

## Non-Linear Control Laws for Electric Power System Stabilization

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**Abstract:** Derivation of nonlinear synchronous generator excitation controllers for improving power system stability is described in this paper. These controllers are based on the existing universal Higher-Order Sliding Mode control Structure (HSMCS), furnishing the system with robustness quality in the presence of disturbance. A significant aspect of the paper is the modification of the HSMCS for enhanced system dynamic performance and better robustness. The parameters of the controllers are carefully selected by simulation using MATLAB® software and various results showing system performance under network fault conditions are presented. The results show the ability of the modified controller to withstand longer fault durations of 14.7 and 14.5 cycles for a three-phase symmetrical fault located at an infinite bus and generator terminal, respectively.

**Key words:** Fault cycle, control law, power system model, power system stability, Sliding Mode Control (SMC)

### INTRODUCTION

Electric power systems are complex and highly non-linear with response characteristics that are a function of the behavior of a number of its components (Kundur *et al.*, 2004). It is very desirable that they operate at all times with high security and reliability in order to meet the consumer demands that they serve; particularly, in a deregulated large-scale power system environment, it is of paramount importance that the power system has the ability to meet stringent performance requirements which arise as an upshot of, among other things, stressed operating conditions and uncertain power flow paths (Chow *et al.*, 2005). Chief among the security and reliability criteria of electric power networks is stability which is a dynamic attribute that has to be ensured and maintained before the operation of any power system can be considered satisfactory. Practically, generating unit controllers such as the prime mover and excitation systems are being employed in the power systems, besides the power and voltage controllers at transmission and distribution levels, not only to realize safe operation of an electric power system, but also to enhance system dynamic performance (Mariani and Murthy, 1997). Specifically, the generating unit excitation control system plays a pivotal role in enhancing systems stability and dynamic response to major network disturbances. And, conventionally, the combination of an Automatic Voltage Regulator (AVR) and a Power System Stabilizer (PSS) are used to provide constant output voltage and damp low-frequency oscillations in power systems (Steinmetz,

1920). But because their structures are linear, the AVR and PSS have limited dynamic responses (Ortega *et al.*, 2005). In the literature, several configurations of the PSS are available to provide positive damping torque to improve the overall generator rotor damping (Mello *et al.*, 1978; Ghandakly and Farhoud, 1992; Irving *et al.*, 1979; Kasturi and Doraraju, 1970; Kundur *et al.*, 1989; Larsen and Swann, 1981; Lim and Elangovan, 1985). Also, these various PSS forms are generally based on the use of control design methods with underlying linearized models of the power systems considered. So improving power system stability by damping electromechanical oscillations using these PSS arrangements and other similar ones (Falkner and Heck, 1995; Magid *et al.*, 1999) has been found to be limited due to the highly nonlinear nature of the system (Chiang, 2011). In furtherance of alternative control strategies for better stabilization of electric power systems, this paper considers a universal high-order sliding mode control structure which is appropriately tuned to damp oscillations due to network disturbance and give improved stability and overall dynamic response. An advantage of this structure is that it preserves the nonlinear nature of the power system and a significant aspect of the paper is the modification of the structure for enhanced system dynamic performance and better robustness.

### MATERIALS AND METHODS

**Power system model:** The model of the power system network considered in this research is that of a single

machine connected to an infinite bus, popularly called SMIB. Even though it is a simplified representation of the system, it allows complex nonlinear system analysis and design to be carried out for the system and provide fundamental understanding and results upon which further stability analyses can be based. The general structure of a SMIB system is shown in Fig. 1 (Kundur, 1994). A typical configuration is given in Fig. 2 (Mahmud *et al.*, 2011) with its simplified representation depicted in Fig. 3.  $Z_E$  in Fig. 2 is the equivalent impedance between the transformer terminal and the infinite bus and is expressed as

$$Z_E = R_E + jX_E \quad (1)$$

where  $R_E$  and  $X_E$  are the equivalent transmission line resistance and reactance, respectively. The values of parameters  $R_E$  and  $X_E$  are lumped together with that of the generator and transformer. In other words,  $R_E$  is added to the generator armature resistance to form the overall resistance, while the sum of  $X_E$  and  $X_T$  (transformer reactance) is added to each generator reactance to get the appropriate overall reactance. The mathematical representation describing the dynamic behavior of the SMIB is given by the following third-order non-linear state-space model (Sauer *et al.*, 1988; Kokotovic and Sauer, 1989):

$$\frac{d\delta}{dt} = (\omega - \omega_s) \quad (2)$$

$$\frac{d\omega}{dt} = A_1 + \frac{F_2 V^2}{2} \sin 2\delta - A_4 V E'_q \sin \delta - T'_{d0} \frac{F_3 F_1}{M} V^2 \cos^2 \delta (\omega - \omega_s) \quad (3)$$

$$\frac{dE'_q}{dt} = -B E'_q + B_2 V \cos \delta + \frac{1}{T'_{d0}} E_f \quad (4)$$

