Journal of Engineering and Applied Sciences 11 (8): 1714-1720, 2016
ISSN: 1816-949X

Utility Marked To Market Optimal Asset Allocation

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Abstract: This study discusses optimal asset allocation strategy of a utility-maximizing investor dynamic in time as new market information becomes available to the investor. The objective is to find the optimal strategy that maximizes the expected total discounted log-utility of consumption over finite life time. Trading is assumed to take place between stock and risk-free bond (money account) paying constant interest rate. The underlying uncertainty in the stock price is governed by binomial process based on simple Markov chain approximation of diffusion process. The problem is solved using stochastic dynamic programming approach. In contrast to the continuous-time counterpart, the optimal trading and consumption strategies are found to be time-dependent in recursive manners. Sufficient conditions for short selling are given in terms of physical and martingale probabilities of the stock price. The result is then applied to Indonesia's stock data.

Key words: Dynamic optimal portfolio, stochastic programming, binomial, diffusion, counterpart

INTRODUCTION

The problem of finding optimal asset allocation strategy has been an interesting theoretical topic and of practical importance on itself. It goes back to the seminal work of Markowitz (Solem, 2012) in which the optimal strategy is done by minimizing the volatility of investor wealth for given level of target return. The Markowitz problem is solved in the static way using the Lagrange multiplier technique. Due to the nature of the problem the optimal solution is of time independent so that the strategy is unable to be adjusted as new market information arrives. This drawback was taken further and improved by within discrete-time setting in which it was assumed the existence of risk-free (money account) asset paying constant rate of interest and risky asset having continuous distribution. The objective of that study was to find optimal trading strategy that maximizes investor's expected value of total discounted utility of intertemporal consumption by applying stochastic dynamic programming. The solution to the investment-consumption problem was found in terms of simultaneous nonlinear equations for weight of risky asset to satisfy. Later on Merton (Joseph, 2013; Bradski and Kaehler, 2008) extended the Samuelson's model within continuous-time framework under which stock price process was driven by Ito process. The objective was to obtain optimal asset allocation strategy that maximizes expected discounted future life-time consumption and terminal wealth under utility function of Hyperbolic Absolute Risk Aversion (HARA). Under the HARA utility function and due to separable property of the portfolio value function Merton was able to obtain closed form solution to the investment-consumption problem with time-dependent effect in the portfolio weights on stocks. This result was quite opposite to those obtained earlier by Samuelson. However, when applied applied to power utility or log-utility function, Merton's result would be similar to Samuelson's results; the portfolio weight would be just a constant. It was noted in discrete-time by Kuo and Hsu (1996) and Swain and Ballard (1991) that this findings could be improved by updating the conditional joint distribution of asset returns every time period. By doing so, the optimal portfolio strategy obtained would be able to adapt to the new market information when it becomes available to the investor over time. Ohkubo and Kobayashi (2008) adapted similar technique to various utility. continuous-time and Samuelson's model, the optimal portfolio weight obtained for the stock is time inhomogeneous in terms of recursive equations. The short selling constraints are given explicitly in terms of expected log-return and volatility of the stock price process. The time inhomogeneity nature of the optimal solution allows us to apply the result to real data so that it is possible to adapt the investment position when the market information arrives.

MATERIALS AND METHODS

The market model and portfolio dynamics: Assumed that market model consists of two assets; bond as a risk-free asset and stock as a risky asset. The bond price is set as

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P1, grows at a constant interest rate of R where P_1 (t+1) = (1+R) P_1 (t), for every time of t and the stock price is set as P_2, evolves using a simple model of Markov chain:

$$P_2(t+1) = \begin{Bmatrix} u \times P_2(t) \text{ with probability } p_u \\ d \times P_2(t) \text{ with probability } p_d \end{Bmatrix} \quad (1)$$

where, $u \geq d$ and $p_u + p_d = 1$ In many literatures, the probability and are often considered as the true probability or physical probability. Using model (MeIntyre, 2002), the stock price will always be positive, because its nature is multiplicative. The expected return and the variance of price change logarithm of the real stock return can be matched with the model by choosing the parameters of p, u and d given by:

$$p_u = \frac{1}{2} + \frac{1}{2}\left(\frac{\mu}{\sigma}\right)\sqrt{\Delta t}$$
$$u = e^{\sigma\sqrt{\Delta t}} \quad (2)$$
$$d = e^{\sigma\sqrt{\Delta t}}$$

where, Δt is the period of length, μ and σ are respectively:

$$\mu = E\left\{\log \frac{P_2(1)}{P_2(0)}\right\}$$

$$\sigma^2 = \text{var}\left\{\log \frac{P_2(1)}{P_2(0)}\right\} \quad (3)$$

where, $P_2(0)$ and P_2 (1) are, respectively the initial stock price and the end price of period. The readers will be suggested to read Luenberger (Brettel *et al*., 1997) for more details. Nelson and Ramaswanny (Jeffries, 1880), shows that the Binomial Model by MeIntyre (2002) and Rigden (1999) converges, when goes to infinitesimally small to the simple geometric Brownian motion:

$$\frac{dP_2(t)}{P_2(t)} = (\mu + \frac{1}{2}\sigma^2)dt + \sigma dB(t)$$

where, B = (B(t): $\geq$o) is the standard Brownian motion. In this market model, the absence of arbitrage opportunity is necessary and sufficient to prevent stock return dominates the return on the bond and vice versa which is given as follows: $d \leq (1+R) \leq u$ The interest reader should read (Nathans *et al*., 1986; Neitz and Neitz, 2000) for more details. He also says that condition, Neitz and Neitz (2000) that (1+R) is convex combination of u and d, defined by: $(1+R) = q_u \times uq_d + \times d$ where q_u and q_d are the martingale probabilities and defined by:

$$q_u = \frac{(1+R)-d}{u-d}$$
$$q_d = \frac{u-(1+R)}{u-d} \quad (4)$$

The investor trades two assets between stock and bond, meanwhile she consumes part of it for her living. In this trading process, assuming that there is no injection of exogenous capital into the portfolio and no withdrawal of money, this is called as self-financing portfolio. Further, the wealth process of the investor over time t is represented with X(t). The portion of bond and stock that investor holds is respectively represented with $\omega_1(t)$ and $\omega_2(t)$ and it is also satisfied the constrain of: $\omega_1(t)+\omega_2(t)$ Therefore, the shares number for each is given by:

$$\theta_1 = \frac{\omega_1(t)X(t)}{P_1(t)} \text{ and } \frac{\omega_2(t)X(t)}{P_2(t)}$$

Assumes that the number of assets held is infinitesimally divisible which means that the number of shares could be fractional number. Further, the wealth of investor over time t and for each asset can be defined in terms of the asset prices and the number of shares as follows:

$$X(t) = \theta_1(t)P_1(t) + \theta_2(t)P_2(t) + \theta(t)^T P(t) \quad (5)$$

For every $t \in Z_T$ where $\theta(t) = (\theta_1(t), \theta_2(t)^T$ and $P(t) = (P_1(t), P_2(t)^T$. The portfolio dynamics is further derived by first deriving the portfolio constraint where the investor can withdraw the amount of consumption from the portfolio. Define C(t) as the consumption proportion that is drawn over period t to t+1, the constraint of portfolio budget can be written as follows.

Proposition 2.1 budget constraint: For every $t \in Z+$ the consumption C(t) is given as:

$$\theta(t+1)^T P(t+1) = \theta(t)^T P(t+1) - C(t) \quad (6)$$

Proof at time t+1 the value of investor's wealth is equal with the value of old portfolio subtracted with the consumption. The remaining value is reinvested in the assets. By inserting Eq. 6 into the portfolio, the wealth process of the investor can be described as follows.

