converges rapidly to the exact solution if such a solution exists. The HPM does not involve discretization of variables as most of numerical methods. Hence, it is free from rounding off errors causing loss of accuracy. The HPM has significant advantage in that it provides an approximate analytical solution to a wide range problems arising in different fields. The HPM is applied to nonlinear equation (Li, 2009; Noor and Khan, 2012) system of linear equations (Yusufoglu, 2009) time-dependent differential equation (Babolian *et al*., 2009) hyperbolic partial differential equation (Biazar and Ghazvini, 2008) coupled non-linear partial differential equations (Sweilam and Khader, 2009) two-dimensional non-linear differential equation (Ghasemi *et al*., 2007) linear wave equation and non-linear diffusion equation (Chun *et al*., 2009) Schrodinger equation and nonlinear schrodinger equation (Mousa and Ragab, 2008) fractional non-linear schrodinger equation (Baleanu *et al*., 2009) and two-dimensional non-linear schrodinger equation.

There are many physical phenomena in engineering and physics can be described by some nonlinear differential equation. The Non-Linear Schrodinger (NLS) equation with cubic non-linearity in the form:

$$i\frac{\partial U}{\partial \tau}+\mu_1\frac{\partial^2 U}{\partial \xi^2}+\mu_2|U|^2U=0,\quad i^2=-1 \tag{1}$$

Where:

- U = The complex-valued function of the spatial coordinates ξ and time t
- μ_1 and μ_2 = Real parameters correspond to a focusing (μ_1 μ_2>0) or defocusing (μ_1 μ_2<0) effects of the non-linearity

The NLS Eq. 1 arises in various physical contexts in the description of dispersive nonlinear waves including nonlinear optics, hydrodynamics, plasma physics, quantum mechanics, water waves and super conductivity. The NLS Eq. 1 is the simplest representative equation describing the self-modulation of one dimensional monochromatic plane waves in dispersive media. It exhibits a balance between the non-linearity and dispersion.

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Due to its application on arterial mechanism the modulation of small-but-finite amplitude pressure waves in a fluid-filled distensible, linear elastic tube has been studied by Ravindran and Prasad (1979) and they obtained the NLS Eq. 1. Demiray and co-workers studied the modulation of nonlinear wave in fluid-filled elastic tube (Antar and Demiray, 1999) and viscoelastic tube (Akgun and Demiray 1999) filled with an inviscid fluid. They obtained the NLS Eq. 1 and Dissipative Non-Linear Schrodinger (DNLS) Eq. 2 as follows:

$$i\frac{\partial U}{\partial \tau}+\mu_1\frac{\partial^2 U}{\partial \xi^2}+\mu_2|U|^2U+i\mu_4U=0 \quad (2)$$

where, $i\mu_4$ denotes the dissipative term. Recently, Choy (2014) studied the nonlinear wave modulation of a thin elastic tube with a symmetrical stenosis. By using an approximate equation of a viscous fluid, she showed that the governing equations can be reduced to the Dissipative Nonlinear Schrodinger with Variable coefficient (DNLSV) Eq. 3 as follows:

$$i\frac{\partial U}{\partial \tau}+\mu_1\frac{\partial^2 U}{\partial \xi^2}+\mu_2|U|^2U-\mu_3h_1(\tau)U+i\mu_4U=0 \quad (3)$$

where $\mu_3h_1(\tau)$ U denotes the variable coefficient term. Notice that when $\mu_3h_1(\tau)U=0$ and $i\mu_4U=0$ both Eq. 2 and 3 reduce back to the NLS Eq. 1. The NLS Eq. 1 had been solved by various numerical methods such as Crank-Nicolson finite-difference method (Taha and Ablowitz, 1984) split-step finite-difference method (Wang, 2005) compact split-step finite-difference method (Dehghan and Taleei, 2010), Adomian Decomposition Method (ADM) (Sayed and Kaya, 2006; Bratsos *et al.*, 2008) and HPM (Mousaa and Ragab, 2008).