Where:

- $\delta$  = The rotor or torque angle in radians
- $\omega$  = The rotor speed in radians/s
- $E'_q$  = The q-axis voltage which is proportional to the field winding flux linkage
- $V$  = The magnitude of the voltage of the infinite bus
- $\omega_s$  = The synchronous speed of the generator
- $E_f$  = Represents the excitation coil voltage
- $M$  =  $2H/\omega_s$  is the moment of inertia
- $H$  = The generator inertia constant in seconds
- $T'_{d0}$  and  $T''_{d0}$  = The d-axis and q-axis open-circuit transient time constants, respectively

All the other parameters in the model, i.e.,  $A_1$ ,  $A_4$ ,  $B_1$ ,  $B_2$ ,  $F_1$ - $F_3$  are given in the Eq. 5-7 (Table 1).

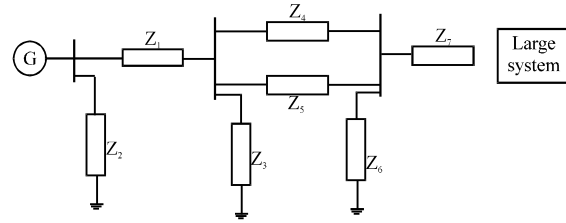


Fig. 1: General representation of a SMIB

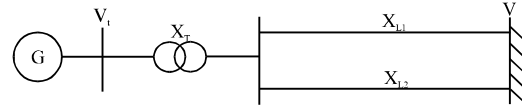


Fig. 2: Simplified representation of a SMIB

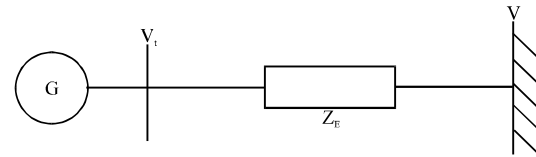


Fig. 3: Equivalent representation of a SMIB

$$A_1 = \frac{T_m}{M}; A_4 = \left( \frac{1}{X'_d + X_E} \right) \frac{1}{M} \quad (5)$$

$$B_1 = \frac{(X_d + X_E)}{T'_{d0} (X'_d + X_E)}; B_2 = \frac{(X_d + X'_d)}{T'_{d0} (X'_d + X_E)} \quad (6)$$

$$F_1 = \frac{(X_q + X'_q)}{(X_q + X_E)}; F_2 = \left( \frac{1}{X'_d + X_E} - \frac{1}{X_q + X_E} \right) \frac{1}{M} \quad (7)$$

$$F_3 = \left( \frac{1}{X_q + X_E} \right)$$

The last term in Eq. 3, defined as the damper-winding torque component, equals  $D/M$ , where  $D$  is called the damping constant. Thus, Eq. 2-4 are equivalent to the popular flux-decay model (Anderson and Fouad, 2003; Fusco and Russo, 2012; Kundur, 1994).

$$\frac{d\delta}{dt} = \omega - \omega_s \quad (8)$$

$$\frac{d\omega}{dt} = A_1 - \frac{D}{M} (\omega - \omega_s) + \frac{F_2 V^2}{2} \sin 2\delta - A_4 V E'_q \sin \delta \quad (9)$$

$$\frac{dE'_q}{dt} = -B E'_q + B_2 V \cos \delta + \frac{1}{T'_{d0}} E_f \quad (10)$$

