Journal of Engineering and Applied Sciences 12 (18): 4595-4601, 2017
ISSN: 1816-949X

Employees Timetabling Simulation using Integer Linear Programming Technique

Sudradjat Supian, R. Sudrajat, Eman Lesmana and Amalia Farida
Department of Mathematics, FMIPA, Universitas Padjadjaran, Bandung, West Java, Indonesia

Abstract: A problem to allocate n number of employees in a directorate/division into a given set of organizational shifts is the Employee Timetabling Problem (ETP). The main purpose of employee timetabling in this research is the allocation of n employees who should be assigned each day so that no idle workers. To solve the ETP and the main purpose of this research in create mathematical modeling represented 0-1 Integer Linear Programming (ILP) Model. The ETP is solved by considering two criteria as the constraints, there are the organizational rules and employee's requirement with the objective function that is to minimize the number of employees to minimize labor costs. About 0-1 ILP Model obtained from process organization data input and employee data input. Employee data input can be done manually or automatically from database with the help of Software Excel. The 0-1 ILP Model solved by framework SCIP optimization suite which then implemented into application of ETP simulation that are developed by Software Microsoft Visual Studio.

Key words: Mathematical modeling, employee timetabling problem, optimization, integer linear programming, employee, ETP

INTRODUCTION

Industrial world will not escape from timetabling problems such as production timetabling, delivery timetabling, timetabling ordering goods includes the employee timetabling problems that would arise in a company.

Employee timetabling involves many consideration factors. First human resources is one of the most important things required from the company. For that, the rights and obligations of employees must be considered such as a holiday shift, the number of working days as well as the number of shifts to be taken by them. In addition, the availability of resources should be utilized as much as n employees as much as possible by the company to assign all employees each day into shifts accordingly. The second shift work or working hours is a solution of the problem needs of companies that may reach 24 h but will not be able to be done by employees. The third factor that made the company rules can be provided that all employees should be assigned every day, many provisions of shifts within one working day and the other provisions according to the needs of the company.

Integer linear programming is a method that can be implemented in the employee timetabling problems into consideration factors that serve as obstacles and variables worth 0-1 decision. Research has been done in the implementation of these issues including (Aloul *et al*., 2013) to discuss the settlement of employee timetabling problems using integer linear programming techniques and 0-1 Boolean satisfiability using a wide range of frameworks that help find a wide range of optimal solutions. Sondita (2013) to discuss the simulation of mathematical models of employee timetabling using integer linear programming models were implemented into the software with the aim for limitation number of employees so that personnel costs can be reduced. Kridanto *et al.* (2013) to discuss the use of integer linear programming optimization heuristic method for timetabling of part-time by implementing the method proposed by Iriani (2009) to discuss the optimization of timetabling exams with highly use branch and bound method.

Powered from the mentioned study, the authors conducted a further study by Sondita (2013) regarding timetabling simulation n employees should be assigned into shift work that has been provided each day so that no idle employees using Microsoft Visual Studio Software to create models 0-1 integer linear programming which were solved with the help of the framework SCIP optimization suite.

Employee timetabling problem: Timetabling related to decision-making that are used on a regular basis in some manufacturing industries, production systems, transportation, distribution arrangements and the service industry (Pinedo, 2012). Timetabling is the allocation of limited resources into jobs or activities with the aim to optimize one or more performance benchmarks. Timetabling purposes may include minimizing the number of workers end (Joseph, 2000).

Corresponding Author: Sudradjat Supian, Department of Mathematics, FMIPA, Universitas Padjadjaran Bandung, West Java, Indonesia

Employee timetabling is the assignment of resources in the form of workers or employees on the job or activity in a set of shifts over a period of time which is usually fixed for 1 week. Employee timetabling goals are to maximize employee satisfaction and minimize labor costs. More specifically the employee timetabling problems began with a given number of employees and shift each shift consists of 8 h of work with the purpose of assigning employees in shifts that satisfies all constraints and constraints of the company employees. Constraints of employees referred to the rights and obligations such as employee satisfaction or requiring employees to choose weekdays and work shifts with the duty of every employee is required to work as many days or hours specified by the company. Regulatory constraints of the enterprise that made the company such as the number of employees in one shift, the worker shall work according to the number of days and hours specified and so forth.

MATERIALS AND METHODS

Mathematical modeling (Mayer, 2004): Mathematical modeling is an attempt to describe some part of the real world in mathematical term. A model is an object or concept that is used to represent something else. A mathematical modeling is a model whose parts are mathematical concept such as constants, variables, functions, equations, inequalities, etc.

