

Formulation and Evaluation the Efficiency of a Heuristics Method for Solving the Vehicle Routing Problems

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Abstract: In this study, a heuristic algorithm called Arya method has been proposed for solving classic problems of VRP and it is based on the selection of the nearest neighbor. This innovative method has been developed and then compared to another well-known method of solving routing classic problems which is proposed by Clarke and Wright. These two methods are firstly coded by a programmer and then 295 randomly generated problems have been solved by these algorithm and the time and quality of their solutions were compared with each other.

Key words: VRP, heuristic algorithm, efficacy evaluation, Arya method, the Clarke and Wright method

INTRODUCTION

Solving nonlinear programming problems has always had its own complexities. When dimensions of the problems goes higher than a certain threshold, its manual solving becomes very time consuming; by increasing the size of the problem, it will be almost impossible to solve it manually. To fix this issue in recent decades, a wide range of software packages have been provided for solving these problems. The developed algorithms can be classified into two main categories of deterministic and probabilistic. The method is classified in the group of deterministic problems. On the other hand, the routing problems are classified as NP-hard problems and in such problems, finding the optimal solution will be difficult by increasing the dimensions of the problem. For this reason, heuristic and meta-heuristic methods are often utilized when solving routing problems.

Heuristic methods themselves are divided into two categories of generator and improver methods (Eshghi and Karimi, 2012). In this study, the first method which has been proposed by the author (Arya method) is of generator type and the second method which is proposed by Clarke and Wright method is of both generator and improver types; these two methods have been compared in the present study. The generator methods are very important. Since many of the exact, heuristics and Meta-heuristics methods requires an initial acceptable solution; thus, the generator methods which can provide acceptable and near-optimal answer in minimum time are attractive for programmers and researchers.

Problem statement: Routing issues are one of the issues raised in the field of operations research and distribution in supply chains that many researchers have studied in this context. This is important since the conducted surveys show that using the routing and optimization techniques has caused 5-20% saving in the cost of transportation of distribution systems. In general, there are always one or several service providers and service receivers in the routing problems that the service providers should provide service to clients in such a way that each service receiver is met once by a the service providers and also service providers return to their primary station at the end of his mission. Naturally, moving between service providers and service receivers has some costs and in optimization of these issues, we are mainly looking to reduce costs and minimize it. In general, in different types of routing problems, the following assumptions are usually considered (Reed *et al.*, 2014):

- Each of the service receiver (customers) is located on a network and there are perfect connections between customers
- The demands of each service receiver are defined in network nodes and demand will be fulfilled by the service provider
- A certain time period can be considered to provide a service which in this case, the problem will be more complicated
- Several service providers may be available to meet the demands of clients

According to the different definitions about the type of application, type of service providers and the other parameters of the problem, we will be faced with a variety of routing problems. For example, one of the different types of routing problems is the Capacitated Vehicle Routing Problems (CVRP). In this kind of problems, notable demand is generated by the service receivers and in contrast, the capacity of service providers is also limited (Reed *et al.*, 2014). Therefore, in these problems, the goal is to find the best route which in addition to covering all clients, looking minimization of the total handling costs of the transportation vehicles and the demands has been met in each route is not more than the capacity of each transportation vehicles (service provider). Another type of routing issues faced with the fact that each client must meet his/her demands in a certain time period. This sort of problems is called Vehicle Routing Problem with Time Windows (VRPTW). However, several problems in this area can be defined and problems can be developed in terms of adding constraints, the type of objective function etc. But in this study, the easiest type of routing problem is considered and the main features of these problems are as follows:

- There is one service center
- Customer demand is certain and is insignificant in terms of the volume
- Service receivers are located on the network nodes
- Service providers have no capacity constraints
- There is no time limit to provide services
- The number of service providers is more than one and assumed to be equal to m
- The number of service receivers is assumed to be equal to n
- Not much time spent to provide service to clients and it is assumed to be almost zero
- Access routes between clients and service center are perfect
- The defined routes are two-way and both the track and the track back has equal costs
- There is no priority between service providers their costs are equal

According to the terms mentioned above, routing problem, try to find a solution that firstly, all clients are to be covered and secondly, to minimize the total cost of the service.

