

Assessment of Non-Linear Interpolators to Construct Digital Elevation Models

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Abstract: Digital elevation models are numerical data structures that represent spatial elevation distribution over the land surface. Characterization of complex land topographic features traditionally has been performed from the Triangular Irregular Network (TIN) interpolator developed by obeying a linear function while the geometry of nature does not. This researches ought to evaluate the quality of the interpolation of distinct non-linear algorithms through the cross-validation technique without ignoring the results of investigations that have worked profoundly on the structure (TIN) during the last 30 years with good products, like the hybrid structures between raster and break lines. The results revealed that the minimum curvature interpolator (SPL) presented more fidelity of the surface's topographic features. In the density distribution analysis of interpolation errors, it was noted that these do not fit a gaussian distribution, rather a logistics distribution.

Key words: DEM, interpolation error, cross-validation, topographic modeling, distribution

INTRODUCTION

A Digital Elevation Model (DEM) is a data numerical structure that represents the spatial distribution of a terrain's surface elevation (Felicisimo, 1994). The DEM have been used broadly to model, analyze and show phenomena related to topography and other surfaces (Ai and Li, 2010). This type of land modeling and its derived products have had an unquestioned leading role in the geo-information environment in the last 25 years with diverse purposes: Civil engineering and infrastructure (Petrie and Kennie, 1987), planning in management of natural resources (Fisher, 1996) modeling of potential erosion processes (Mitasova *et al.*, 1996; Ren *et al.*, 2011) hydrologic modeling (Jana *et al.*, 2007) construction of multivariate predictive models of sites of archeological potential (Vaughn and Crawford, 2009), military engineering (Fleming *et al.*, 2009; Maio *et al.*, 2013), detection of geomorphological changes (James *et al.*, 2012) and climate impact studies (Yan Hong *et al.*, 2005; Marques *et al.*, 2013).

Due to their structure, DEM permit storage and analysis without having to research directly on the real surface. The majority of GIS users do not keep in mind the complexity of the phenomenon captured, obviating fundamental elements, like break lines, cutoff zones, dimensional points and most importantly the interpolation algorithm (Interpolation is the mechanism that permits estimating elevation in zones where altimetry data has not

been captured. It is based on the principle of spatial self-correlation which measures the degree of relation or dependence between near and distant elevations in terms of surface representation, the raster-type structure is considered a functional surface, given that for any position x, y only its z value is stored). Such is selected more because of presentation in terms of its visual appearance than because of reasons obeying to the relation existing between the geometry of the shape to be represented and its interpolator (Peucker *et al.*, 1977).

This complexity poses a challenge to automated DEM techniques. The problem raised is that of wanting to compare the earth's skin that behaves as a continuous function of infinite points and which in terms of modeling will be defined incompletely-with another that comes from discrete intervals (finite sampling points) that will be generalized and consequently, cannot be determined exactly. This generalization produces loss of information which affects DEM error and that will be transmitted to those derived products that can be applied at local scale.

The principal motive for these differences (reality vs. model) lies on the subjective assessment of the DEM this model is constructed with spatial data that due to its nature has errors intrinsic to the measurements and to its propagation in the very model which should be treated through geo-statistic procedures, quantifying their error and seeking to dimension their uncertainty.

According to Goodchild *et al.* (1994) and Hunter *et al.* (1995) calculation of the total error in DEM cannot be quantified because it is impossible to determine the true value for each topographic accident or phenomenon represented in a set of geographic data. Among the factors affecting DEM precision, the most penetrating are sampling density, data distribution, the interpolation algorithm and finally, its spatial resolution (Ley, 1986; Li, 1990; Li *et al.*, 2005; Fisher and Tate, 2006). A DEM's precision may be defined as the mean of the vertical errors of all potentially interpolated points. This is achieved by calculating the Root-Mean-Square Error (RMSE) that measures the dispersion of the frequency distribution of the deviations between original elevation data and DEM data (Ackermann, 1996; Weng, 2002). Kumlér (1994) developed a methodology to study the cause of error in TIN structures through distinct methods of selecting vertices (Gao, 1997) studied the resolution and precision of the terrain's representation through a micro-scale regular grid (Rees, 2000) studied the precision of DEM interpolated at high resolutions and demonstrated that the simple bilinear interpolation produce results appropriate for DEM applications. Li and Zhu (2000) systematically discussed the DEM theory, especially in the precision of the analysis of the models and deduced their precision based on raster structures. Kidner (2003) argued that the higher-order interpolation techniques were always more precise than those generated by the bilinear interpolation. Marquez (2004) sustained that in the representation of surfaces, information will always be imprecise due to the modeling's own simplification and hence, it is necessary to know and control error uncertainty to determine the reliability of the results obtained. Deng *et al.* (2007) documented error variation in a DEM in terms of the dependence of the raster resolution in function of the analysis of the landforms.

