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Implementation of Half-Sweep Age Method Using Seikkala Derivatives Approach for 2D Fuzzy Diffusion Equation

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Abstract: In this study, the iterative methods particularly families of Alternating Group Explicit (AGE) methods are used to solve finite difference algebraic equation arising from fuzzy diffusion equation is examined. For the proposed problems, family of AGE methods namely Full-Sweep AGE (FSAGE) and Half-Sweep AGE (HSAGE) has been considered to be the generated linear solver. The formulation and implementation of these two proposed methods were also presented. In addition, numerical results by solving two test problems are included and compared with the Full-Sweep Gauss Seidel (FSGS), FSAGE and HSAGE methods to show their performance.

Key words: Two-stage iteration, implicit scheme, fuzzy parabolic equation, solver, full-sweep

INTRODUCTION

The Alternating Group Explicit (AGE) method is one of the widely used and successful two-stage iterative methods to solve sparse linear system. The AGE method employs the fractional splitting strategy which is applied alternately at each intermediate step on linear system. In a series of studies, the effectiveness of the AGE and its variants methods were studied and tested by solving a variety of scientific problems, for instance refer (Dahalan *et al*., 2015a, b, 2013, 2014; Mohanty and Talwar, 2012; Feng, 2008; Feng and Zheng, 2009; Bildik and Ozlu, 2005). Besides that, the concept of half-sweep iteration has been initiated by (Abdullah, 1991) via the Explicit Decoupled Group (EDG) method for solving two-dimensional Poisson equations. The basic idea of the half-sweep iteration approach is to speed-up the computational time by reducing the computational complexity of the solution method.

In this study, performance of the half-sweep iteration with AGE method, i.e., Half-Sweep Alternating Group Explicit (HSAGE) method will be investigated for solving linear systems generated from the fuzzy heat equation. The performance of HSAGE method will be compared with the existing standard Gauss-Seidel (GS) and AGE methods. The standard GS and AGE methods are also known as Full-Sweep Gauss-Seidel (FSGS) and Full-Sweep Alternating Group Explicit (FSAGE) methods respectively.

MATERIALS AND METHODS

Finite Difference approximation equations: Let x and y be two fuzzy subset of real numbers. They are characterized by a membership function evaluated at t, written x(t) and y(t) respectively as a number in [0,1]. The membership function can be used to identified the fuzzy numbers. For, the α cut of x and y which is denote as a crisp number, can be written as x(a) and y(a) in $\{x \mid \tilde{x}(t) \geq \alpha\}$ and $\{x \mid \tilde{y}(t) \geq \alpha\}$, respectively. Since, they are always closed and bounded interval for all (Allahviranloo, 2002), the α cut of fuzzy numbers can be written as:

$$\tilde{x}(\alpha) = [\underline{x}(\alpha), \overline{x}(\alpha)]$$

And:

$$\tilde{y}(\alpha) = [\underline{y}(\alpha), \overline{y}(\alpha)]$$

Suppose $(\underline{x}, \overline{x})$ and $(\underline{y}, \overline{y})$ be parametric forms of fuzzy functions x and y, respectively. An arbitrary positive integer n and m subdivided the interval $a \leq t \leq b$ where $x_i = a+ih$ (i = 0, 1, 2, ..., n) and $y_i = a+j_1$ (i = 0, 1, 2, ..., n) for I and j, respectively. The step size h and l are define by h = b-a/n and h = b-a/n.

For further discussions on formulating the full-and half-sweep finite difference approximation equations, consider the interval that is divided uniformly as shown in Fig. 1.

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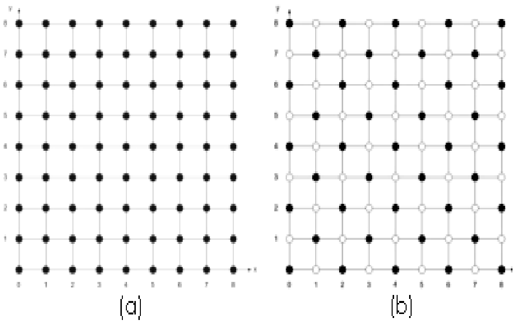