Problem formulation: As previously mentioned, our studied problem is one of the operations research problems and a sub-branch of the integer models. The

following symbols will be used in this model: d_{ij} = the cost of movement between two clients of i and j and $X_{ijk} = \{1$ if the distance i to j is assigned to the server K , 0 in other mode.

According to these symbols, the objective function will be defined as follows:

$$\text{Min } Z = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^m d_{ij} \times X_{ijk}$$

This relationship suggests that the problem is seeking the routes that have the lowest amounts and on the other hand, satisfy the other constraints of the problem. The main constraints of the problem are provided below: the constraint which ensures that every service receiver will be met only by a service provider:

$$\sum_{j=0}^n \sum_{k=1}^m X_{ijk} + X_{jik} = 2 \quad i = 1, 2, 3, \dots, n \quad i \neq j$$

Constraints about the absence of a node in the two different paths:

$$\sum_{k=1}^m X_{ijk} \leq 1 \quad j, i = 1, 2, 3, \dots, n \quad i \neq j$$

$$X_{ijk} \sum_{p=0}^n \sum_{i=1}^n X_{ipk} \quad k \neq 1 \quad i = 1, 2, \dots, n$$

$$j = 1, 2, \dots, n, \quad k = 1, 2, \dots, m$$

All clients must be met by service providers:

$$\sum_{i=0}^n \sum_{j=0}^n X_{ijk} = n + m \quad i \neq j$$

The constraint stating that all service providers must return to service centers after finishing their tasks:

$$\sum_{j=0}^n \sum_{k=0}^m X_{ojk} + X_{iok} = 2m$$

As you can see, the problem is a zero and one programming problem and it significantly expands by increasing the number of clients and the number of service providers and it will be unsolvable in very large scales.

The brief review of Literature: Various methods have been proposed to resolve these kinds of problems. As stated previously, the methods of solving these types of

problems are divided into two general categories. The exact methods that use mathematical formulas and methods to provide accurately the optimum solution and the approximate or heuristic methods that have higher speed of solving problems than the exact methods in case of the complexity of problems in higher dimensions and they do not necessarily provide the optimal production.

One of the methods in this context is the routing problem of traveling salesman problem and its solution and again extracting routes from the answers obtained from solving the traveling salesman problem (Laporte *et al.*, 1986; Laporte, 1992). Another method for these problems is the central tree algorithm, in which the exact solution can be obtained by spending considerable time for categorizing bows and providing Mathematical Model. For each of these categories (Christofides *et al.*, 1991). The dynamic programming is also another method to solve routing problem. In one of these methods, the M-TSP4 problem is solved using dynamic programming (Eilon *et al.*, 1971). These methods are time-consuming and they are not applicable in large-scaled problems with the high number of service providers.

The variety of methods is also provided in the second category and given that many of the real problems trying to find a "feasible solution", these methods are more taken into account because of their high speed in solving problem. Gondrao proposed a heuristic method to solve a particular routing problem taking into account the capacity constraints (Gendreau *et al.*, 1994). The graphical methods are also used to solve these problems; the Petal method is one of them which has been developed by Ryan *et al.* (1993). The paths in this method are elected in such a way that ultimately, there is no service place or client in the routes. Two other researcher in this area also proposed a heuristic method based locating methods that can be used in the debates about locating equipment (Bramel and Simchi-Levi, 1996). Another method that is fairly old is a heuristics method based on improving routes and it is known as cost saving or storage methods. It actually started from a feasible solution and gradually considers the problem constraints (Clarke and Wright, 1964). Some meta-heuristic methods are also used in this area in recent years. Chang and Chen (2007) have used genetic algorithms to solve the kind of routing problems.