The aim of this study was to search for the relation between the surface geometry modelled through a DEM and the errors derived when using distinct interpolation techniques in terms of the sensitivity of the terrain's variations. For this, six interpolation techniques were applied (Inverse Distance Weighted (IDW), Kriging (KRG), Natural Neighbor (NN), Spline (SPL), Topo to Raster (T2R) and Triangular Irregular Networks (TIN)) commonly used modeling of topographic surfaces (Bater and Coops, 2009; Achilleos, 2011; Chen *et al.*, 2013). Interpolator performance was evaluated through an ASTER GDEM image with 30 m spatial resolution, 541 rows by 541 columns which is approximately equivalent to 300.000 data items. The study area is located in Colombia between the departments of Cauca and Narino with latitudes 01° and 02°N and longitudes 77° and 78°W, covering nearly 263 km². This zone is characterized for

having elevations from 500-1600 m.a.m.s.l with great hydric wealth, besides housing xerophyte vegetation and ecosystems rich in flora and fauna declared biosphere reserve by UNESCO.

Besides the impact of the interpolation techniques on the topographic modeling, this study suggests a strong correlation between DEM error and the morphometric parameters: slope and curvatures.

MATERIALS AND METHODS

Methodological approach: The methodology proposed derived from a technique known as crossed validation. It has been used in environments of topographic modeling by diverse researchers (Rees, 2000; Kidner, 2003; Hancock and Hutchinson, 2003; Wise, 2011) the technique consists in eliminating raster (temporarily) elevation values, executing the interpolation algorithm and estimating the values interpolated in the temporary withdrawal positions. The error was calculated by comparing the value estimated to the real value. This simple but recursive, technique produces altimetry information derived at distinct resolutions from a same data source which will permit identifying the difference of heights in a same position thus calculating the interpolation error and its incidence on the representation of topographic features through the interpolation techniques evaluated. This approach sought to re-sample a DEM from the ASTER sensor with 30 m spatial resolution to a point such that the data re-sampled at low density permit detecting the error trend in the models interpolated. This study considered as reference DEM information from the ASTER sensor. For this reason, the results of the six interpolation models evaluated were compared against said DEM by selecting a study area with nearly 300,000 points which permitted the DEM to be re-sampled by powers of three to conserve a reasonable data set with which to interpolate.

The object of the methodology proposed was to implement a model that permits determining which of the algorithms evaluated best represents the surface without compromising the altimetry quality of its representation.

The first step consisted in extracting the elevations of the DEM from the ASTER sensor image. There after the re-sampling procedure was applied to determine new values of the raster's cells in function of the levels indicated in Table 1 (Kidner, 2003). Then, the

Table 1: Re-sampling errors and resolutions to create the DEM under assessment

| Re-sampling | Size of cell (m) | No. of data |
|-------------|------------------|-------------|
| 0 | 30×30 | 298.681 |
| 1 | 60×60 | 73.442 |
| 2 | 90×90 | 32.761 |
| 3 | 120×120 | 18.496 |
| 4 | 150×150 | 11.881 |