Modeling cannot be done mechanically. Nevertheless, there are some guidelines for how to go about it. Modeling process can divide into three main steps that is formulation problem (atating the question, identifying relevant factors, mathematical description), mathematical manipulation, evaluation. Mathematical modeling are often used to help us make decisions. When use a mathematical model to select the best alternative out of a large number of possibilities is called optimization.

Linear programming (Rao, 2009): Model of linear programming problems can be expressed by the equation:

$$\begin{aligned} &\text{minimize } f(x_1, x_2, \ldots, x_n) = c_1x_1 + c_2x_2 + \ldots + c_nx_n \\ &\text{subject to } a_{11}x_1 + a_{12}x_1 + \ldots + a_{1n}x_n = b_1 \\ &\qquad a_{21}x_1 + a_{22}x_1 + \ldots + a_{2n}x_n = b_2 \\ &\qquad \vdots \\ &\qquad a_{m1}x_1 + a_{m2}x_1 + \ldots + a_{mn}x_n = b_m \\ &\qquad x_1, x_1, \ldots, x_n \geq 0 \end{aligned} \tag{1}$$

where, c_j, b_j and a_{ij} (i = 1, 2, ..., m; j = 1, 2, ..., n) is a constant known and x_j is decision variable. Linear programming model consists of: decision variable is a component in the system that must be determined to decide how the amount or value of any variable that must be taken in a matter which form x_j, j = 1, ..., n. Objective function which form $f(x_1, x_2, ..., x_n) = c_1x_1+c_2x_2+...+c_nx_n$ is linear function representing the cost, benefit or other purposes to be maximum or minimum. Coefficients c_j, j = 1, ..., n known as the coefficient of cost, showed an increase f that would result from any increase in the decision variable x_j, j = 1, ..., n.

Constraint function $a_1x_1+a_2x_2+...+a_{mn}x_n = b_m$ is a form of constraint function where a_{ij}, i = 1, ..., n, j = 1, ..., n is called coefficient technology that the amount of resource i consumed by each unit of activity j and b_i, i = 1, ..., m is the number of resources available to be allocated to activities with decision variable $x_j \geq 0$, j = 1, ..., n is called non-negative constraints.

The purpose of linear programming problem is to find feasible solution. Linear programming problem can be solved by the simplex method that gives an algorithm to check the feasible solution in which if an optimal solution cannot be found then it will look for the closest possible solutions.

Integer linear programming (Hillier and Lieberman, 2001): A linear programming problem that requiring integer value is Integer Programming problem (IP). The mathematical model for integer programming is the linear programming model with the one additional restriction that the variables must have integer values. If only some of the variables are required to have integer values, this model is referred to as Mixed Integer Programming (MIP). Integer programming problem that the only two possible choices are yes or no can be represent such decision variables that are restricted to just two values, say 0 and 1. Thus, the jth yes or no decision would be represented:

$$x_{ijk} = \begin{cases} 1, & \text{If decision j is yes} \quad (j = 1, 2, 3, ..., n) \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Integer programming problem that contain only binary variables sometimes are called Binary Integer Programming (BIP) problems or 0-1 integer programming problems). Model 0-1 integer linear programming can be expressed by equation:

$$\begin{aligned} &\text{minimize } f(x_1, x_2, \ldots, x_n) = c_1x_1 + c_2x_2 + \ldots + c_nx_n \\ &\text{subject to } a_{11}x_1 + a_{12}x_1 + \ldots + a_{1n}x_n = b_1 \\ &\qquad a_{21}x_1 + a_{22}x_1 + \ldots + a_{2n}x_n = b_2 \\ &\qquad \vdots \\ &\qquad a_{m1}x_1 + a_{m2}x_1 + \ldots + a_{mn}x_n = b_m \\ &\qquad x_j \geq 0, x_j \leq 1 \\ &\qquad x_j \text{ integer, } j = 1, \ldots, n \end{aligned} \tag{3}$$

Branchand bound method (Hillier and Lieberman, 2001): The method used to find the solution of the problem of binary integer linear programming is branch and bound. The basic concept underlying the branch and bound technique is to divide and conquer. Since, the original large problem is too difficult to be solved directly, it is divided into smaller and smaller problems until these subproblems can be conquered.

A subproblem is fathomed (dismissed from further consideration) if meet one test, first test its bound $\leq Z^*$ or second test its linear programming relaxation has no feasible solution or third test is the optimal solution for its linear programming relaxation is integer (if this solution is better than the incumbent, it becomes the new incumbent and first test is reapplied to all unfathomed subproblems with the new larger Z^*).