But since the method developed in this paper is a heuristic method, so it has been compared with Clarke and Wright method which is also a heuristic method.

MATERIALS AND METHODS

Introduction of Arya method: As mentioned above, Arya method is a generator method which is developed to solve

routing problems. In this method, the branches generated are two times of transportation vehicles. To create the branch, distance between nodes should be sorted in ascending order branches; from the top of the list, the first route will be assigned to the first branch. Similarly, the next routes will be allocated to the next branches as far as all branches receive their first route. In next step, we should calculate the length of the branches that has the least distance will be selected. After re-allocation the length of all branches will be calculated and again, the shortest branch is selected for the next assignment. This process is repeated until all routes between nodes allocated to one of the branches. At this point, the branches should be connected mutually and loops are formed. For this purpose, the distance between the last nodes of each branch and the other branches will be calculated; the two branches which their nodes have the least distances will be firstly connected to each other. This procedure will also be repeated until all branches are connected mutually. To illustrate this algorithm, the following signs are used: d_{ij} = price (distance) between two nodes of i and j in the routes network, T_k = the total length between branches, $k = 1, 2, 3, \dots, 2m$, TDI = total edges length of the route l , $l = 1, 2, 3, \dots, m$, TD = total edges length of all routes, m = number of service providers, n = number of service receivers, the zero (0) symbol is used to show the service center. With respect to the above symptoms, the main steps of the procedure will be as follows:

- Step 1: Sort customers based on their distance from the service center in ascending order
- Step 2: Select $2m$ (m = number of service providers) of customers from the beginning of the list prepared in Step 1 and create a branch for each of them and assign a number to every branch
- Step 3: Calculate T_k (total edge length of branch k) for each branch
- Step 4: Select the branch which has the lowest T_k and consider the last customer assigned to this branch; among the customer who have not yet been assigned to any branch, add the customer which has the lowest distance to the mentioned customer to the branch
- Step 5: Are all customers assigned to branches? In this case go to the next step, otherwise go to step 3. Repeat the process until all customers assigned to branches