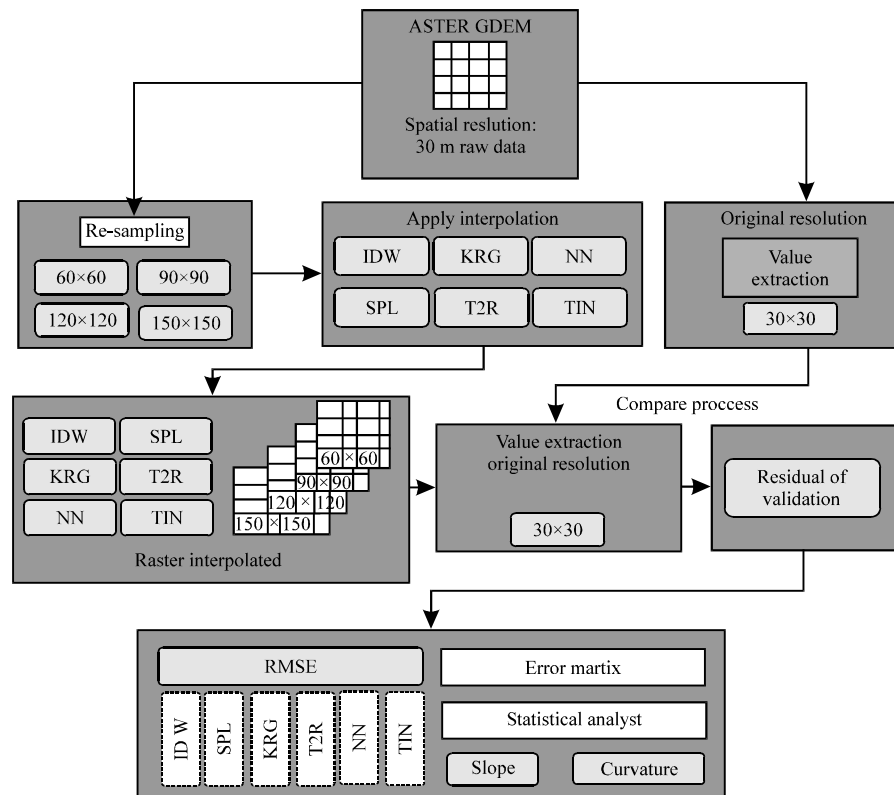


Fig. 1: Methodological flow

interpolation algorithms evaluated (IDW), (KRG), (NN) (SPL), (T2R) and (TIN) were applied, obtaining for each interpolator six results of the surface with the resolutions indicated in Table 1. To obtain the residual elevation value, altimetry information was extracted from the surfaces interpolated at the original resolution (30x30) which permitted calculating RMSE through the difference between the original value and the interpolated value. A multiple-factor. Analysis of Variance (ANOVA) was performed, defining the RMSE as dependent variable and the four re-sampling errors and the six interpolation algorithms as independent variables (Fig. 1).

Interpolation techniques: This study selected interpolation techniques with local neighborhood or geo-statistic approaches, like Inverse Distance Weighted (IDW), Kriging (KRG), Natural Neighbors (NN), Minimum Curvature (SPL), Topo to Raster (T2R) and Triangular Irregular Network (TIN). All these methods are based on the principle of spatial self-correlation which measures the degree of dependence between near and distant elevations to predict the altimetry position of a point in a position not measured. Inverse Distance Weighted (IDW) interpolation is based on the assumption that the value of a point not sampled can be approached as a weighted mean of the close values sampled and that each point

influences on the resulting surface up to a finite distance (Mitas and Mitasova, 1999). This technique suggests that the result predicted reduces its incidence on the measurement that increases the separation between the point to evaluate and the points from its environment (Burrough *et al.*, 2015) there by the points closest to the centroid have higher weight in the calculation of elevation (KRG) is a geo-statistic interpolation technique that makes the prediction in function of three big processes: structural calculation, calculation of the regionalized variable and determination of the residual error. This model seeks to minimize the variance of error and bring the error mean of the values estimated to zero to avoid overestimations or underestimations.

The (NN) is an interpolation technique developed by Sibson (1980) who indicates that the local natural regions generated around each point are used to choose and weigh the elevation of neighboring points. This interpolation technique has the advantage of not requiring any additional parameter and the results are acceptable in sampling points with an irregular distribution. However, according to Morillo *et al.* (2002) this eventual ease for the process may be an inconvenience if we bear in mind the scarce number of decision parameters used by the technique.

According to Burrough *et al.* (2015), (SPL) adjusts the function to a limited amount of points, generating a line passing exactly through the original samples, ensuring continuity in the junction of the distinct curves. To interpolate surfaces, a special type of spline is used called minimum curvature which passes exactly through the sampling points, managing to adjust the terrain to the shape of an elastic membrane. This method is recommended when surface changes have slight variations. However, it is not appropriate if many changes exist in very short horizontal distances because the estimated values may be exceeded, introducing anomalies not found in the original surface (Lam, 1983).

The (T2R) is a finite iterative differential interpolation technique. It has been optimized to achieve the computational efficiency of local interpolation methods, like IDW without losing surface continuity of global interpolation methods, like KRG and SPL. It is essentially a fine discretization technique in which roughness is modified to allow the DEM adjustment process to follow abrupt changes on the terrain, like water collector and divider lines (Hutchinson, 2008).

The (TIN) is supported on the creation of a network conformed by irregular adjacent triangles whose vertices define the terrain adjusting to a plain at three near non-collinear points (Zeiler, 1999). Several algorithms exist to systematize the procedure which select the best way of triangulating; the best known is that by Delaunay supported on the Thiessen polygons (Ali and Mehrabian, 2009). This condition mentions that the circumradius of each triangle of the network should not contain any vertex from another triangle, indicating that the points are connected with their two nearest neighbors. This interpolator does not require the statistic continuity of the surface to model; its results can be improved through the integration of distinct types of structural lines that permit the definition of break lines, collection or division of water, slope demarcation or communication paths. In spite of those benefits, there is also a big disadvantage: the topographic features do not have linear simplicity; rather they have a large amount of complex and dynamic irregularities, like concave, convex or mixed curvatures that are not described through surfaces produced with linear interpolators.