Initialization algorithm branch and bound is set $Z^* = \infty$. Apply the bounding step, fathoming step and optimality test described below to the whole problem. If not fathomed, classify this problem as the one remaining "subproblem" for performing the first full iteration below. Steps for each iteration.

Branching: Among the remaining (unfathomed) subproblems, select the one that was created most recently (Break ties according to which has the larger bound). Branch from the node for this subproblem to create two new subproblems by fixing the next variable (the branching variable) at either 0 or 1.

Bounding: For each new subproblem, obtain its boundby applying the simplex method to its LP relaxation and rounding down the value of Z for the resulting optimal solution.

Fathoming: For each new subproblem, apply the three fathoming tests summarized above and discard those subproblems that are fathomed by any of the tests.

Optimality test: Stop when there are no remaining subproblems; the current incumbentis optimal. Otherwise, return to perform another iteration.

Microsoft visual studio: Microsoft visual studio is a microsoft product that is used to simplify the user in creating or developing applications in the form of console applications, Windows applications or Web applications microsoft visual studio provides a compiler, Integrated Development Environment (IDE), Software Development kit (SDK) and the MSDN Library (Haris, 2012). Microsoft visual studio is used in this study is a version of microsoft visual studio 2013 ultimate with license 9FJCR-X7BKD-G84RY-QYH4P-VYYB7 with programming language C# and Windows forms application as interface elements that are used.

SCIP optimization suite: SCIP (Solving Constrain Integer Programs) is a software framework for Constrain Integer Programming (CIP) (Achterberg, 2009). SCIP is currently one of the fastest non-commercial Mixed Integer Programming (MIP) and Mixed Integer Programming (MINP) (Berthold *et al*., 2012) without the help of other tools provided in the format of MPS, LP, flatzinc, CNF, OPB, WBO, PIP or CIP (13) which can be obtained on the official website SCIP is scip.zib.de. Constrain integer programming (C, I, c) is a generalization of MIP are defined as follows (Achterberg, 2009):

$$(\mathrm{CIP})\, c^* = \min\left\{c^T x | C(x), x \in \mathbb{R}^n, x_j \in \mathbb{Z} \text{ for all } j \in I\right\} \quad (4)$$

with a finite set $C^m = \{C_1, ..., C_m\}$ of constraints $C_i = \mathbb{R}^n \rightarrow \{0, 1\}$, i-1, ..., m, a subset $1 \subseteq N$-{1, ..., n} of a variable index set and an objective function vector $c \in \mathbb{R}^n$.

If given the constraint integer program (C, I, c) and linear programming with equation is a generalization of MIP are defined as follows:

$$(\mathrm{LP})\, \breve{c} = \min\left\{c^T x | Ax \leq b,\ x \in \mathbb{R}^n\right\} \quad (5)$$

is called an LP relaxation of (CIP) if:

$$\left\{x \in \mathbb{R}^n | Ax \leq b\right\} \supseteq \left\{x \in \mathbb{R}^n\ C(x), x_j \in \mathbb{Z} \text{for all } j \in I\right\} \quad (6)$$

The format used by SCIP to reading problems in research is OPB format because of problems in the form of a model that is worth 0-1 integer. OPB format stored in notepad ++ application and the following is an example of writing OPB format. After modeling format ILP with OPB, OPB steps to resolve the ILP by SCIP method is as follows (Fig. 1):

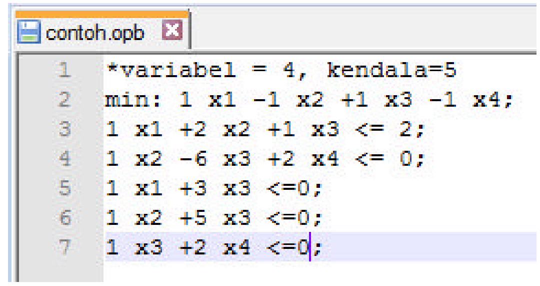

Fig. 1: Example file with format OPB

- OPB save the file in the same folder as the application SCIP
- File OPB read by SCIP with the command “read” for example “read contoh.opb”
- Finding out the optimal solution is done with the command “optimize”
- If a feasible solution, the solution is stored in a TXT file with the command “write solution” for example “write solusi.txt solution”

RESULTS AND DISCUSSION

Timetabling criteria: Timetabling criteria such as corporate governance and employee needs. These criteria serve the function of constraints into consideration in the linear programming model of employee timetabling problems.