Fig. 1: a, b) Show distribution of uniform node points for the full-and half-sweep cases respectively

criterion is achieved (Abdullah, 1991). Now, consider the following general fuzzy heat equation:

$$\frac{\partial \tilde{U}}{\partial t} = V\left(\frac{\partial^2 \tilde{U}}{\partial x^2} + \frac{\partial^2 \tilde{U}}{\partial y^2}\right), R = \left[0 \le x \le n\right] \times \left[0 \le y \le m\right] \quad (1)$$

with boundary conditions:

$$\tilde{U}(x, y, 0) = \tilde{f}(x, y), (x, y) \in R$$

and initial conditions:

$$\tilde{U}(x, y, t) = g(x, y, t), (x, y, t) \in \delta R \times \left[0 \le t \le T\right]$$

where, δR was an boundary of R. In this study, we derive the formulation of full-and half-sweep finite difference approximation equations based on the implicit scheme, i.e., Backward Time, Centered Space (BTCS). By using BTCS scheme, Eq. 1 can be developed as:

$$\frac{\partial \underline{U}}{\partial t} \approx \frac{\underline{U_{i,j+1,k+1}} - \underline{U_{i,j,k}}}{\Delta t} \quad (2)$$

$$\frac{\partial \overline{U}}{\partial t} \approx \frac{\overline{U_{i,j+1,k+1}} - \overline{U_{i,j,k}}}{\Delta t} \quad (3)$$

With:

$$\Delta t = t_{j+1} - t_j$$

And:

$$\frac{\partial^2 \underline{U}}{\partial x^2} \approx \left[\frac{\underline{U_{i-p,j,k+1}} - 2\underline{U_{i,j,k+1}} + \underline{U_{i+p,j,k+1}}}{(ph)^2}\right] \quad (4)$$

$$\frac{\partial^2 \overline{U}}{\partial x^2} \approx \left[\frac{\overline{U_{i-p,j,k+1}} - 2\overline{U_{i,j,k+1}} + \overline{U_{i+p,j,k+1}}}{(ph)^2}\right] \quad (5)$$

$$\frac{\partial^2 \underline{U}}{\partial y^2} \approx \left[\frac{\underline{U_{i,j-p,k+1}} - 2\underline{U_{i,j,k+1}} + \underline{U_{i,j+p,k+1}}}{(ph)^2}\right] \quad (6)$$

$$\frac{\partial^2 \overline{U}}{\partial y^2} \approx \left[\frac{\overline{U_{i,j-p,k+1}} - 2\overline{U_{i,j,k+1}} + \overline{U_{i,j+p,k+1}}}{(ph)^2}\right] \quad (7)$$

For full-sweep cases, by applying 2-4, lower boundary for (Eq. 1) can be reduced to:

$$\underline{U_{i,j,k+1}} - \underline{U_{i,j,k}} = \frac{V\Delta t}{h^2}\left(\begin{array}{l}\underline{U_{i-1,j,k+1}} + \underline{U_{i+1,j,k+1}} + \underline{U_{i,j-1,k+1}} \\ + \underline{U_{i,j+1,k+1}} - 4\underline{U_{i,j,k+1}}\end{array}\right) \quad (8)$$

for i = 1p, 2p, ..., n-p and j = 1p, 2p, ...m-p. Meanwhile, applying Eq. 3, 5 and 7 into upper boundary for (Eq. 1), it can be shown:

$$\overline{U_{i,j,k+1}} - \overline{U_{i,j,k}} = \frac{V\Delta t}{h^2}\left(\begin{array}{l}\overline{U_{i-1,j,k+1}} + \overline{U_{i+1,j,k+1}} + \overline{U_{i,j-1,k+1}} + \\ \overline{U_{i,j+1,k+1}} - 4\overline{U_{i,j,k+1}}\end{array}\right) \quad (9)$$

Whereas, for half-sweep cases could be written as:

$$\underline{U_{i,j,k+1}} - \underline{U_{i,j,k}} = \frac{V\Delta t}{2h^2}\left(\begin{array}{l}\underline{U_{i-1,j-1,k+1}} + \underline{U_{i-1,j+1,k+1}} + \underline{U_{i+1,j-1,k+1}} + \\ \underline{U_{i+1,j+1,k+1}} - 4\underline{U_{i,j,k+1}}\end{array}\right) \quad (10)$$