**Control law derivation:** This study presents two non-linear excitation control signals based on the model given previously. The first control signal is derived directly from the general higher-order SMC structure (Levant, 2001, 2005):

$$u_1 = -K\varphi_{r-1,r}(h, \dot{h}, \ddot{h}, \dots, h^{(r-1)}) \quad (11)$$

Where:

$$\varphi_{j,r} = \text{sat} \left( \frac{h^{(j)} + \alpha_j M_{j,r} \varphi_{j-1,r}}{M_r}, \varepsilon_j \right)$$

$$\varphi_{0,r} = \text{sat} \left( \frac{h}{|h|}, \varepsilon_0 \right)$$

And:

$$M_{j,r} = \left( |h|^{\frac{q}{r}} + |h|^{\frac{q}{(r-1)}} + \dots + |h|^{\frac{q}{(r-j+1)}} \right)^{\frac{r-j}{q}}$$

$$M_r = \left( |h|^{\frac{q}{r}} + |h|^{\frac{q}{(r-1)}} + \dots + |h|^{\frac{q}{(r-1)}} \right)^{\frac{1}{q}}$$

for  $j = 1, 2, \dots, r-1$ ,  $\varepsilon_j > 0$  and  $q = r!$ .  $K$  and  $\alpha_j$  are the parameters of the controller. The second control signal employs a combination of the saturation and signum functions for control switching in a more effective way so as to make  $u_1$  in Eq.11 settle more quickly to its steady-state value and also endow the system with the ability to withstand greater fault duration. It is given by:

$$u_2 = -K\text{sat}((\Gamma_{r-1,r}, (h, \dot{h}, \ddot{h}, \dots, h^{(r-1)}), \varepsilon) \quad (12)$$

Where:

$$\Gamma_{j,r} = h^{(j)} + \alpha_j M_{j,r} \text{sat}((\Gamma_{j-1,r}), \varepsilon)$$

$$\Gamma_{1,r} = h + \alpha_1 M_{1,r} \text{sign}(\Gamma_{0,r})$$

$$\Gamma_{0,r} = h$$

In Eq. 11 and 12,  $r$  stands for the relative degree of the system with respect to an output function  $h(x)$ .  $h(x)$  must be properly selected to give stable zero dynamics. The control signals  $u_1$  and  $u_2$  which must be finite and bounded, guarantee finite-time stabilization of the

system under uncertainty by ensuring that the condition  $\ddot{h}(x) = \dot{h}(x) = h(x) = \dots = h^{(r-1)}(x) = 0$  is satisfied. By selecting a deviation of the rotor angle from its nominal steady-state value, i.e.,  $\delta_\Delta$  as the output function, thereby yielding  $r = 3$ ,  $u_1$  and  $u_2$  result in the following.

$$u_1 = -K\varphi_{2,3}(h, \dot{h}, \ddot{h}) = -K\text{sat} \left( \frac{\ddot{h} + \alpha_2 M_{2,3} \varphi_{1,3}}{M_3}, \varepsilon \right)$$

With:

$$\varphi_{1,3} = \text{sat} \left( \frac{\dot{h} + \alpha_1 M_{1,3} \varphi_{0,3}}{M_3}, \varepsilon \right)$$

$$\varphi_{0,3} = \text{sat} \left( \frac{h}{|h|}, \varepsilon \right)$$

$$M_{2,3} = \left( |\dot{h}|^2 + |\ddot{h}|^3 \right)^{1/6}$$

$$M_{1,3} = \left( |h|^2 \right)^{1/3}$$

And:

$$M_3 = \left( |h|^2 + |\dot{h}|^3 + |\ddot{h}|^6 \right)^{1/6}$$

Likewise:

$$u_2 = -K\text{sat}((\Gamma_{2,3}, h, \dot{h}, \ddot{h}), \varepsilon) \\ = -K\text{sat}((\ddot{h} + \alpha_2 M_{2,3} \text{sat}(\Gamma_{1,3}, \varepsilon)), \varepsilon)$$

With:

$$\Gamma_{1,3} = \dot{h} + \alpha_1 M_{1,3} \text{sign}(\Gamma_{0,3}) \text{ and } \Gamma_{0,3} = h$$

The system relative degree  $r$  is very imperative to the application of these control laws. And it can be determined by using the flowchart in Fig. 3 together with the MATLAB code given in Appendix B. This chart is produced on the basis of Definition 1 (Isidori, 1995).