Company regulation: Shift work was made to meet the company’s needs and abilities of employees. Moreover, in accordance with Law No. 13 of 2003 on employment, an employee working for 5 days in 1 week with 8 h of work obligations pershift or 6 days in 1 week with work obligations per shift 7 and 40 h in 1 week. Then, shift work that can be made within 1 working day maximum of 3 shifts, namely the morning shift (7:00-15:00), the day shift (15:00-23:00) and the night shift (11:00 p.m. to 7:00 a.m.). In addition, companies need to determine working day 5 or 6 days within 1 week.

Needs of employee: Needs of employees each employee can be required to work at least one shift within one day, got the night shift a maximum of 3 times a week, the right got off shift and alternate shifts. Alternate policy shifts meant employees who work on the night shift should not be working in the morning shift the next day but should work on day shift or night shift.

Built optimization model: Employee timetabling problems implemented within 0-1 integer linear programming models with a decision variable is 0 (true) and is worth 1 (false). Then made 0-1 integer linear programming models consisting of objective function and constraint functions into consideration.

$$x_{ijk} = \begin{cases} 1, & \text{If the employee i work on day j shif k} \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

Based on the decision variables as in Eq. 7, the number of variables in the problem of timetabling n employees is much hmax.n.s with hmax is the maximum working day, n is the number of employees and s is the number of shifts in 1 day.

The objective function or functions of interest in n employees timetabling problems is to minimize the number of employees needed to minimize the cost of labor by the following equation:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{hmax} \sum_{k=1}^{s} x_{ijk} \quad (8)$$

Companies constraint functions that must be met are as follows: the company makes the rules for n/s employees working on each shift that is already available with n number of employees and s the number of shifts, expressed as follows:

$$\sum_{i=1}^{n} x_{ijk} \leq \frac{n}{s}, \forall j \forall k \quad (9)$$

Companies create rules for each employee in one week worked at least five or six working days symbolized by hmax. This constraint is expressed as follows:

$$\sum_{j=1}^{hmax} x_{ijk} \leq hmax, \forall i \forall k \quad (10)$$

The company makes available the regulations all employees of n, required to work every day for hmax corresponding shift provided is expressed as follows:

$$\sum_{i=1}^{n} \sum_{k=1}^{s} x_{ijk} = n, \forall j = \{1, \ldots, hmax\} \quad (11)$$

Constraints of employees contains the rights and obligations of employees that must be met are as follows: regulation of mandatory employee employees who have to work a maximum of one shift a day expressed as follows:

$$\sum_{k=1}^{s} x_{ijk} \leq \forall i \forall j = \{1, \ldots, hmax\} \quad (12)$$

Regulation of the employee in the form of each employee gets a maximum of 3 times a night shift (k = 3) is expressed as follows:

$$\sum_{j=1}^{hmax} x_{ij3} \leq 3, \forall i \quad (13)$$

There are alternate shift to night shift employees get. If officials assumed i got the night shift (k = 3) on day j,

Table 1: Simulation data 30 employee in a division that will be scheduled

Name employees	Ages	Gender	Shift off	Information
A	21	Male		Alternate shifts
B	22	Female		Alternate shifts
C	23	Male	Shift 3	
D	24	Female		Alternate shifts
E	25	Female		Alternate shifts
F	26	Male		Alternate shifts
G	27	Female		Alternate shifts
H	28	Male		Alternate shifts
I	29	Male	Shift 3	
J	21	Female		Alternate shifts
K	22	Male		Alternate shifts
L	23	Female		Alternate shifts
M	24	Female	Shift 3	
N	25	Male		Alternate shifts
O	26	Female		Alternate shifts
P	27	Male		Alternate shifts
Q	28	Male		Alternate shifts
R	29	Female		Alternate shifts
S	21	Male		Alternate shifts
T	22	Female		Alternate shifts
U	23	Female		Alternate shifts
V	24	Male		Alternate shifts
W	25	Female	Shift 3	
X	26	Male		Alternate shifts
Y	27	Male		Alternate shifts
Z	28	Female		Alternate shifts
AA	29	Male		Alternate shifts
BB	21	Female		Alternate shifts
CC	22	Female		Alternate shifts
DD	23	Male	Shift 3	Alternate shifts

then i officers are not allowed to take the morning shift (k = 1) on the day j+1. This constraint is expressed as follows:

$$x_{ij3} + x_{i(j+1)3} \leq 1 \text{ for } i \quad (14)$$

There is a shift holiday for some employees. If the employee i cannot work on a shift k_0 constraints expressed as follows:

$$\sum_{j=1}^{hmax} x_{ijk_0} = 0 \quad \text{for} \quad i \quad (15)$$

In an industrial manufacturing company, directorate/division 3 has three shifts in a day with the number of workers that companies provide for n = 30. The company has rule all employees have to work every day with a maximum working day for 6 days and every shift charged as much as n/s = 30/3 = 10 employees. Employee simulation data are given in Table 1.