Morphometric parameters: The complexity of the topographic features can be described and characterized through diverse numerical parameters derived from the neighborhood or adjacency analysis in a DEM; among the most used there are slope, curvature, roughness, orientation and fractal dimension. This study will only use the first two descriptors mentioned. The slope is

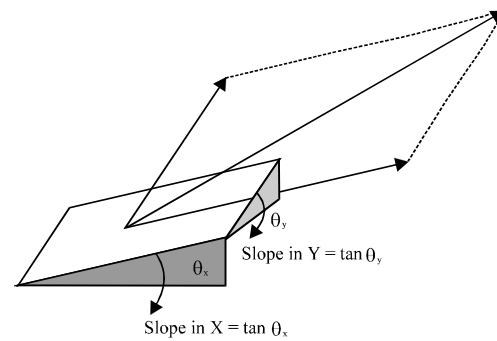


Fig. 2: Angular coefficient of trigonometric tangent (Gorokhovich and Voustianiouk, 2006)

| | | Vertical curvature | | |
|----------------------|-------------|--------------------|-------------|---------|
| | | Convex | Vert. plane | Concave |
| Tangential curvature | Convex | | | |
| | Tang. plane | | | |
| | Concave | | | |

Fig. 3: Geometric assignment of curvatures (Temme *et al.*, 2009)

Table 2: Ranges of slope

| Type of terrain | Slope (°) | Slope (%) |
|-----------------|-----------|-----------|
| Plain | <2 | <4.4 |
| Undulated | 2-6 | 4.4-13.3 |
| Hill | 6-25 | 13.3-55.6 |
| Mountainous | >25 | >55.6 |

a topographic descriptor par excellence used widely in topography and cartography (Wilson and Gallant, 2000) define it as an indicator that measures the rate of change of elevation in the most hill descending direction. To use the slope in terms of the terrain description (Ley, 1986) suggested the following ranges.

The curvature is the first derivate of the slope that is the second derivate of the terrain. It describes the rate of change of the relief in terms of convexity, concavity or flat surface in each cell in the direction of the slope and its transversal direction (Fig. 2). This study associated the RMSE values of each cell to their corresponding slope values indicated in Table 2 and curvature values shown in Fig. 3, to try to elucidate the relations existing between the error generated by the different interpolators at

distinct resolutions and the representation of the topographic features associated to morphometric parameters.

RESULTS

Statistic modeling: Upon calculating the RMSE values for each of the re-samplings through the methodology posed, their statistics were obtained associated to the interpolators proposed (Table 3).

Given that the value of probability in ANOVA was <0.01 , a statistically significant relation exists among RMSE, the re-sampling order and the interpolators at 99% CI. To contrast the hypothesis of independent variables, their regions of criterion were established by calculating snedecor's F statistic distribution in both cases, the F ratio was found on the right side of the statistic, confirming that the null hypothesis is in the rejection zone.

According to Fig. 4, the (SPL) algorithm is closest to the lower RMSE; in contrast (IDW) had the highest error in all the re-sampling errors. It was also possible to infer that no statistically significant difference exists between the (T2R) and (KRG) algorithms, given that the uncertainty value in (KRG) was contained in the range of error of the (T2R) interpolator. The same occurred between (TIN) and (NN) interpolators, although these presented lower altimetry deviations than the two previous interpolators.

Much of the ANOVA potential is the capacity to estimate and test the effects of interaction between the independent variables evaluated (re-sampling) (Fig. 4). Much of the ANOVA potential is the capacity to estimate and test the interaction effects between the independent variables assessed (re-sampling and interpolation algorithms). This analysis tries to determine if interaction existed between both factors, representing the fact that

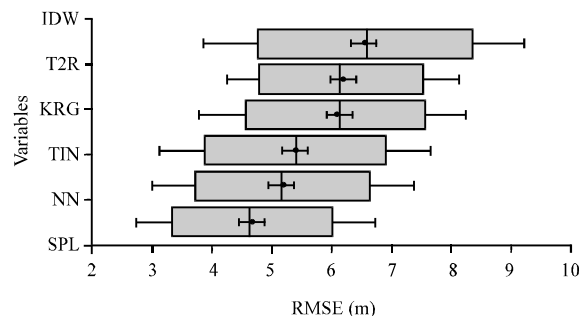


Fig. 4: Box-cox graph and means for interpolation algorithms, 95% Fisher LSD

the RMSE at a factor level were different for each level of the other factor. Figure 5 shows that RMSE did not provide a complete description of the data; usually, the parallel lines indicate little or no interaction among the factors. This means that as re-sampling resolution increased so did its mean errors (as expected) likewise, it is inferred that independent of the re-sampling resolution all the interpolation algorithms had significant errors among them. Due to this, the minimum curvature interpolator (SPL) was identified as that with the highest fidelity of the surface's topographic features. In all cases, the interpolators had lower error as the raster resolution became finer.