Many studies have been devoted to the numerical solution of the NLS Eq. 1 and the HPM solution of the NLS Eq. 1. However, none of the literature has dealt with the HPM solution of the DNLSV Eq. 3. Motivated by the works of wave modulation by Choy (2014) which yielded the DNLSV Eq. 3, numerical works as well as the HPM solution of the NLS Eq. 1 we are going to approximate the DNLSV Eq. 3 using the HPM. By using the conditions from a previous study we then compare the solution with progressive wave obtained by Choy (2014).

MATERIALS AND METHODS

Homotopy Perturbation Method: To illustrate the basic ideas of HPM, we consider the following non-linear differential Eq. 4:

$$A(U)-f(r)=0, \quad r\in\Omega \quad (4)$$

with the boundary condition:

$$B(U,\partial U/\partial n)=0, \quad r\in\Gamma \quad (5)$$

Where:

A = A general differential operator
B = A boundary operator
f (r) = Known as analytical function
Γ = The boundary of the domain Ω

The operator A can be divided into two parts of linear, L and non-linear N. Therefore, Eq. 1 can be rewritten as follow:

$$L(U)+N(U)-f(r)=0 \quad (6)$$

By the homotopy technique, we construct a homotopy V(r, p): Ω×[0, 1]→R which satisfies:

$$H(V,p)=(1-p)\left[L(V)-L(U_0)\right]+p\left[A(V)-f(r)\right]=0 \quad (7)$$

Or:

$$H(V,p)=L(V)-L(U_0)+pL(U_0)+p\left[N(V)-f(r)\right]=0 \quad (8)$$

where p∈[0, 1] is an embedding parameter, u_0 is an initial approximation of Eq. 4 which satisfies the boundary condition. Obviously from Eq. 7and 8, we have:

$$H(V,0)=L(V)-L(U_0)=0 \quad (9)$$

$$H(V,1)=A(V)-f(r)=0 \quad (10)$$

and the changing process of p from zero to unity is just that of V(r,p) changes from $U_0(r)$ to U (r). In topology, this is called deformation $L(V)-L(U_0)$ and A(V)-f(r) is called homotopic. According to the HPM firstly, we use the embedding parameter p as a small parameter and assume that the solution of Eq. 7 can be written as a power series in p:

$$V=V_0+pV_1+p^2V_2+p^3V_3+\cdots \quad (11)$$

setting p = 1, results in approximate solution of Eq. 4:

$$U=\lim_{p\to 1}V=V_0+V_1+V_2+V_3+\cdots \quad (12)$$

The series Eq. 12 is convergent for most cases. However, convergent rate depends on the non-linear operator A(V).

RESULTS AND DISCUSSION

An application of HPM: Consider the following DNLSV Eq. 3 with initial condition given by Choy (2014) as follows:

$$U(\xi,\tau) = a\tanh\left[\left(-\frac{\mu_2}{2\mu_1}\right)^{\frac{1}{2}} a\xi\right]e^{iK\xi} \quad (13)$$

Using HPM, we construct a homotopy in the following form:

$$\begin{aligned} H(V,p) &= (1\text{-}p)\left[i\frac{\partial V}{\partial \tau} \text{-} i\frac{\partial U_0}{\partial \tau}\right] + \\ & p\left[i\frac{\partial V}{\partial \tau} + \mu_1\frac{\partial^2 V}{\partial \xi^2} + \mu_2|V|^2 V \text{-} \mu_3 h_1(\tau)V + i\mu_4 U\right] = 0 \end{aligned} \quad (14)$$