Proposition 2.2 portfolio dynamics: The investor's portfolio dynamics is given by:

$$x(t+1) = (1+R)X(t) + \theta_2\left[P_2(t+1) - (1+R)P_2(t)\right] - C(t) \quad (7)$$

The first term describes the time value of money of the investor's wealth from time t to t+1, the second term describes the proceeds that investor make from trading on stock in excess of the holding period time value of money of the stock when held during time t to t+1. The last term is the withdrawal consumption. Proof during the period t and t+1 following (Brettel *et al.*, 1997):

$$\begin{aligned} &X(t+1) - X(t) = \theta(t+1)^T P(t+1) - \theta(t)^T P(t) \\ &= \theta(t)^T P(t+1) - \theta(t)^T P(t) + \theta(t+1)^T P(t+1) - \theta(t)^T P(t+1) \\ &= \theta(t)^T (P(t+1) - P(t)) + P(t+1)^T (\theta(t+1) - \theta(t)) \\ &= \theta(t)^T \Delta P(t) + P(t+1)^T \Delta\theta(t) \\ &\theta(t)^T \Delta P(t) - C(t) \end{aligned}$$

where, $\Delta P(t)$: = $(\Delta P_1(t), \Delta P_2(t)^T$ and C(t) using model (Poret *et al.*, 2009). From (Brettel *et al.*, 1997) we get that $(\Delta P_1(t) = RP_1(t)$ and $\theta_1(t)P_1(t) = X(t)\text{-}\theta_2(t)P_2(t)$. The wealth process could be further simplified with:

$$X(t+1) = \begin{cases} X^+(t+1) := R_t X(t) + X_2(t)u - C(t) \\ \text{with probability } p_u \\ X^-(t+1) := R_t X(t) + X_2(t)d - C(t) \\ \text{with probability } p_d \end{cases} \quad (8)$$

where, R_t: = 1+R, $\hat{u} = -R_t$, $\hat{d}$ = u-R_t, $X_2(t)$: = $\theta_2(t)\, P_2(t)$.

The asset allocation and consumption optimization: Defining the underlying wealth process by X = (X_t: t∈z_+ which has uncertainty generated in the probability space (Ω, F, P). The system had filtration of F = (F_t, t≤T∈Z_+). The control of (X_2, C) assumed to be F_t adapted in wealth process is required to only allow past observed value of X. To conduct analysis, let X become time-dependent function $X_2 = X_2(t, X_t)$ such and $C_t = C(t, X_t)$ and define the expectation associated with the X law which X∈R_+ at time t is $E_{t,x}$. Hence, in order to maximize the investment and consumption strategy (X_2, C), set of admissible control $A_{(t,T)}$: = (X_2(s), C_s): s = t, t+1, ..., T) must define the t-value of function J which described the expectation of total discounted consumption and terminal wealth at time t under utility μ_1 and μ_2. Giving J-function is following:

$$j^{(x_2,c)}(t,x) = E_{t,x}\left\{\sum_{s=t}^{T-1} D(t,s)\times u_1(C_s) + D(t,T)u_2(X_T)\right\} \quad (9)$$

where, D is the discount factor defined as D(t,s) = $R_t^{(s-t)}$ and R_t = (1+R). The log-utility function being used hroughout this studyr is $N_i(x)$ = log(x), i = 1, 2. Thus, the optimal value function formulation is then become:

$$v(t,x) = (X_2, C)^{\max} \varepsilon A_{[t,T]} j^{(x_2,c)}(t,x) \quad (10)$$

The optimal value function V, optimal asset allocation, $\mu_{1,2}$ and consumption strategies (X_2,C) need to be determined within the admissible control set of $A_{(t,T)}$. In order to satisfy (Neitz and Neitz, 2000), above variables must set to be positive. The discrete time stochastic dynamic programming methodology is to be applied to find investor's investment and consumption strategy (Dowell, 2008; Yang and Ro, 2003).

Proposition: The optimal value function v is given by:

$$V(t,x) = \phi(t)\times\log(x) + u(t) \quad (11)$$

where, ϕ(t) and (t) solves the recursive (Eq. 12):