A map of absolute errors was created associated to the scale of slopes show in Table 1. It was identified that errors 1.28 m were found in zones classified as plain and undulated and errors of greater magnitude (values fluctuating between 1.28 and 3.48 m) were found in zones of hill and mountainous slopes (Fig. 6 and 7). For the curvatures, the error's spatial pattern presented similar behaviors: overestimation was produced in convex zones

Table 3: Summary of elevation error statistics

| Re-sampling | Interp. | RMSE | Media | Min. | Max. |
|-------------|---------|-------|--------|----------|--------|
| 150×150 | IDW | 9.392 | 0.026 | -68.991 | 51.174 |
| | T2R | 4.147 | -1.082 | -154.970 | 43.586 |
| | KRG | 8.372 | 0.031 | -60.125 | 42.275 |
| | TIN | 7.743 | 0.067 | -55.005 | 55.408 |
| | NN | 7.452 | 0.032 | -52.401 | 43.327 |
| | SPL | 6.765 | 0.008 | -48.161 | 41.413 |
| 120×120 | IDW | 7.564 | 0.003 | -44.262 | 42.863 |
| | T2R | 6.981 | -0.941 | -148.157 | 34.856 |
| | KRG | 6.924 | 0.014 | -39.541 | 38.873 |
| | TIN | 6.151 | 0.023 | -39.670 | 38.335 |
| | NN | 5.884 | 0.006 | -35.384 | 37.631 |
| | SPL | 5.263 | -0.027 | -38.651 | 39.192 |
| 90×90 | IDW | 5.667 | 0.023 | -36.701 | 35.916 |
| | T2R | 5.290 | -0.675 | -106.581 | 29.797 |
| | KRG | 5.354 | 0.032 | -34.352 | 32.126 |
| | TIN | 4.567 | 0.005 | -46.501 | 43.503 |
| | NN | 4.364 | 0.027 | -32.891 | 31.005 |
| | SPL | 3.822 | 0.016 | -31.374 | 32.790 |
| 60×60 | IDW | 3.731 | 0.012 | -29.364 | 25.466 |
| | T2R | 4.140 | -0.421 | -138.674 | 29.988 |
| | KRG | 3.648 | 0.027 | -30.281 | 26.453 |
| | TIN | 2.961 | 0.023 | -30.505 | 29.004 |
| | NN | 2.833 | 0.027 | -27.754 | 29.253 |
| | SPL | 2.552 | 0.011 | -32.710 | 38.107 |

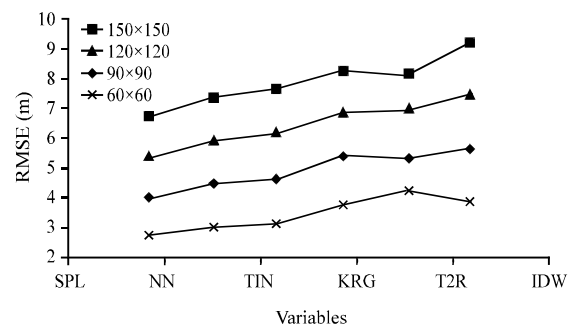


Fig. 5: Interaction among factors

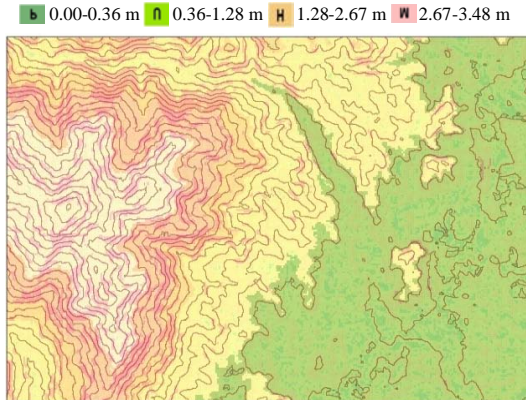


Fig. 6: Map of absolute errors for the SPL interpolator at 30 m resolution

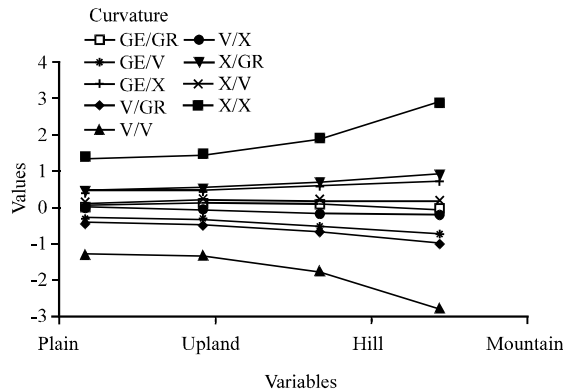


Fig. 7: Interaction among error, curvatures and slopes. SPL interpolator resolution: 300 m