Or:

$$\begin{aligned} H(V,p) &= (1-p)\left[i\frac{\partial V}{\partial \tau} - i\frac{\partial U_0}{\partial \tau}\right] + \\ & p\left[i\frac{\partial V}{\partial \tau} + \mu_1\frac{\partial^2 V}{\partial \xi^2} + \mu_2 V^2\bar{V} - \mu_3 h_1(\tau)V + i\mu_4 U\right] = 0 \end{aligned} \quad (15)$$

where, $U_0(\xi, \tau) = V_0(\xi, 0) = U(\xi, 0), |v|^2 = v\bar{v}$ and $\bar{v}$ is the conjugate of V. Suppose that the series solution of Eq. 14 and its conjugate have the following forms:

$$\begin{aligned} V &= V_0(\xi,\tau) + pV_1(\xi,\tau) + \\ & p^2V_2(\xi,\tau) + p^3V_3(\xi,\tau) + \cdots \end{aligned} \quad (16)$$

$$\begin{aligned} \bar{V} &= \bar{V}_0(\xi,\tau) + p\bar{V}_1(\xi,\tau) + \\ & p^2\bar{V}_2(\xi,\tau) + p^3\bar{V}_3(\xi,\tau) + \cdots \end{aligned} \quad (17)$$

Substituting Eq. 16 and 17 into Eq. 15 and collecting the terms with identical powers of p, leads to:

$$p_0 = i\frac{\partial V_0}{\partial \tau} \text{-} i\frac{\partial U_0}{\partial \tau} = 0 \quad (18)$$

$$\begin{aligned} p_1 &= i\frac{\partial v_1}{\partial \tau} + \mu_1\frac{\partial^2 V_0}{\partial \xi_2} + \mu_2 V_0^2\bar{V}_0 \text{-} \\ & \mu_3 h_1(\tau)V_0 + i\mu_4 V_0 + i\frac{\partial U_0}{\partial \tau} = 0 \end{aligned} \quad (19)$$

$$\begin{aligned} p_2 &= i\frac{\partial V_2}{\partial \tau} + \mu_1\frac{\partial^2 V_1}{\partial \xi^2} + \mu_2\left(V_0^2\bar{V}_1 + 2V_0V_1\bar{V}_0\right) \text{-} \\ & \mu_3 h_1(\tau)V_1 + i\mu_4 V_1 = 0 \end{aligned} \quad (20)$$

The given initial value admits the use of:

$$V_i(\xi,0) = \begin{cases} a\tanh\left[\left(-\frac{\mu_2}{2\mu_1}\right)^{\frac{1}{2}} a\xi\right]e^{iK\xi}, & i = 0 \\ 0, & i = 1, 2, 3, \cdots \end{cases} \quad (21)$$

The solutions read:

$$V_0(\xi,\tau) = a\tanh\left(\sqrt{-\frac{\mu_2}{2\mu_1}}a\xi\right)e^{iK\xi} \quad (22)$$

$$\begin{aligned} V_1(\xi,\tau) &= a\tanh\left(\sqrt{-\frac{\mu_2}{2\mu_1}}a\xi\right)\left[i\left(a^2\mu_2 - \mu_1K^2 - \mu_4\right)\right]\tau e^{iK\xi} - \\ & \frac{10}{3}ia\mu_3\tanh\left(\sqrt{-\frac{\mu_2}{2\mu_1}}a\xi\right)\arctan\left[\sinh(0.3\tau)\right]e^{iK\xi} - \\ & 2a^2\mu_1\sqrt{-\frac{\mu_2}{2\mu_1}}\operatorname{sech}^2\left(\sqrt{-\frac{\mu_2}{2\mu_1}}a\xi\right)\tau Ke^{iK\xi} \end{aligned} \quad (23)$$

and so on. The solution of Eq. 3 can be obtained by setting p = 1 in Eq. 11:

$$V = V_0 + V_1 + V_2 + \cdots \quad (24)$$

Thus, we have:

$$\begin{aligned} U(\xi,\tau) &= V(\xi,\tau) = V_0(\xi,\tau) + V_1(\xi,\tau) + \cdots = \\ & a\tanh\left(\sqrt{\text{-}\frac{\mu_2}{2\mu_1}}a\xi\right)e^{iK\xi} + \\ & a\tanh\left(\sqrt{\text{-}\frac{\mu_2}{2\mu_1}}a\xi\right)\left[i\left(a^2\mu_2 \text{-} \mu_1K^2 \text{-} \mu_4\right)\right]\tau e^{iK\xi} \text{-} \\ & \frac{10}{3}ia\mu_3\tanh\left(\sqrt{\text{-}\frac{\mu_2}{2\mu_1}}a\xi\right)\arctan\left[\sinh(0.3\tau)\right]e^{iK\xi} \text{-} \\ & 2a^2\mu_1\sqrt{\text{-}\frac{\mu_2}{2\mu_1}}\operatorname{sech}^2\left(\sqrt{\text{-}\frac{\mu_2}{2\mu_1}}a\xi\right)\tau Ke^{iK\xi} \end{aligned} \quad (25)$$

The progressive wave solution (quite close to exact solution) of the DNLSV Eq. 3 is given by Choy (2014) as follow:

$$\begin{aligned} U(\xi,\tau) &= ae^{-\frac{2}{3}\mu_4\tau} \\ & \tanh\left[\sqrt{\text{-}\frac{\mu_2}{2\mu_1}}ae^{-\frac{2}{3}\mu_4\tau}(\xi \text{-} 2\mu_1K\tau)\right]e^{i(K\xi-\Omega\tau)} \end{aligned} \quad (26)$$

Table 1: The absolute error of the DNLSV equation for different spatial values, ξ at τ = 0.0001

Spatial parameter (ξ)	-4	-3	-2	-1	0	1	2	3	4
L_∞	0.0045	0.0045	0.0045	0.0045	0	0.0045	0.0045	0.0045	0.0045

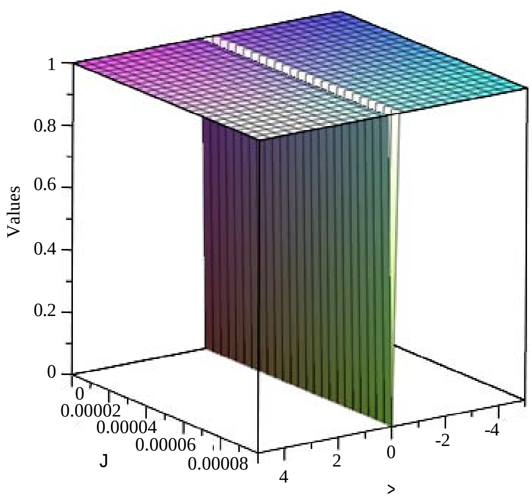

Fig. 1: 3D-plot of the HPM solution of the DNLSV Eq. 3 under initial condition (Chun *et al*., 2009)