**Definition 1:** Consider the general nonlinear system:

$$\dot{x} = f(x, t) + g(x)u$$

The relative degree of this system with respect to an intuitively pre-selected output function  $y(x)$  is the value of  $k$  such that:

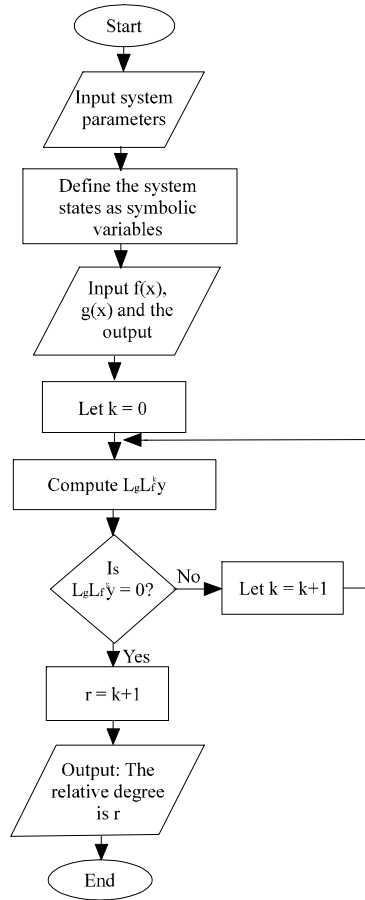


Fig. 4: Flowchart for computing the relative degree of a non-linear system

$$L_g y(x) = L_g L_f y(x) = L_g L_f^2 y(x) = \dots = L_g L_f^{k-2} y(x) = 0$$

But:

$$L_g L_f^{k-1} y(x) \neq 0$$

within a region of  $x = x_0$ ; where  $L_g L_f^i y(x)$  represents the Lie derivative of  $L_f^i y(x)$  along the function  $g(x)$ .

## RESULTS AND DISCUSSION

The performances of the control laws are examined in this study. A solid symmetrical three-phase fault which is simulated by a sudden reduction of the infinite bus voltage to zero, is applied to create a temporary mismatch between electromagnetic torque ( $T_e$ ) and input mechanical torque ( $T_m$ ); post-fault and pre-fault conditions are assumed to be the same. Two fault locations are examined: one at the infinite bus and the other at the generator terminals. Also, the fault clearance time is varied to show the action of the control laws in retaining system stability and improving damping. The values of both the system and control law parameters used are provided in Eq. 2-7. The system operating point for a constant excitation voltage of 1.5603 pu is  $(\delta_0, \omega_s, E_{q0}') = (38.7621, 314.2857, 1.1300)$ . Figure 4-7 present sets of waveforms of system variables for a three-phase fault at the infinite bus with fault durations of 7.9 and 14.7 cycles,

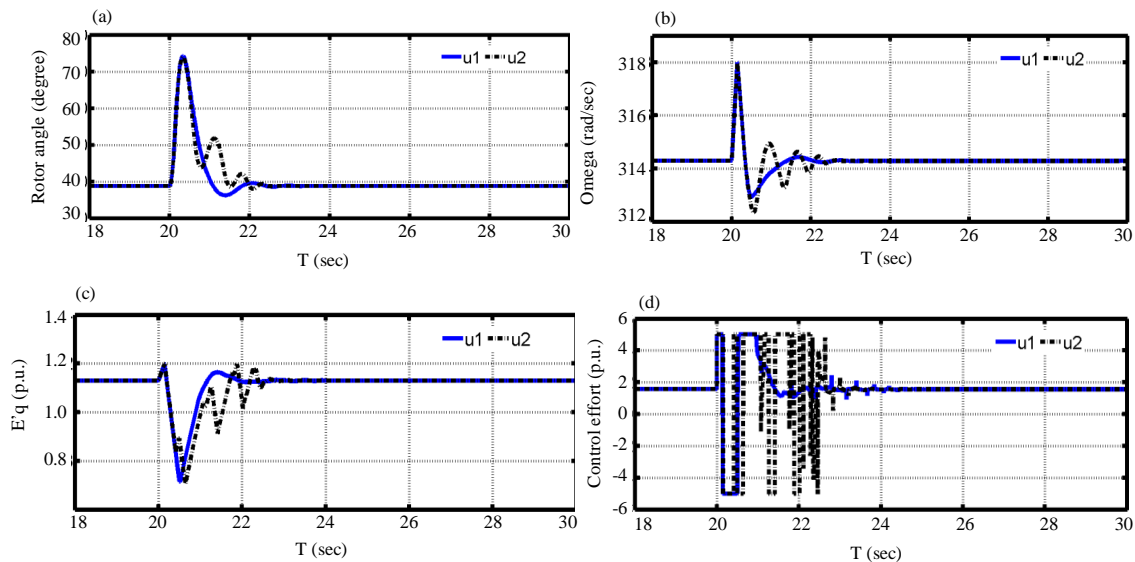


Fig. 5: Waveforms comparing the performances of control laws (u1, u2) for an infinite bus fault cleared after 7 cycles