Based on the timetabling criteria in the form of company regulations and the needs of employees, the decision variables that can be formed with I = {1, ..., 30} for n = 30 employees, J = {1, ..., 6} for 6 days and K = {1, 2, 3} for three shifts in a day.

$$x_{ijk} = \begin{cases} 1, & \text{If the employee } i \in I \text{ works on day } j \in J \text{ shift } k \in K \\ 0, & \text{otherwise} \end{cases} \quad (16)$$

0-1 integer linear programming models for timetabling problems n = 30 employees is as follows:

$$\begin{aligned} &\text{minimize } f(x) = \sum_{i=1}^{30}\sum_{j=1}^{6}\sum_{k=1}^{3} x_{ijk} \\ &\text{s.t } \sum_{i=1}^{30}\sum_{k=1}^{3} x_{ijk} = 30, \forall j \in J \\ &\sum_{j=1}^{6} x_{ijk} \leq 6, \forall i \in I, \forall k \in K \\ &\sum_{i=1}^{30} x_{ijk} \leq 10, \forall i \in I, \forall k \in K \\ &\sum_{k=1}^{3} x_{ijk} \leq 1, \forall i \in I, \forall j \in J \\ &\sum_{j=1}^{6} x_{ijk} \leq 3, \forall i \in I \end{aligned} \quad (17)$$

$$\begin{aligned} &x_{i13} + x_{i23} + x_{i33} + x_{i43} + x_{i53} + x_{i63} + \\ &x_{i73} = 0, \forall i \in \{3, 9, 13, 23, 30\} \\ &x_{ij3} + x_{i(j+1)1} \leq 1, \forall i \in \{1, \ldots, 30\} \backslash \{3, 9, 13, 23, 30\}, \\ &\forall j \in J \ \forall x_{ijk} \in \{0, 1\} \text{integer} \end{aligned}$$

n employees in the timetabling process simulation application is divided into four parts, namely the input of company data, employee data input section that can be done manually or automatically, the execution section which consists of making 0-1 integer linear programming models, finding solutions aided framework SCIP and the results of employee schedules that will be displayed at the employee schedule. Timetabling criteria that have been described previously in the form of company regulations inputted in the data input company regulations as in Fig. 2 and the needs of employees inputted in the data input service rules as in Fig. 3.

In the execution shown in Fig. 4 consists of Buton "Make Optimization Model" which serves to make the model 0-1 integer linear programming by reading the input data and then the application will make OPB format file with the filename "Model Penjadwalan Pegawai.opb. Buton "SCIP ILP" function to process the framework SCIP Optimization Suite to display the interactive framework which will then send the syntax of a string as a command. The results of the solution of SCIP saved in TXT format file. Buton "Schedule" function to facilitate the reading solution by displaying a schedule to read the files solusi.txt line per line then translate and store employee ID that appear on the employee schedule as shown in Fig. 5.