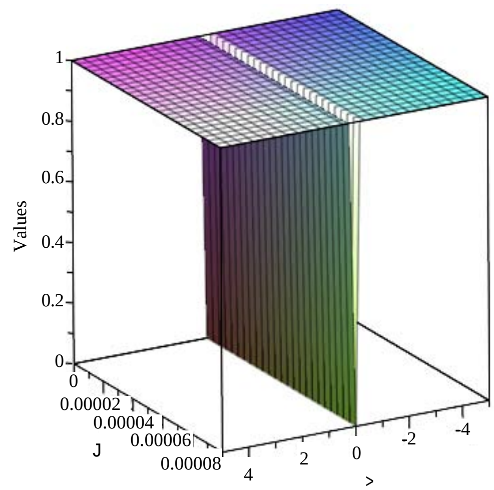

Fig. 3: 3D-plot of the progressive wave solution of the DNLSV Eq. 3

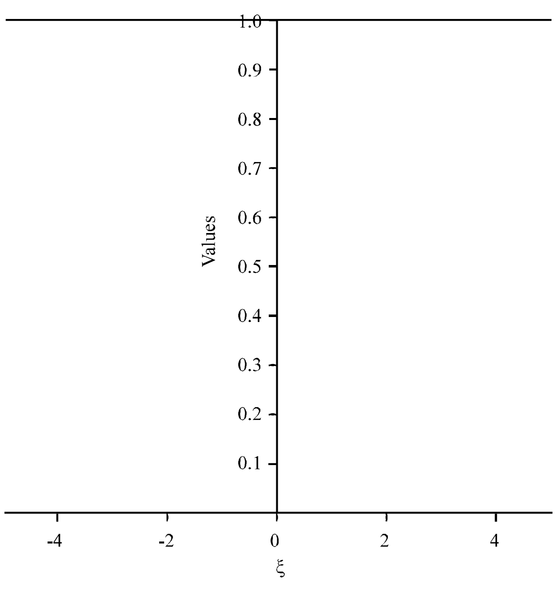

Fig. 2: 2D-plot of the HPM solution of the DNLSV Eq. 3 under initial condition (Chun *et al*., 2009)

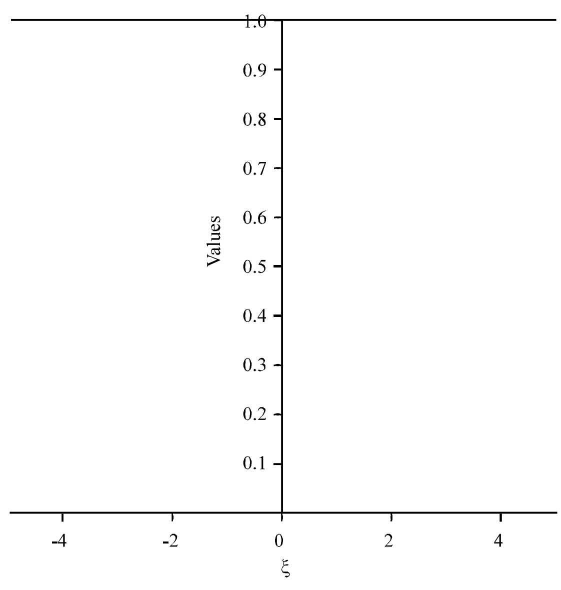

Fig. 4: 2D-plot of the progressive wave solution of the DNLSV Eq. 3

where, $\Omega = \mu_1 K^2 - \mu_2 a_0^2 e^{-4/3\mu_4\tau} + \mu_3 h_1(\tau)$. Given that these numerical values of the coefficient by Choy (2014) $h_1(\tau)$ = sech (0.3τ), μ_1 =-0.1548, μ_2 = 26.4295, μ_3 = 7.3572, μ_4 = 0.1082 provided a = 1 and K = 2.

Figure 1 and 3 show the 3D-plot of the HPM solution and progressive wave solution of the DNLSV Eq. 3 with spatial parameter, ξ and time, τ, respectively while Fig. 2-4 show the 2D-plot of HPM solution and the progressive wave solution of the DNLSV Eq. 3 with spatial parameter, ξ at time τ = 0.0001, respectively. Figure 5 shows the absolute error of the DNLSV Eq. 3. Table 1 shows the absolute error between the progressive wave and HPM solutions for certain spatial points, ξ and at time, τ = 0.0001.

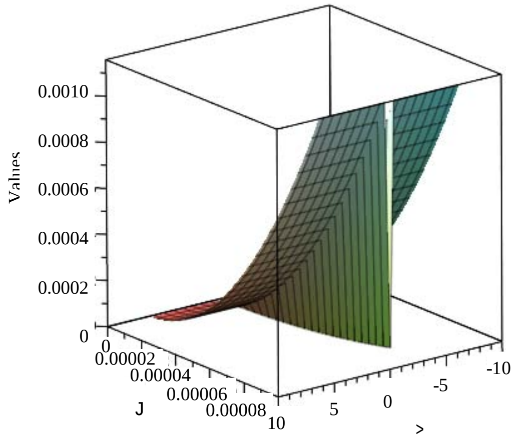

Fig. 5: 3D-plot of the absolute error of the DNLSV Eq. 3

CONCLUSION

In this researche, HPM is successfully applied to solve the DNLSV Eq. 3. The solution obtained by HPM is an infinite series for appropriate initial condition that can be expressed in a closed form to the exact solution. The solution obtained by HPM is found to be a powerful mathematical tool which can be used to solve nonlinear partial differential equations.

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