And:

$$\overline{U_{i,j,k+1}} - \overline{U_{i,j,k}} = \frac{V\Delta t}{2h^2}\left(\begin{array}{l}\overline{U_{i-1,j-1,k+1}} + \overline{U_{i-1,j+1,k+1}} + \\ \overline{U_{i+1,j-1,k+1}} + \overline{U_{i+1,j+1,k+1}} - 4\overline{U_{i,j,k+1}}\end{array}\right) \quad (11)$$

Eventhough, Eq. 10 and 11 have the same form in terms of the equation but based on the interval of the α-cuts, the differences identified in the upper and lower bound, thus it can be rewritten as:

$$U_{i,j,k+1} - U_{i,j,k} = \beta\begin{pmatrix} U_{i-1,j,k+1} + U_{i+1,j,k+1} + U_{i,j-1,k+1} + \\ U_{i,j+1,k+1} - 4U_{i,j,k+1} \end{pmatrix} \quad (12)$$

with $\beta = V\Delta t/h^2$ for full-sweep and:

$$U_{i,j,k+1} - U_{i,j,k} = \beta\begin{pmatrix} U_{i-1,j-1,k+1} + U_{i-1,j+1,k+1} + \\ U_{i+1,j-1,k+1} + U_{i+1,j+1,k+1} - 4U_{i,j,k+1} \end{pmatrix} \quad (13)$$

with $\beta = (V\Delta T/2h^2)$ for half-sweep cases. Moreover, 14 and 15 can be represented in matrix form as follows:

$$A\underset{\sim}{U}_{j+1} = \underset{\sim}{b}_{j} \quad (14)$$

Implementation of the BTCS scheme requires to solve a linear system at each time step and it is unconditional stable.

Family of alternating group explicit iterative methods: Based on the splitting of the matrix into the sum of its constituent symmetric and positive definite matrices, consider a class of methods be mentioned by Eq. 16, (Evans, 1997) as follows:

$$A = G_1 + G_2 + G_3 + G_4 \quad (15)$$

where, G_1 and G_2 are the forward and backward differences in the x-plane. Then, diag (G_1) = diag (G_2) = 1/4 diag (A) with (Fig. 2). By reordering the points column-wise along y-direction, G_3 and G_4 literally have the same structure as G_1 and G_2 respectively, (Fig. 3). Then, Eq. 17 becomes:

$$\left(G_1 + G_2 + G_3 + G_4\right)\underset{\sim}{U}_{j+1} = \underset{\sim}{b}_{j} \quad (16)$$

Thus, the explicit form of AGE method can be written as:

$$\underset{\sim}{U}\left(k+\frac{1}{4}\right) = (r_1I + G_1)^{-1}\left[2f + (r_1I + G_1 - 2A)\right] \quad (17)$$

$$\underset{\sim}{U}^{\left(k+\frac{1}{2}\right)} = \left(r_1I + G_2\right)^{-1}\left[G_2\underset{\sim}{U}^{(k)} + r_1\underset{\sim}{U}^{\left(k+\frac{1}{4}\right)}\right] \quad (18)$$

$$\underset{\sim}{U}^{\left(k+\frac{3}{4}\right)} = \left(r_2I + G_3\right)^{-1}\left[G_3\underset{\sim}{U}^{(k)} + r_2\underset{\sim}{U}^{\left(k+\frac{1}{2}\right)}\right] \quad (19)$$

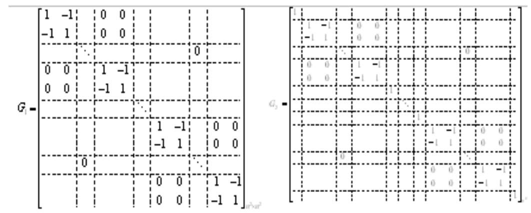

Fig. 2: Family of alternating group explicit iterative methods

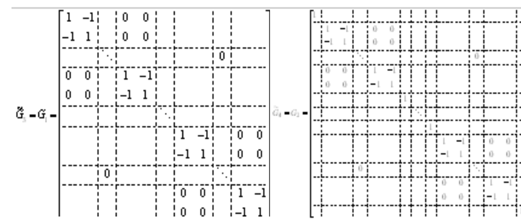

Fig. 3: Coulmn-wise direction

And:

$$\underset{\sim}{U}^{(k+1)} = \left(r_2I + G_4\right)^{-1}\left[G_4\underset{\sim}{U}^{(k)} + r_2\underset{\sim}{U}^{\left(k+\frac{3}{4}\right)}\right] \quad (20)$$

Therefore, from Eq. 18-20, the implementation of FSAGE and HSAGE methods to solve corresponding full- and half-sweep BTCS approximation equations is presented in Algorithm 1.