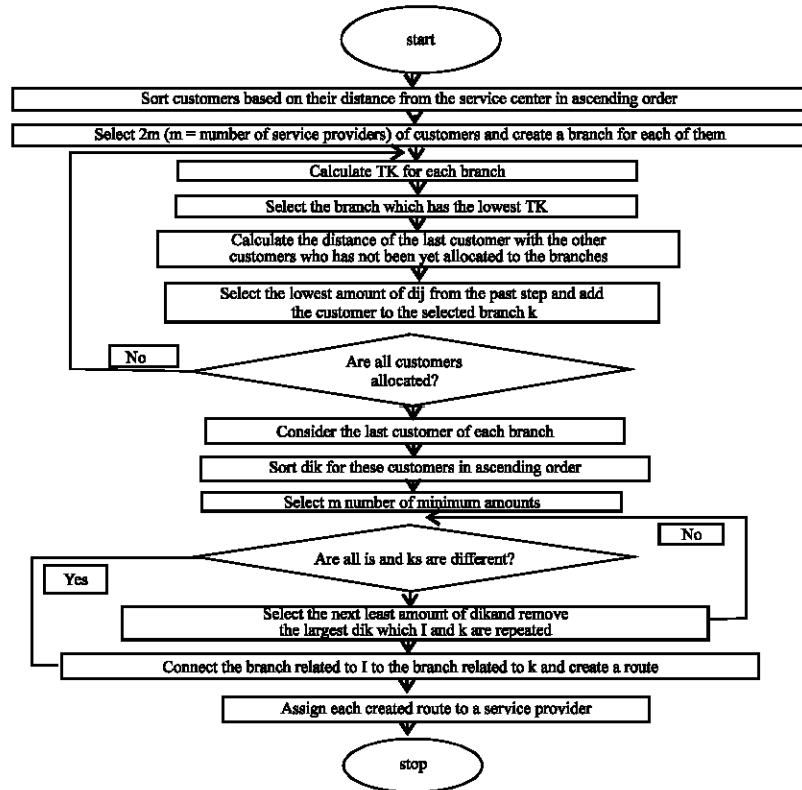


Fig. 1: Arya method algorithm flowchart

6. Connect the created branches with the following commands until m routes will be created
7. Consider the customers located at the end of each branch
8. Calculate the distances between all the customers determined in step 7 (d_{ik} = distance between the two clients i and k.); sort these distances in ascending order
9. Select m number of minimum amounts specified in step 8. If i and k values for these intervals were different, then connect i to k to create the desired routes and go to step 11; otherwise, go to Step 10
10. Select the next least amount of d_{ik} and act according to Step 9
11. Assign each created route to a service provider
12. Sum up the edge length of all routes (TD). This sum will be considered as the objective function of the problem (TDL = total edge length of the route L)

$$TD = \sum_{j=1}^m TD_{ij}$$

The steps of the method are represented in Fig. 1 according to the above explanations.

Clarke and Wright method: Clarke and Wright method is a well-known method which is discussed in various resources including source number. This method works on the basis of cost savings. In this method, a route is firstly created for each customer. In this case, n routes will be created at the beginning of the algorithm. Given that usually the number of vehicles (m) is less than the number of customers, the initial answer of this method is unacceptable. But in the next steps we try to provide an acceptable solution to the problem by changing and combining the route. The combination of the routes will be continued until all constraints will be covered and the number of vehicles is equal to the number of paths.

Research method: To compare these two methods, a developer was asked to simultaneously and with a method of coding convert the two algorithms into the codes. Then random problems in different sizes in terms of the number of nodes and vehicles were regenerated. These problems are classified and finally, the number 295 problems were created. The results of these studies contain significant points that have been mentioned as.

Table 1: Comparing two methods based on the number of the service receivers

No. of customer	No. of solved problems	Time superiority				Quality superiority			
		Ar method		CW method		Ar method		CW method	
		Amount	Percent	Amount	Percent	Amount	Percent	Amount	Percent
90	25	25	100	0	0	10	40	15	60
80	25	25	100	0	0	11	44	14	56
70	25	25	100	0	0	10	40	15	60
60	25	25	100	0	0	5	20	20	80
50	25	25	100	0	0	6	24	19	76
45	25	25	100	0	0	5	20	20	80
40	25	25	100	0	0	4	16	21	84
35	25	20	80	5	20	8	32	17	68
30	25	20	80	5	20	10	40	15	60
25	25	20	80	5	20	5	20	20	80
20	20	15	75	5	25	5	25	15	75
15	15	10	67	5	33	3	20	12	80
10	10	10	10	0	0	0	0	10	10
total	295	270	92	25	8	82	28	213	72

Table 2: comparing two methods based on the service providers

No. of service providers	No. of solved problems	Time superiority				Quality superiority			
		Ar method		CW method		Ar method		CW method	
		Amount	Percent	Amount	Percent	Amount	Percent	Amount	Percent
20	50	35	70	15	30	3	6	47	94
15	55	50	91	5	9	0	0	55	100
10	60	55	92	5	8	2	3	58	97
5	65	65	100	0	0	20	31	45	69
1	65	65	100	0	0	57	88	8	12
total	295	270	92	25	8	82	28	213	72

RESULTS AND DISCUSSION

Reviewing the results of solving sample problems show that Arya algorithm has the absolute excellence in terms of the speed compared to the Clarke and Wright method and almost in 92% of cases it has the high speed in obtaining the solution. Therefore, it can be proposed to the researchers looking for an acceptable primary solution to developing their algorithms. The following tables compare the time and quality of the solutions obtained from these two methods. As can be seen in Table 1 and 2, CW method is not superior to Ar in terms of the quality of the solutions obtained; because, Ar method shows superiority in the low number of service providers and high number of customers. Thus, although the method of Clarke and Wright seems to provide a better quality of solutions but Arya procedure has also provided a better answer in certain circumstances.