$$\begin{aligned} &\phi(t) = \frac{R_t}{R_t - 1}\left(1 - \frac{R_t^{\,t}}{R_{t^T}}\right) \\ &\phi(T) = 0 \\ &\Psi(t) = \frac{\Psi(t+1)}{R_t} + \ln\left(\frac{R_t^{\,2}}{R_t + \phi(t+1)}\right) = + \frac{(1-\alpha)\times\phi(t+1)}{R_t} \\ &= .\ln R_t\left(1 + \left[\frac{[(p_u - 1)d - (p_u)u]}{u}\times\frac{\phi(t+1)}{R_t + \phi(t+1)}\right]\times X\right) \\ &= -\frac{R_t^{\,2}}{R_t + \phi(t+1)} + \frac{(\alpha)\times\phi(t+1)}{R_t} \\ &= .\ln R_t\left(1 + \left[\frac{[(p_u - 1)d - (p_u)u]}{d}\times\frac{\phi(t+1)}{R_t + \phi(t+1)}\right]\times X\right) \\ &= -\frac{R_t^{\,2}}{R_t + \phi(t+1)}\Psi(T) = 0 \end{aligned} \quad (12)$$

The optimal asset allocation and consumption strategies are solved:

$$X_2^* = \left[R_t \times \frac{[(p_u - 1)d - (p_u)u]}{d\times u}\times\frac{\phi(t+1)}{(R_t + \phi(t+1))}\right]\times X \quad (13)$$

$$c_t^* = \frac{R_t^{\,2}}{(R_t + \Psi)} \times X \tag{14}$$

Short selling of stock asset is strictly prohibited with $p_u > q_u$. Satisfying the prohibition of short selling on bond, thus the required condition is $p_u < qu.u/(1+R)$. Satisfying the prohibition of short selling on bond and stock, thus the required condition is:

$$q_u < p_u < \frac{q_u \times u}{(1+R)}$$

Proof: Using the standard stochastic dynamic programming argument and for further details) and the Bellman equation of the optimal value function, V is described:

$$v(t, x) = (X_2(t)^{max}, C(t))E_{t,x} \begin{Bmatrix} u_1(C(t)) + \\ R_t^{-1}V(t+1, X(t+1)) \end{Bmatrix}$$

Substituted from Eq. 11 and 15 and rearranging the equation:

$$\begin{aligned} \phi(t)\times \ln(X) + \psi(t) &= \left[1 + \frac{\phi(t+1)}{R_t}\right]\times \ln(X) \\ &+ \left[\begin{matrix} \ln - \left(\frac{R_t^{\,2}}{R_t + \phi(t+1)}\right) + \frac{\rho u \times \phi(t+1)}{R_t} \times \\ \ln\left(\begin{matrix} R_t + \\ \left(R_{t'}\, \frac{[(p_u - 1)d - (p_u)u]}{d\times u} \times \frac{\phi(t+1)}{(R_t + \phi(t+1))}\right)\times u \end{matrix}\right) \end{matrix}\right] \\ &+ \frac{(1-\alpha)\times\phi(t+1)}{R_t}\times \ln\left(\begin{matrix} R_t + R_t \times \frac{[(p_u - 1)d - (p_u)u]}{d\times u} \\ \times \frac{\phi(t+1)}{(R_t + \phi(t+1))}\times d \end{matrix}\right) \\ &- \frac{\phi(t+1)}{R_t}\times\left(\frac{X\times R_t^{\,2}}{R_t + \phi(t+1)}\right) + \frac{\Psi(t+1)}{R_t} \end{aligned} \tag{15}$$

Applying Euler condition on static optimization with the respect to X_2 gives:

$$\begin{aligned} 0 &= \frac{p_u\phi(t+1)}{X^{(+)}(t+1)}\hat{u} + \frac{p_d\phi(t+1)}{X^{(-)}(t+1)}\hat{d}_n \\ 0 &= p_u\phi(t+1)\hat{u}\times X^{(-)}(t+1) \\ &\quad + p_d\phi(t+1)\hat{d}\times X^{(+)}(t+1) \end{aligned}$$

$$\begin{aligned} 0 &= p_u\phi(t+1)\hat{u}\times\left[\left(R_t\times X(t) - C(t)\right) + X_2(t)\times\hat{d}\right] \\ &\quad + p_d\phi(t+1)\hat{d}\times\left[\left(R_t\times X(t) - C(t)\right) + X_2(t)\times\hat{u}\right] \\ 0 &= \phi(t+1)\left(R_t\times X(t) - C(t)\right)\left[p_u\hat{u} + p_d\hat{d}\right] + \\ &\quad X_2(t)\hat{u}\hat{d}\left[p_u\phi(t+1) + p_d\phi(t+1)\right] \\ 0 &= \phi(t+1)\left(R_t\times X(t) - C(t)\right)\left[p_u\hat{u} + p_d\hat{d}\right] + \\ &\quad X_2(t)\hat{u}\hat{d}\phi\,(t+1) \end{aligned} \tag{16}$$