Alogrithm 1: Families of age methods:

Initialize $\underset{\sim}{U}^{(0)} \leftarrow 0$ and $\varepsilon \leftarrow 10^{-10}$.
First sweep
Compute:

$$\underset{\sim}{U}\left(k+\frac{1}{4}\right) = (r_1I + G_1)^{-1}\left[2\underset{\sim}{f} + (r_1I + G_1 - 2A)\right]$$

Second sweep
Compute:

$$\underset{\sim}{U}\left(k+\frac{1}{2}\right) = \left(r_1I + G_2\right)^{-1}\left[G_2\underset{\sim}{U}^{(k)} + r_1\underset{\sim}{U}\left(k+\frac{1}{4}\right)\right]$$

Third sweep
Compute:

$$\underset{\sim}{U}\left(k+\frac{3}{4}\right) = \left(r_2I + G_3\right)^{-1}\left[G_3\underset{\sim}{U}^{(k)} + r_2\underset{\sim}{U}\left(k+\frac{1}{2}\right)\right]$$

Fourth sweep
Compute:

$$\underset{\sim}{U}^{(k+1)} = \left(r_2I + G_4\right)^{-1}\left[G_4\underset{\sim}{U}^{(k)} + r_2\underset{\sim}{U}\left(k+\frac{3}{4}\right)\right]$$

Convergence teast. If the convergence criterion i.e., $\left\|\underset{\sim}{U}^{(k+1)} - \underset{\sim}{U}^{(k)}\right\|_{\infty} \le \varepsilon$ is satisfied, go to step. Otherwise go back to step
Display approximate solutions

RESULTS AND DISCUSSION

Numerical experiments: In order to compare the performance of the HSAGE method, the following fuzzy heat equations were used as the test problems.

Test problem 1 (Kadalbajoo and Rao, 1997):

$$\frac{\partial \tilde{U}}{\partial t}(x,y,t) = \frac{\partial^2 \tilde{U}}{\partial x^2}(x,y,t) + \frac{\partial^2 \tilde{U}}{\partial y^2}(x,y,t), \quad 0 \le x,\ y \le 1,\ t \ge 0 \tag{21}$$

where $\tilde{k}[\alpha] = [\underline{k}(\alpha), \overline{k}(\alpha)] = [0.75 + 0.25\alpha, 1.25 - 0.25\alpha]$ with the initial condition $\tilde{U}(x,y,0) = \sin(\pi y)\sin(\pi x)$. The boundary conditions $\tilde{U}(x,0,t) = \tilde{U}(x,1,t) = 0$ and $\tilde{U}(0,y,t) = \tilde{U}(1,y,t) = 0$. The exact solution for:

$$\frac{\partial \underline{U}}{\partial t}(x,y,t;\alpha) = \frac{\partial^2 \underline{U}}{\partial x^2}(x,y,t;\alpha) + \frac{\partial^2 \underline{U}}{\partial y^2}(x,y,t;\alpha) \tag{22}$$

And:

$$\frac{\partial \overline{U}}{\partial t}(x,y,t;\alpha) = \frac{\partial^2 \overline{U}}{\partial x^2}(x,y,t;\alpha) + \frac{\partial^2 \overline{U}}{\partial y^2}(x,y,t;\alpha) \tag{23}$$

Are:

$$\underline{U}(x,y,t;\alpha) = \underline{k}(\alpha)\sin(\pi y)\sin(\pi x)e^{-\pi^2 t} \tag{24}$$

And:

$$\overline{U}(x,y,t;\alpha) = \overline{k}(\alpha)\sin(\pi y)\sin(\pi x)e^{-\pi^2 t} \tag{25}$$

respectively.