of tangential/vertical combination (X/X) hence, most errors occurred along the water dividing lines. Underestimation (or errors with negative sign) occurred in concave areas of tangential/vertical combination (V/V).

Convex/concave (X/V) plain/plain (GE/GR) and concave/convex (V/X) curvatures had errors close to zero without showing sensitive changes among the four types of slope. The opposite occurred with plain curvatures (X/X, V/V) given that as slopes increased so did their error.

Finally, to characterize the error, the magnitudes from Table 3 were modelled under a gaussian distribution. Chi-squared, Kolmogorov-Smirnov and normality goodness of fit tests, performed in all cases, showed they were significantly different from said distribution with probability values for the test 0.05 which rejects the idea that error behavior presents a normal distribution. Differences in the distributions found through the distinct methods interpolated were very slight

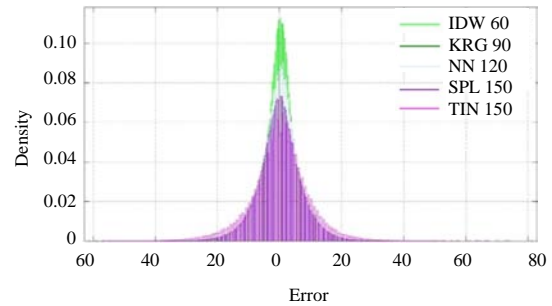


Fig. 8: Density distribution of errors in interpolators

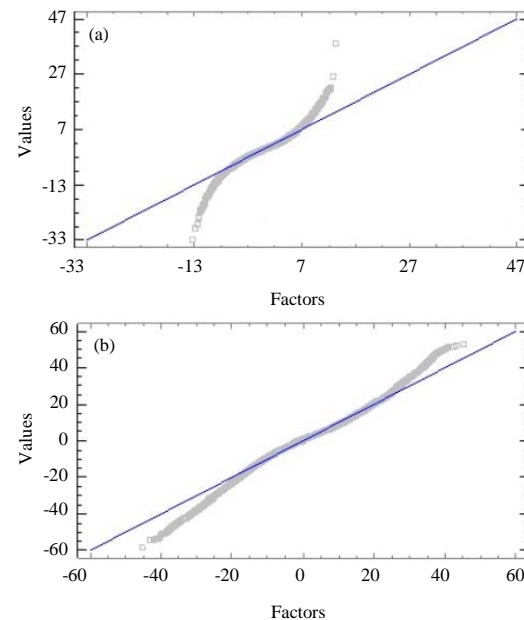


Fig. 9: Quantile-quantile graph for error produced in: a) 60x60 SPL re-sampling and b) 150x150 KRG re-sampling

compared to those produced by the different re-sampling errors as evidenced in Fig. 8 where the only visible divergence visible between the graphs are the heights of the curve peaks (Fig. 8).

However, an analysis of the quantile-quantile graphs (Fig. 9) and the distribution statistics showed interesting results. Interpretation of the results at first sight, suggests a normal distribution but detailed observation shows long tails and high kurtosis values which keeps the data from adjusting to a gaussian distribution instead of a logistic distribution. Figure 9 shows two quantile quantile graphs, revealing the non-extreme normality: when re-sampling is low and extreme or closest to the normal: when re-sampling is high. When modeling analysis, the

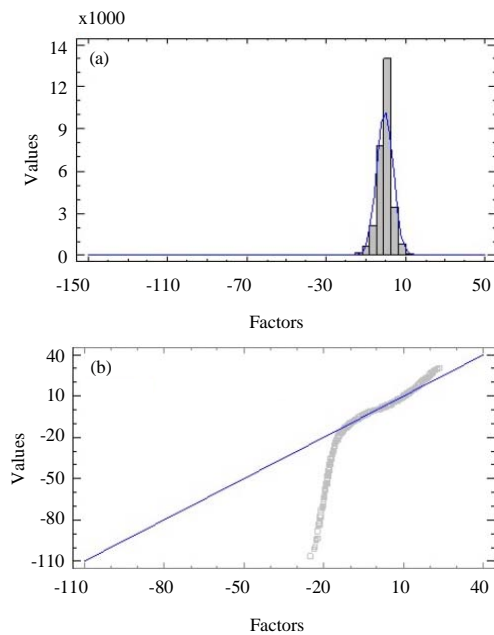


Fig. 10: T2R algorithm errors: a) frequency distributions and b) quantile-quantile graph