and the solution can be obtained:

$$X_2(t) = R_t \frac{[(p_u - 1)d - (p_u)up]}{d\times up} \times \frac{\phi(t+1)}{\left(R_t + \phi(t+1)\right)} \times x \tag{17}$$

The same procedure of application of Euler condition with the respect to C gives:

$$\begin{aligned} 0 &= \frac{1}{C(t)} - \frac{p_u\phi(t+1)}{R_t\times X^{(+)}(t+1)} - \frac{p_d\phi(t+1)}{R_t\times X^{(-)}(t+1)} \\ 0 &= R_t X^{(+)}(t+1)\times R_t X^{(-)}(t+1) - C(t)\phi(t+1) \\ &\left[p_u X^{(-)}(t+1) + p_d X^{(+)}(t+1)\right] \\ 0 &= R_t\left[\left(R_t\times X(t) - C(t)\right) + X_2(t)\times\hat{u}\right] \\ &\left[\left(R_t\times X(t) - C(t)\right) + X_2(t)\times\hat{d}\right] \\ &- C(t)\phi(t+1)\begin{Bmatrix} p_u\left[\left(R_t\times X(t) - C(t)\right) + X_2(t)\times\hat{d}\right] \\ + p_d\left[\left(R_t\times X(t) - C(t)\right) + X_2(t)\times\hat{u}\right] \end{Bmatrix} \end{aligned} \tag{18}$$

Then, the equation of algebra can be obtained:

$$\begin{aligned} 0 &= R_t^{\,2}\times X(t)\left[1 - \frac{p_u\hat{u} + p_d\hat{d}}{\hat{d}}\right]\times\left[1 - \frac{p_u\hat{u} + p_d\hat{d}}{\hat{u}}\right] \\ &- C(t)\begin{Bmatrix} R_t\times\left[1 - \frac{p_u\hat{u} + p_d\hat{d}}{\hat{d}}\right]\times\left[1 - \frac{p_u\hat{u} + p_d\hat{d}}{\hat{u}}\right] \\ + \phi(t+1)\left[\left(1 - \frac{p_u\hat{u} + p_d\hat{d}}{\hat{u}}\right)\left(-\frac{p_u\hat{u} + p_d\hat{d}}{\hat{d}}\right)\right] \end{Bmatrix} \end{aligned} \tag{19}$$

From which the expression of in follows. The optimal asset allocation decision (t) is obtained after replacing by which indeed proves the claim. Inserting the equation into the Bellman's equation, then the simple form can be obtained:

$$c(t)\times\ln(X)+\psi(t)=$$
$$\left[1+\frac{\phi(t+1)}{R_t}\right]\times\ln(x)+\left[\begin{array}{l}\ln\left(\dfrac{R_t^2}{R_t+\phi(t+1)}\right)+\dfrac{a\times\phi(t+1)}{R_t}\times\ln R_t\\ \left(1+\left(\begin{array}{c}\dfrac{\left[(p_u-1)\hat{d}-(p_u)\hat{u}\right]}{\hat{d}\hat{u}}\\ \times\dfrac{\phi(t+1)}{(R_t+\phi(t+1))}\end{array}\right)\times\hat{u}\right)\end{array}\right]+$$
$$\frac{(1-a)\times\phi(t+1)}{R_t}\times\ln\left(1+\frac{\left[(p_u-1)\hat{d}-(p_u)\hat{u}\right]}{\hat{d}\hat{u}}\times\frac{\phi(t+1)}{(R_t+\phi(t+1))}\times\hat{d}\right)$$
$$-\frac{\phi(t+1)}{R_t}\times\left(\frac{X\times R_t^2}{R_t+\phi(t+1)}\right)+\frac{\psi(t+1)}{R_t} \tag{20}$$

The proof for claims in is complete by checking $\omega_2(t)>0$, $\omega_1(t)>0$ and $0<\omega_2(t)<1$ are satisfied under the condition on p_u.