Test problem 2:

$$\frac{\partial \tilde{U}}{\partial t}(x,y,t) = \frac{\partial^2 \tilde{U}}{\partial x^2}(x,y,t) + \frac{\partial^2 \tilde{U}}{\partial y^2}(x,y,t), \quad 0 \le x,\ y \le 1,\ t \ge 0 \tag{26}$$

where $\tilde{k}[\alpha] = [\underline{k}(\alpha), \overline{k}(\alpha)] = [0.75 + 0.25\alpha, 1.25 - 0.25\alpha]$ with the initial condition $\tilde{U}(x,y,0) = \sin(\pi y)\sin(\pi x)$. The boundary conditions $\tilde{U}(x,0,t) = \tilde{U}(x,1,t) = 0$ and $\tilde{U}(0,y,t) = \tilde{U}(1,y,t) = 0$. The exact solution for:

$$\frac{\partial \underline{U}}{\partial t}(x,y,t;\alpha) = \frac{\partial^2 \underline{U}}{\partial x^2}(x,y,t;\alpha) + \frac{\partial^2 \underline{U}}{\partial y^2}(x,y,t;\alpha) \tag{27}$$

And:

$$\frac{\partial \overline{U}}{\partial t}(x,y,t;\alpha) = \frac{\partial^2 \overline{U}}{\partial x^2}(x,y,t;\alpha) + \frac{\partial^2 \overline{U}}{\partial y^2}(x,y,t;\alpha) \tag{28}$$

Are:

$$\underline{U}(x,y,t;\alpha) = \underline{k}(\alpha)\sin\left(\frac{1}{2}\pi y\right)\sin\left(\frac{1}{2}\pi x\right)e^{(-\pi^2/2)\,t} \tag{29}$$

And:

$$\overline{U}(x,y,t;\alpha) = \overline{k}(\alpha)\sin\left(\frac{1}{2}\pi y\right)\sin\left(\frac{1}{2}\pi x\right)e^{\left(-\frac{\pi^2}{2}\right)t} \tag{30}$$

respectively.

For numerical results, three parameters, i.e., number of iterations, execution time (in seconds) and Hausdorff distance (as mention in definition 1) were measured and considered for comparative analysis.

Definition 1: (Nutanong *et al.*,011): Given two minimum bounding rectangles P and Q, a lower bound of the Hausdorff distance from the elements confined by P to the elements confined by Q is defined as:

$$\text{HausDistLB}(P,Q) = \text{Max}\left\{\text{MinDist}(f_\alpha, Q) : f_\alpha \in \text{FacesOf}(P)\right\} \tag{31}$$

The computations are performed on a PC with Intel(R) Core(TM) 2 (1.66, 1.67 GHz) and 1022 MB RAM and the programs were compiled by using C language. Throughout, the numerical experiments, the convergence test considered $\varepsilon = 10^{-10}$ and carried out on several different values of n. Moreover, numerical results of FSGS for solving full-sweep BTCS approximation equations are also included for comparison purpose. All results of numerical simulations obtained from implementation of the FSGS, FSAGE and HSAGE methods for test problems 1 and 2 have been tabulated in Table 1-5. Table 6 described the percentage gains in terms of number of iterations and execution time FSAGE and HSAGE methods compared to FSGS method for both test problems.

Table 1: Numerical results of FSGS, FSAGE and HSAGE methods at α = 0.00

	n			
Methods	32	64	128	256
Test problem 1 (No. of iterations)				
FSGS	181	546	1500	2134
FSAGE	60	186	569	1553
HSAGE	35	106	328	966
Execution time				
FSGS	3.83	29.74	322.58	4137.79
FSAGE	1.63	14.03	168.97	2269.53
HSAGE	0.81	5.00	52.89	684.92
Hausdorff distance				
FSGS	9.1334e-04	9.1338e-04	9.1351e-04	9.1401e-04
FSAGE	9.1333e-04	9.1335e-04	9.1339e-04	9.1352e-04
HSAGE	9.1328e-04	9.1334e-04	9.1336e-04	9.1343e-04
Test problem 2; No. of iterations				
FSGS	561	1884	6186	19449
FSAGE	173	585	1971	6477
HSAGE	95	317	1077	3596
Execution time				
FSGS	8.62	67.43	773.95	9764.77
FSAGE	3.42	36.09	396.16	5524.52
HSAGE	1.55	11.07	131.04	1847.37
Hausdorff distance				
FSGS	9.5198e-07	9.2103e-07	8.0616e-07	3.9726e-07
FSAGE	9.5848e-07	9.4891e-07	9.1788e-07	7.9618e-07
HSAGE	1.9622e-06	9.5754e-07	9.3897e-07	8.7667e-07