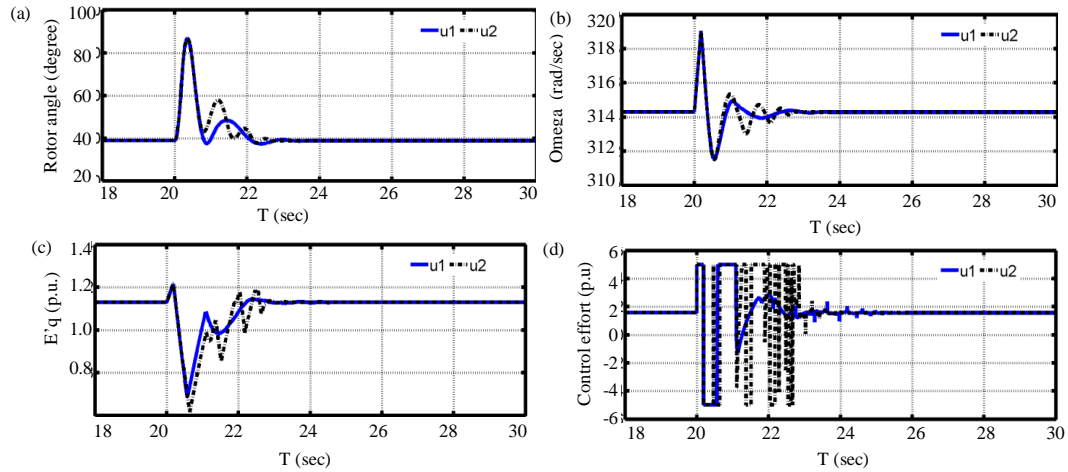


Fig. 6: Waveforms comparing the performances of control laws ( $u_1$ ,  $u_2$ ) for an infinite bus fault cleared after 9 cycles

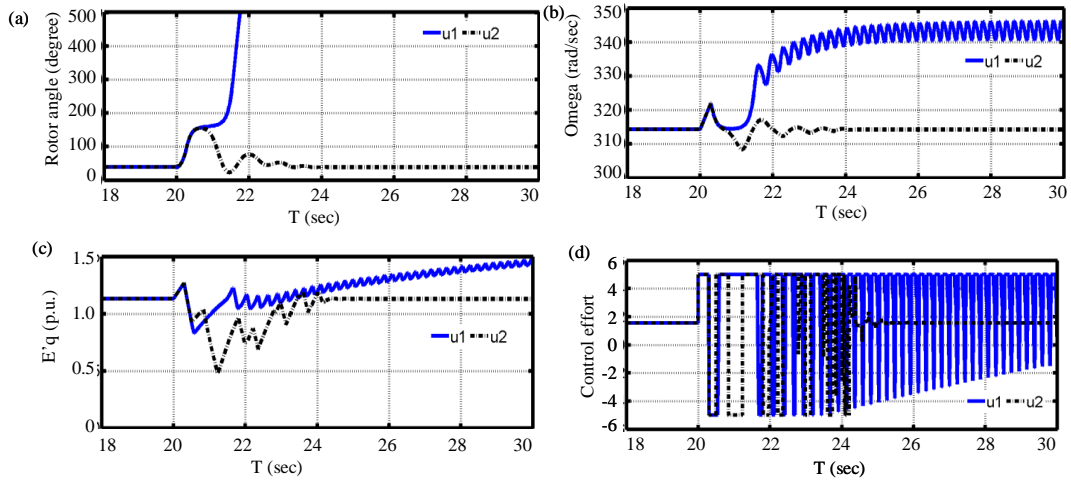


Fig. 7: Waveforms comparing the performances of control laws ( $u_1$ ,  $u_2$ ) for an infinite bus fault cleared after 14.7 cycles

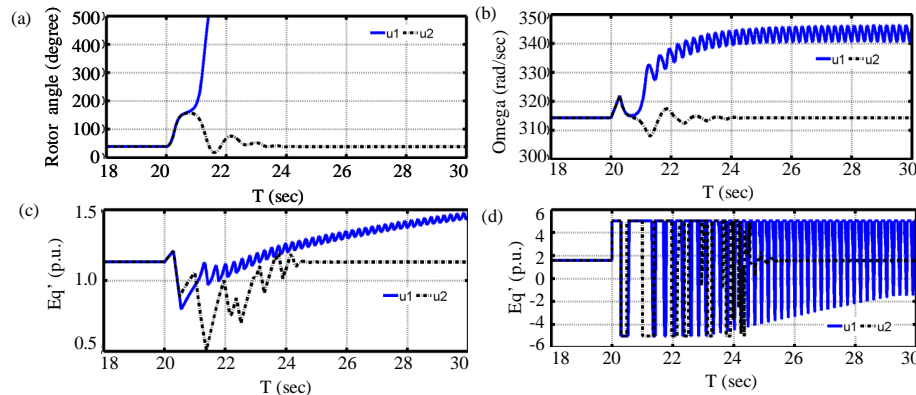


Fig. 8: Waveforms comparing the performances of control laws ( $u_1$ ,  $u_2$ ) for a generator terminal fault cleared after 14.5 cycles