RESULTS AND DISCUSSION

The numerical example to be implied into this model are those that satisfying the proposition constraints, which already described in Eq. 1-4. The Indonesian risky asset that satisfied those constrains is MAYORA stock (MYOR.JK) in Rupiah (IDR), based on weekly basis price (from January 1st , 2008 to January 1st, 2012. As for the bond, none of the Indonesian bonds satisfy those constrains, instead we search for other Asian bonds and one that suitable is the Japanese Government Bond with 2 year yield selected with R = 0.23%.

Portfolio initial investment is Rp. 1,000,000. The stock prices movement is taken from closing prices and started from 01-01-2008-01-01-2013. It could be seen in Fig. 1. The growth rate and volatility of MYOR. JK are respectively, u = 0.95%, σ = 9.7%. and The risk-free asset of which Japanese Government Bond with 5 year yield been selected has annual return of 0.24%.

Following proposition, the value function of the portfolio shown in Fig. 2 below is decreasing from Rp. 2,114-0 as the time goes to maturity and forms a concave function. The consumption increases from Rp. 3,9178-500,000. The composition of the stock weight and the bond are varying respectively from 88.26-0 and 11.74-100% as it can be seen that the portfolio is dominated by bond at the end of period. In Fig. 3, the function of and are is also fulfilling the proposition of 3.1.

Further is the simulation of the wealth process X(t) using the data of MYOR.JK, the amount of consumption C(t) and the wealth portion invested in bond $X_1(t)$ and stock $X_2(t)$ and the portfolio time-value process V(t). The simulation is shown in Fig. 4. Due to the withdrawal of the consumption by the investor which goes increasingly, results that the wealth process of X(t) is going up and down in decreasing manner. The portfolio V(t) is also moving randomly up and down. These all findings resemble with the theoretical facts in Proposition 3.1.

Model sensitivity: The optimal model will be perturbed to ensure that the solution of optimal consumption C*(t) and optimal trading strategy $X_2^*(t)$ is the highest giving wealth and consumption for Investor. Compared with another number in parameter $X_2(t)$, one is held constant at 50% and one is $X_2^*(t)$-0.15 for which the wealth dynamics as well as consumption rate are about to derived. The results are depicted in Fig. 5 and 6.

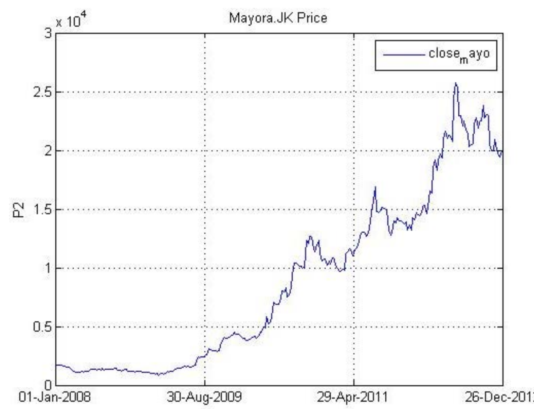

Fig. 1: Mayora stock (MYOR.JK) stock price movement from 01-01-2008 to 01-01-2013; the data is taken from www.finance.yahoo.com accessed at 26 September 2013

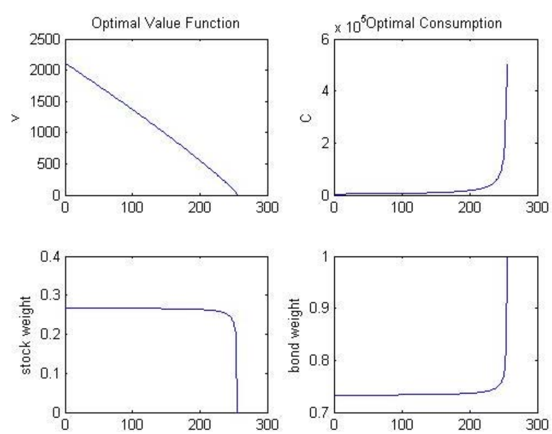

Fig. 2: Optimal Portfolio of Consumption and Weight, Based on Proposition 3.1 (Model)