Table 2: Numerical results of FSGS, FSAGE and HSAGE methods at α = 0.25

	n			
Methods	32	64	128	256
Test problem 1 (No. of iterations)				
FSGS	183	554	1543	2284
FSAGE	61	188	577	1601
HSAGE	35	106	333	986
Execution time				
FSGS	3.85	29.75	324.20	3952.68
FSAGE	1.64	14.03	169.04	2286.40
HSAGE	0.81	5.13	53.53	682.43
Hausdorff distance				
FSGS	8.3723e-04	8.3727e-04	8.3740e-04	8.3790e-04
FSAGE	8.3722e-04	8.3724e-04	8.3728e-04	8.3741e-04
HSAGE	8.3718e-04	8.3723e-04	8.3725e-04	8.3732e-04
Test problem 2; No. of iterations				
FSGS	565	1901	6251	19708
FSAGE	174	589	1988	6545
HSAGE	95	320	1086	3631
Execution time				
FSGS	8.54	67.64	794.78	9877.22
FSAGE	3.41	32.58	397.57	5522.91
HSAGE	1.55	11.10	133.25	1741.90
Hausdorff distance				
FSGS	8.7188e-07	8.4120e-07	7.2648e-07	3.2823e-07
FSAGE	8.7840e-07	8.6898e-07	8.3799e-07	7.1663e-07
HSAGE	1.7987e-06	8.7741e-07	8.5913e-07	7.9689e-07

Table 3: Numerical results of FSGS, FSAGE and HSAGE methods at α = 0.50

	n			
Methods	32	64	128	256
Test problem 1 (No. of iterations)				
FSGS	183	560	1570	2416
FSAGE	62	190	583	1629
HSAGE	35	107	335	997

Table 3: Continue

	n			
Methods	32	64	128	256
Execution time				
FSGS	3.88	29.88	325.38	3960.04
FSAGE	1.65	14.22	170.43	2290.82
HSAGE	0.83	5.04	53.46	685.02
Hausdorff distance				
FSGS	7.6112e-04	7.6116e-04	7.6129e-04	7.6179e-04
FSAGE	7.6111e-04	7.6113e-04	7.6116e-04	7.6130e-04
HSAGE	7.6107e-04	7.6111e-04	7.6114e-04	7.6121e-04
Test problem 2; No. of iterations				
FSGS	567	1910	6291	19873
FSAGE	175	592	2000	6588
HSAGE	96	320	1092	3652
Execution time				
FSGS	8.62	68.65	783.15	9788.74
FSAGE	3.43	32.83	405.44	5498.81
HSAGE	1.54	11.12	124.70	1923.16
Hausdorff distance				
FSGS	7.9178e-07	7.6131e-07	6.4681e-07	2.6243e-07
FSAGE	7.9838e-07	7.8908e-07	7.5819e-07	6.3700e-07
HSAGE	1.6351e-06	7.9716e-07	7.7919e-07	7.1709e-07

Table 4: Numerical results of FSGS, FSAGE and HSAGE methods at α = 0.75

	n			
Methods	32	64	128	256
Test problem 1 (No. of iterations)				
FSGS	184	562	1585	2698
FSAGE	61	190	586	1645
HSAGE	36	108	337	1003
Execution time				
FSGS	3.83	30.73	328.76	4075.89
FSAGE	1.78	14.13	169.33	2298.88
HSAGE	0.80	5.02	53.39	687.14
Hausdorff distance				
FSGS	6.8501e-04	6.8505e-04	6.8517e-04	6.8567e-04
FSAGE	6.8500e-04	6.8502e-04	6.8505e-04	6.8518e-04
HSAGE	6.8496e-04	6.8500e-04	6.8503e-04	6.8510e-04
Test problem 2; No. of iterations				
FSGS	569	1916	6314	19966
FSAGE	176	594	2005	6613
HSAGE	95	322	1094	3664
Execution time				
FSGS	8.68	69.99	805.99	9910.91
FSAGE	3.42	32.55	400.02	5536.48
HSAGE	1.60	11.38	124.83	1987.29
Hausdorff distance				
FSGS	7.1176e-07	6.8152e-07	5.6736e-07	2.0107e-07
FSAGE	7.1831e-07	7.0923e-07	6.7828e-07	5.5761e-07
HSAGE	1.4716e-06	7.1698e-07	6.9921e-07	6.3724e-07