Another point that should be noted is that although the CW method is better in terms of the number of solutions provided but the quality of the results produced by this method is on average 12.5% better than Ar method that this value in the best conditions is 32%. The

qualitative edge of CW method is not dramatic and absolute. Summary results of these solutions are given in Table 3. As can be seen, in terms of number of customers, the Clarke and Wright method had not produced better solutions and in other cases, by decreasing the number of customer, the quality of Arya answer is reduced and the quality of Clarke and Wright solution is added.

Also in Table 4, it can be seen that by reducing the number of service providers, again, Arya method generates better solutions. So that if a server is present, the results produced by the Arya method are even better than Clarke and Wright method. But, by increasing the number of service providers, the average solutions of Clark and Wright method will be better. But, these results are at the best case, up to 25% better than the Arya method. On the other hand, the speed of reaching answer in "Ar" method is on average 76% better than using CW and in the best condition, this value is 99%; this superiority is significant unlike the qualitative superiority of the answer. And somehow, it can be claimed that Arya methods is superior to Clarke method in terms of speed of reaching an acceptable solution. Details are given in Table 5 and 6.

Table 3: Comparing the quality of the results of Clarke and Arya methods based on the number of customers

No of service receivers	No of solved problems	Average quality superiority of Clarke method than Arya method (%)
9	25	0.0
80	25	3.0
70	25	8.0
60	25	11.0
50	25	14.0
45	25	15.0
40	25	20.0
35	25	12.0
30	25	7.0
25	25	12.0
20	20	14.0
15	15	13.0
10	10	32.0
Average 25 total	295	12.5

Table 4: Comparing the quality of the Clarke method results to Arya method on the number of service providers

No of service providers	No of solved problems	Average time superiority of Clarke than Arya method
20	50	25.0
15	55	25.0
10	60	21.0
5	65	10.0
1	65	-19.0
Average total	295	12.5

Table 5: Comparing the speed of reaching a solution based on the number of service receivers using Arya method and to Clarke method

No of service providers	No of solved problems	Average quality superiority of Clarke method than Arya method (%)
90	25	99
80	25	99
70	25	98
60	25	97
50	25	94
45	25	91
40	25	86
35	25	68
30	25	63
25	25	30
20	20	37
15	15	52
10	10	71
Average total	295	76

Table 6: Comparing the speed of reaching a solution based on the number of clients using Arya method and Clarke and Wright method

No of service providers	No of solved problems	Average quality superiority of Clarke method than Arya method (%)
20	50	48
15	55	63
10	60	79
5	65	86
1	65	97
Average total	295	76

CONCLUSION

In general, since the answer of Arya method is close to the answer of the Clarke and Wright method and in

average, in 12% of cases provides low quality solutions, it always is more acceptable in terms of time to answer than Clarke method; therefore, the method is recommended to use in situations where speed is important and we are merely looking for an acceptable solution in the shortest possible time.

SUGGESTIONS

As mentioned above, this method is related to solving routing public issues. The author has developed this method for the issues with capacity that it will publish in the near future. As mentioned above, this method is among the causer methods; the author has extracted more quality answers from the method by adding an improver stage to it that its results will be published in the future. The researcher now has the capacity to expand this approach to issues that it will publish in the near future. But the development of this method for other routing issues is the other research path that researchers can keep it in mind. The method is extensible for routing problems with time window constraints, product delivery and customer's order reception routing problems, state where in addition to vehicles capacity constraints, service should be offered to customers at a certain time etc. Also, our fundamental condition in this method is the perfect graph between customers; the method can be developed by removing this condition.

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