respectively. The relative action of the two control laws is clearly depicted, demonstrating that the effort of control law  $u_1$  to damp oscillations and restore the system

variables to their pre-fault operating values degrades as the fault duration elongates. But for much smaller fault durations,  $u_1$  exhibits better performance from power

oscillation point of view with minimum adjustment of  $E'_q$  (adjustment needed in the generator EMF through manipulation of the excitation system to counter the demagnetization effect of the armature reaction during fault). Figure 8 show similar waveforms for a fault at the generator terminal with a duration of 14.5 cycles. Because of the severity of the fault at this location,  $u_i$  could only restore system variables for up to below a 14.5-cycle fault (unlike the previous case). But generally, the system variables settle to their steady-state values within 2.4 sec after the removal of the fault.

## CONCLUSION

This study has presented two non-linear synchronous generator excitation control laws based on the universal higher-order sliding mode control structure (HSMCS). Whereas the first control law is a direct application of the HSMCS, the second law is a modified form of the first, derived to minimize the computational time (since fewer calculations are required) and also to make the system able to withstand longer fault duration. System simulations under the action of these control laws for a fault condition tested with various durations of time have also been provided to show the relative performance of the laws. Meanwhile, this kind of control signals can be implemented in a static exciter configuration having a very fast response. And it is very necessary to employ a stable and robust online algorithm for obtaining the time derivatives of the selected system output function. The values of the system parameters used in this study are as (Anderson and Fouad, 2003; Sauer *et al.*, 1988).

## APPENDIX

MATLAB Code for finding the relative degree of a general affine non-linear system:

```
%This function Relative Degree = RELDEG (F, G, H, S) is used to
determine the relative degree of any given affine non-linear SISO system
dx/dt = f(X)+g(X)u, y = h(X), where X represents the % states (x1,
x2,...,xn) of the system. F, G, H and S are symbolic %expressions for f(x),
g(x), h(x) %and the states, respectively; f and g vector functions and h is a
scalar function. % RelativeDegree is a positive integer between 1 and the
order (i.e., n) of the system. Note that %the order of the system must be at
least 2. ALSO, NOTE THAT THE STATES IN F, G AND %H APPEAR
AS x1, x2, x3,..., xn with THESE , OF COURSE, HAVING BEEN
%DEFINED AS SYMBOLIC VARIABLES. For example, the system
dx(1)/dt = x(1)sin %x(2)+20x(1)-2u, dx(2)/dt = cos %x(1)+ 10u and y =
x(1) + x(2) having steady-state values %x0(1) = 0.5 and x0(2) = 2 is
%created as: syms x1 x2 f g h
% f=[x1*sin(x2)+ 20*x1 cos(x1)+10];g=[-2 10];h=x1+x2;x=[x1 x2];
function reldegResult=reldeg(f,g,h,x)
sysorder=length(f);
m=zeros(1,sysorder);d=zeros(1,sysorder);
LfHx=sym(m);LgLfHx=sym(d);
LfHx(1)=h; % the first element of LfHx
% Compute the other elements of LfHx
if sysorder==2
```

```
LfHx(sysorder)=jacobian(LfHx(sysorder-1),x)*f;
else
for k=2:sysorder
LfHx(k)=jacobian(LfHx(k-1),x)*f;
end
end
% Compute the elements of LgLfHx
for k=1:sysorder
LgLfHx(k)=jacobian(LfHx(k),x)*g;
end
% Find the relative degree of the system
input('Enter all the n steady-state values as : x1 = ; x2 = ; x3 = ... ; xn
= ; ')
input('Enter the values for all the system parameters if any or press the return
key ')
LgLfHx_comp=subs(LgLfHx);
p=find(LgLfHx_comp);
RelativeDegree=p(1);
% Output the result
reldegResult=['the relative degree of the system is: ' num 2st
r(RelativeDegree)];
```

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