(T2R) interpolator had atypical data with respect to the other interpolation algorithms evaluated. For all re-sampling errors, this algorithm in its distribution elevation error, always assume it is spatially self correlated; this is a reasonable presumption given that the terrain tends to have the slight variations in short distances so that neighboring pixels probably have similar height values and because the interpolation in itself has the probability of producing similar values for the closest points. Kurtosis values (Table 4) indicated that the distributions became much less leptokurtic as the re-sampling level increased that is it tried to normalize as data density diminished. As a special case within this presented a much longer tail toward the left side than toward the right which indicates that the model overestimates in most cases. Toward the right side that is overestimation errors behaved as data quite close to the normal distribution (Fig. 10).

Finally, seeking to explore the error's spatial self correlation, moran's index was calculated from the four orthogonal neighbors of each pixel (Table 5). Values for the highest re-sampling levels were quite high which indicates strong degree of spatial self-correlation in the error pattern.

Fidelity of the interpolation techniques in representing land surface: Representation of topographic features of a surface through a DEM will always contemplate absolute errors given that it is incapable of totally

Table 4: Standardized Kurtosis for the distribution of elevation errors

| Re-sampling | Interpolator | Kurtosis |
|-------------|--------------|-----------|
| 150×150 | IDW | 179.1440 |
| | T2R | 1067.1930 |
| | KRG | 125.3130 |
| | TIN | 218.8660 |
| | NN | 155.2210 |
| 120×120 | SPL | 146.9430 |
| | IDW | 145.1930 |
| | T2R | 2759.8060 |
| | KRG | 103.9070 |
| | TIN | 181.9210 |
| 90×90 | NN | 131.3130 |
| | SPL | 141.1620 |
| | IDW | 115.1640 |
| | T2R | 1460.4730 |
| | KRG | 91.4350 |
| 60×60 | TIN | 157.1420 |
| | NN | 122.1150 |
| | SPL | 159.3810 |
| | IDW | 99.0371 |
| | T2R | 8180.6370 |
| | KRG | 92.2430 |
| | TIN | 159.0120 |
| | NN | 122.1220 |
| | SPL | 258.6430 |

Table 5: Spatial self-correlation. Moran's I

| Re-sampling | Interpolator | Moran's I |
|-------------|--------------|-----------|
| 150×150 | IDW | 0.953 |
| | T2R | 0.954 |
| | KRG | 0.966 |
| | TIN | 0.971 |
| | NN | 0.965 |
| 120×120 | SPL | 0.953 |
| | IDW | 0.912 |
| | T2R | 0.913 |
| | KRG | 0.951 |
| | TIN | 0.923 |
| 90×90 | NN | 0.941 |
| | SPL | 0.940 |
| | IDW | 0.721 |
| | T2R | 0.743 |
| | KRG | 0.691 |
| 60×60 | TIN | 0.712 |
| | NN | 0.651 |
| | SPL | 0.672 |
| | IDW | 0.608 |
| | T2R | 0.397 |
| | KRG | 0.482 |
| | TIN | 0.396 |
| | NN | 0.415 |
| | SPL | 0.372 |

representing the surface of the terrain in its true form. This model will never be defined absolutely for this, it would need to contain much information, like the geometry of the surface sought to represent which is inconceivable given that it is a continuous surface with infinite variations. In any case, these errors caused by modeling also tend to being self-correlated and the combination of these factors lead to moran's index values approaching their maximum value: 1. However, in the lowest re-sampling errors, when the DEM was generated from many points, the spatial self-correlation in the error diminished and became quite changing among the interpolation methods. This

suggests that in some cases the simple calculation of a single self-correlation error can truly be a dangerous indicator because it may be judged that the error does not have significant correspondence on the surface studied.

The (KRG) interpolator produced acceptable results when sufficient data was available to estimate the semi-variogram because noise is treated as part of the signal. In any case, interpolation precision through this method was better than that presented by (IDW) although upon modeling the surface it had a tendency to generate concentric patterns around the original points, making it the study's interpolator with greatest RMSE. This is argued through the weight given to the particular variation of the value of a sampling point over those around it.