Table 5: Numerical results of FSGS, FSAGE and HSAGE methods at α = 1.00

	n			
Methods	32	64	128	256
Test problem 1 (No. of iterations)				
FSGS	184	564	1590	2774
FSAGE	62	190	586	1650
HSAGE	36	108	336	1006
Execution time				
FSGS	3.86	29.58	327.86	4119.54
FSAGE	1.63	14.31	171.39	2298.56
HSAGE	0.86	5.04	53.47	689.51

Table 5: Continue

	n			
Methods	32	64	128	256
Hausdorff distance				
FSGS	6.0890e-04	6.0894e-04	6.0906e-04	6.0956e-04
FSAGE	6.0889e-04	6.0891e-04	6.0894e-04	6.0907e-04
HSAGE	6.0886e-04	6.0889e-04	6.0892e-04	6.0898e-04
Test problem 2; No. of iterations				
FSGS	570	1918	6322	19996
FSAGE	176	594	2008	6620
HSAGE	96	322	1096	3668
Execution time				
FSGS	8.65	69.75	794.75	9640.57
FSAGE	3.39	32.54	401.98	5516.17
HSAGE	1.55	11.41	125.36	1928.24
Hausdorff distance				
FSGS	6.3181e-07	6.0165e-07	4.8972e-07	1.4564e-07
FSAGE	6.3830e-07	6.2929e-07	5.9846e-07	4.7822e-07
HSAGE	1.3081e-06	6.3683e-07	6.1934e-07	5.5755e-07

Table 6: Percentage gains for FSAGE and HSAGE methods compared to FSGS method

α	Methods	Execution time (%)	No. of iterations (%)
Test problem 1			
0.00	FSAGE	45.15-57.44	27.23-66.85
	HSAGE	78.85-83.60	54.73-80.66
0.25	FSAGE	42.16-57.40	29.90-66.67
	HSAGE	78.96-83.49	56.83-80.87
0.50	FSAGE	42.15-57.47	32.57-66.12
	HSAGE	78.61-83.57	58.73-80.89
0.75	FSAGE	43.60-54.02	39.03-66.85
	HSAGE	79.11-83.76	62.82-80.78
Test problem 2			
1.00	FSAGE	44.20-57.77	40.52-66.31
	HSAGE	77.72 83.69	63.73-80.85
	FSAGE	43.42-60.32	66.70-69.16
0.00	HSAGE	81.08-83.58	81.51-83.17
	FSAGE	44.08-60.07	66.79-69.20
0.25	HSAGE	81.85-83.59	81.58-83.19
	FSAGE	43.83-60.21	66.85-69.14
0.50	HSAGE	80.35-84.08	81.62-83.25
	FSAGE	44.14-60.60	66.88-69.07
0.75	HSAGE	79.95-84.51	81.65-83.30
	FSAGE	42.78-60.81	66.89-69.12
1.00	HSAGE	80.00-84.23	81.66-83.21

CONCLUSION

In this study, the family of AGE iterative methods was used to solve linear systems arise from the discretization of fuzzy diffusion equation using the implicit difference scheme. The results show that HSAGE method is more superior in terms of the number of iterations, execution time and Hausdorff distance compared to the FSAGE and FSGS methods. Since, AGE is well suited for parallel computation, it can be considered as a main advantage because this method has groups of independent task which can be implemented simultaneously. It is hoped that the capability of the proposed method will be helpful for the further investigation in solving any multi-dimensional fuzzy partial differential equations (Farajzadeh *et al.*, 2010). Other family of AGE methods as in (Mohanty *et al.*, 2003; Evans and Yousif, 1988; Yousif and Evans, 1987; Sukon, 1996) also can be used as linear solvers in solving the same problem. Apart from the concept of the full and half-sweep iterations, further investigation of quarter-sweep (Sulaiman *et al.*, 2009, 2004, 2010; Othman and Abdullah, 2000) iteration can also be considered in order to speed up the convergence rate of the standard proposed iterative methods.

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