Fig. 2: Display data input company regulation

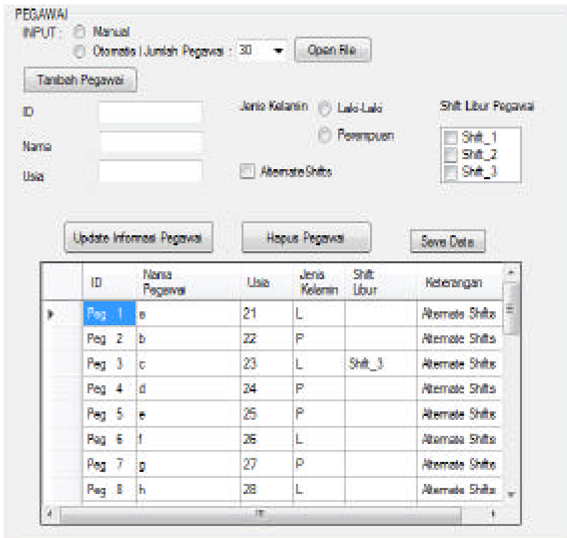

Fig. 3: Display employee data input

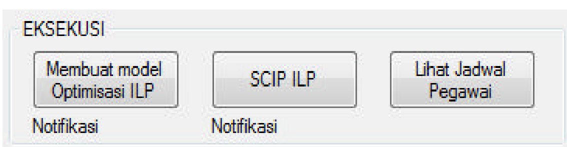

Fig. 4: Display execution

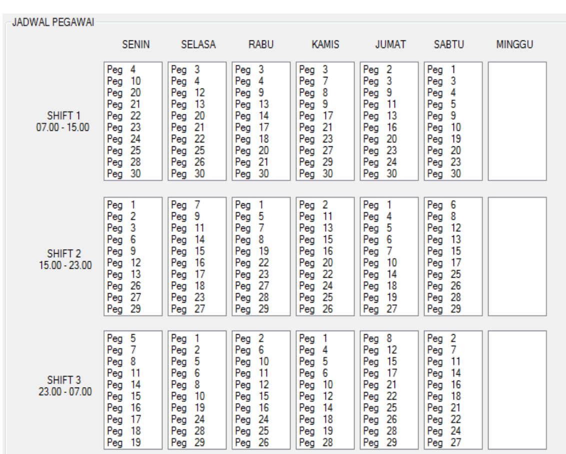

Fig. 5: Display solution timetables 30 employee

CONCLUSION

Based on the above discussion, it can be concluded that the problem of timetabling employees may be modeled into a 0-1 integer linear programming models. By using the objective function to minimize the number of employees needed to minimize labor costs and constraint functions that implement the scheduling criteria that the company regulations and the needs of employees. Employees of n were provided companies for directorate/division can be optimally assigned by assigning all employees every day so there are no idle employees. n employees timetabling simulation can be developed with Microsoft Visual Studio 2013 as an interactive tool or the user interface elements. About 0-1 integer linear programming models can be solved with the help of the framework SCIP optimization suite to get the optimal solution. n employees timetabling simulation that have been developed can be used in timetabling n employees.

REFERENCES

Achterberg, T., 2009. SCIP: Solving constraint integer programs. Math. Program. Comput., 1: 1-41.

Aloul, F.A., S.Z. Zahidi, A. Al-Farra, B. Al-Roh and B. Al-Rawi, 2013. Solving the employee timetabling problem using advanced SAT and ILP techniques. J. Comput., 8: 851-858.

Berthold, T., G. Gamrath, A.M. Gleixner, S. Heinz and T. Koch *et al.*, 2012. Solving mixed integer linear and nonlinear problems using the SCIP optimization suite. Zuse Institute Berlin, Berlin, Germany.

Haris, R., 2012. [Creating application windows phone game-based education foundation primary school in Al-Falah]. Master Thesis, Pasca Sarjana Universitas Komputer Indonesia, Bandung, Indonesia. (In Indonesian)

Hillier, and G.J. Lieberman, 2001. Introduction to Operations Reseacrh. 7th Edn., McGraw-Hill, New York, USA., ISBN: 9780071213448, Pages: 1214.

Iriani, T.A., 2009. [Optimization exam schedule in college with branch and bound method]. Proceedings of the National Seminar on Mathematics and Mathematics Education, December 5, 2009, Yogyakarta State University, Yogyakarta, Indonesia, ISBN:978-979-16353-3-2, Pages: 1162-1173 (In Indonesian).

Joseph, Y.T.L., 2000. Handbook of Schedulling: Algorithms, Models and Performance Analysis. Chapman & Hall/CRC, New York, USA.

Kridanto, A., A. Saikhu and R. Soelaiman, 2013. [Use of integer linear programming with heuristic method for scheduling optimization part time (In Indonesian)]. J. Eng. Pomits, 2: 1-5.

Mayer, W.J., 2004. Concepts of Mathematical Modeling. Dover Publications, New York, USA.

Pinedo, M.L., 2012. Scheduling: Theory, Algorithms and Systems. 4th Edn., Springer, New York, USA., ISBN:978-1-4614-1986-0, Pages: 672.

Rao, S.S., 2009. Engineering Optimization Theory and Practice. 4th Edn., John Wiley&Sons, Hoboken, New Jersey, USA., ISBN:978-0-470-18352-6, Pages: 814.

Sondita, A., 2013. [Employee scheduling simulation technique using integer linear programming]. Master Thesis, Padjadjaran University, Bandung, Indonesia. (In Indonesian).