The (TIN) interpolator produced an unsmoothed surface which caused discontinuous slopes on the edges of the triangulation. This algorithm only considers the spatial distribution of the original points and not the shape of the surface generated and accommodates the creation of triangles that alter the geometry of the expected surface. This is because it is a linear interpolator and the forms of the land surface, generally do not correspond to linear models; however (TIN) has been the interpolator par excellence to represent topographic forms, given that it is considered a local and exact method based on Delaunay's triangulation. According to the relation between contrasts no significant difference existed between (TIN) and (NN) which although operating similarly to the (IDW) method uses a local adjustment that reduces the effect of concentric circles, given its smoothing parameter in addition to bearing in mind the anisotropy granting different weights along the search axes.

The (T2R) was characterized for having excessive smoothing of the surface as well as omitting a great deal of its detail which led to loss of planimetric precision in the disposition of the forms that do not correspond to drainage networks. The behavior of underestimations with this interpolator was because the algorithm has as principal function continuity in the hydrologic network and it is precisely in zones of greater depression where the error had its maximum magnitudes.

The (SPL) technique adjusted to the surface through the original points, preserving surface tendency and making the best fit in all degrees of variability of the topographic gradient.

DISCUSSION

The DEM's altimetry representation error, caused by the interpolators evaluated, showed a strong link with the slope; it increased as its degree of variability increased

which coincides with results reported by Tahmassebpour (2016) who after analyzing distinct interpolation models in zones of different relief concluded that the altimetry error increased as the topographic gradient increased. According to the results obtained, the pattern of elevation errors showed a clear bond between the geometry of the terrain, slope and curvature, exposing under estimation for concave curvatures and overestimation for convex curvatures. It was also proven that a high spatial self correlation index exists as the raster resolution became less dense. The differences of the RMSE achieved by the interpolators evaluated indicated that (SPL) has the most stable variation pattern in all the re-sampling levels (Fig. 5). It retained topographic features that none of the other interpolators could maintain. This coincided with findings by who holds that methods based on spline best fit the sampling points to the representation of the surface they model given that they expose higher self-correlation than those algorithms that base their estimation in function of the closeness of points. This also agrees with the results obtained by the (IDW) interpolator that displayed the greatest errors induced by the interpolation.

The convex/concave (X/V) plain/plain (GE/GR) and concave/convex (V/X) curvatures indicated low level error sensitivity which implies that in studies of flow acceleration, erosion, deposit and transit of materials this descriptor has a high degree of fidelity without being affected by the topographic gradient.

In the DEM field, it is usual to assume that errors in spatial data are normally distributed (Li *et al.*, 2005; Goovaerts, 1997). Many researchers, Temme *et al.* (2009), Jeffrey *et al.* (2001) and Atafar *et al.* (2013) have modelled their propagation under Monte Carlo simulations where the error is assumed under a gaussian behavior there in deducing the characteristics of its propagation. However, findings from this study (Table 4) report that errors have high kurtosis values, implying greater variance caused by infrequent deviations in the long tails of the distribution, notoriously removing it from its normal behavior.

In many cases, the value of data re-sampled at 150×150 was not far from the zero value of a gaussian distribution this suggests that with diminished number of re-sampling points, the behavior of the errors tends to normalize. According to Chiles and Delfiner (2012) in spite of the wide use of crossed validation to evaluate interpolation algorithms we must be aware of their limitations, especially we tend to overestimate the interpolation error because prediction is calculated in places where the data are available, hence, estimation of the crossed validation can be altered by eliminating the validated point. In practice, these problems are inevitable but upon increasing the number of input points, there is lesser impact on the result.

CONCLUSION

In conclusion, from the arguments exposed, their discussion and the literature cited, the DEM produced have high degree of sensitivity to the interpolation method used. The (SPL) technique functioned better in comparison to the others evaluated here, having the best fit in the different degrees of sensitivity of the topographic gradient. The low level of error sensitivity of the combination of vertical/tangential curvatures: X/V, GE/GR, V/X was not associated to increased errors of the slope.

The election of the interpolation algorithm in the construction of a DEM must be linked strongly to the final use for which it is destined. Likewise, this selection requires knowledge of the type of structure that stores the altimetry information, the spatial resolution, topographic descriptors of the area to be modelled, tolerance in the magnitude of the final representation error and finally the way to measure this error. Land surface is continuous and a DEM is a set of discrete measures. The fidelity with which a DEM models the true surface is related to morphometric parameters, like slope and curvature, besides the relative error of the DEM's data source given that there will always be details revealed in a finer scale than those measured in the DEM's original resolution.

To round off the idea of the affectation occurring, selection of the interpolation algorithm in surface representation shows that an ideal interpolation method does not exist for all purposes. This is true because each was created to estimate information that best characterizes the phenomenon limited to assumptions inherent in: the design of the algorithm, type of surface, its representation structure and tolerance of expected errors.

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