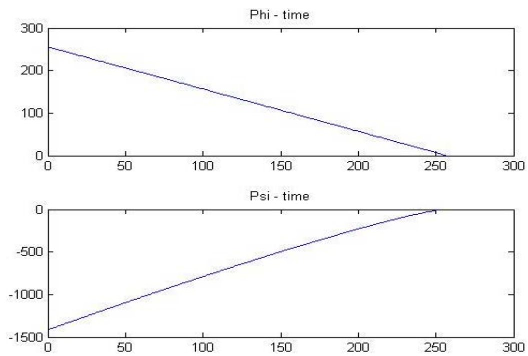

Fig. 3: Function of φ(t) and ψ(t) based on proposition 3.1. Results has shown that optimal consumption C*(t) and optimal trading strategy X_2*(t) is the highest giving wealth and consumption for Investor after 84 day of investment period

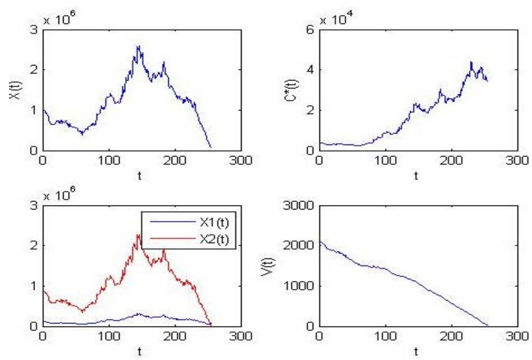

Fig. 4: Upper left showed total investor wealth process with optimal trading strategy while lower left showed the wealth process for stock and bond, respectively. Upper right showed the consumption rate of investor which reach optimally at the end of period. Lower right showed value function decreasing as time reach maturity

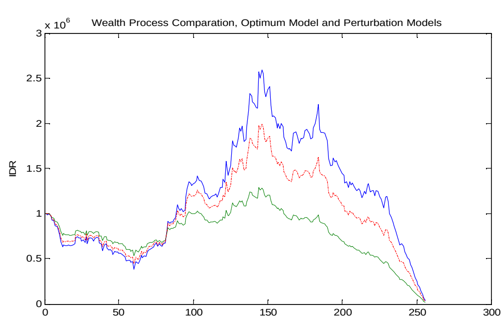

Fig. 5: The wealth process of optimum model (the solid line) and perturbation models (the dash line is the model held 50% for X_2 and the dot line is the model with X_2 = X_2*-0.15)

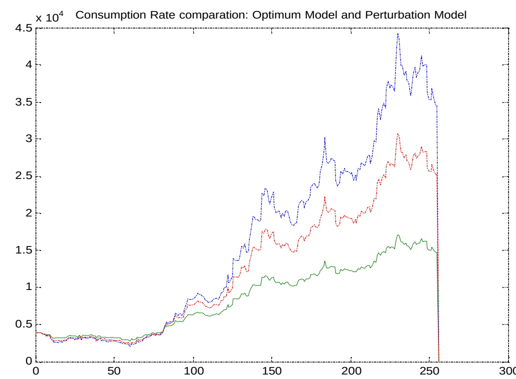

Fig. 6: The consumption rate of optimum model (the solid line) and perturbation models (the dash line is the model held 50% for X_2 and the dot line is the model with X_2 = X_2*-0.15)

CONCLUSION

In this study, we have derived the optimal asset allocation strategy of a utility maximized investor dynamic in life-time investment. Closed form expressions were obtained for the optimal portfolio strategy that maximize the expected total discounted log-utility of consumption over finite life time while short selling is not allowed for both stock and bond. In this model, the investor is constrained from the assumption that the injection of exogenous capital into portfolio is not allowed. But the consumption is designed so that investor can withdrawal some portion from trading gain. The optimal portfolio was achieved from an investment in bond and stock which respectively grows at constant rate and using simple model of Markov chain. The model was implemented into Mayora stock from Indonesian Stock Exchange and 2 times of Japanese

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