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Design Nonlinear Model Predictive Controller with Contractive Constraint in Fast Terminal Sliding Mode of Plate Heat Exchanger

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Abstract: In this study, a model predictive control algorithm based on rapid terminal contractive constraint is considered for the first time. The advantage of the proposed terminal contractive constraint is that while ensuring the stability of the closed-loop control system, the speed of system responsiveness convergence to the reference value is increased. To evaluate the performance, the proposed algorithm on a plate heat exchanger that has constraint input variables and states is implemented with dynamic discretization. The simulation results show the effectiveness of this approach in meeting the constraints, increase speed of convergence and improve the performance of the system.

Key words: Model predictive control, fast terminal sliding mode, contractive constraint, plate heat exchanger

INTRODUCTION

Model predictive control is a control method that the optimization problem solving with limited horizons will be done online at any time of sampling process. In practice, all processes are limited for example, the limitation of non saturation of operator, state restrictions to set the environmental conditions and high product quality (Hong and Frank, 1999). One of specifications of model predictive control is its capacity to systems with high limitations on control inputs and states (Hong and Frank, 1999). Considering these limitations try to determine the control command so that system act in determined area.

Many researchers in the field of process control have used the ability to control the model predictive against nonlinear and constraint processes and many chemical and industrial processes have used in this method, meanwhile, the MPC theory has expanded increasingly (Li *et al*., 1999; Rawlings and Muske, 1993; Michalska and Mayne, 1993).

To ensure the stability of closed-loop system for nonlinear dynamic, the various constraints are provided to model predictive control. The simplest method is to be added the terminal equality constraint to optimization problem (Keerthi and Gilbert, 1998).

It does require is that the state is limited in the steps precisely converge to zero. This is a conservative approach and optimization problem may be impossible .To increase the feasibility of response, the inequality constraint may be used in return of equality so that terminal states forced to enter an area with equilibrium point instead of placing on the point of balance. By combining the final cost function and the terminal unequal constraint, the control strategy of quasi-unlimited model predictive was introduced in which the infinitive horizons performance with minimum upper band of the final cost function is shown. According to Oliveira and Morari (2000), the contractive constraint is suggested that by adding a final contractive constraint in the optimization, the system stability is guaranteed. One of the most important control methods is Sliding mode. The basis for this method is to change the control law through the trajectory of state variables and placing variables on the stable sliding surface asymptotically.

Control algorithm that combines model predictive control and sliding mode control is called (SM_MPC) that by Zhou *et al.* (2000) a systems with state-space model was proposed in which sliding coefficient is used as a new variable and will stabilize using Dual mode MPC.

In this study, a Fast Terminal Sliding Mode combined the Model Predictive Control (FTSM-MPC) and similar to (Zhou *et al*., 2000), this study is defined sliding coefficient as a new variable but unlike it, the contractive constraint has used for sliding coefficient and also to stabilize the nonlinear chemical process, the constraint discrete-time has used. This decreases the sliding coefficient to zero step by step.

Chemical process used in this study is heat exchanger system. The heat exchange is occurred between two liquids of different temperatures that are separated by the hard wall in many engineering processes. The tool is used to implement this exchange is

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called heat exchanger. It means that the heat exchanger is used to implement the exchange. Heat exchangers are usually classified based on fluid flow configuration or structure (plate, tube or stratum and skin). In this study, the temperature control of heat exchangers have been raised and the main purpose of controllers is to set the temperature of the outlet water as desire temperature. Many studys have focused on heat exchangers control systems and offered some more efficient procedures for finding an appropriate control methods that the (Marselle *et al.*, 1982; Calandranis and Stephanopoulos, 1988; Huang and Fan, 1992; Mathisen *et al.*, 1992) can be named as the most important of them.

According to Aguilera and Jacinto (1998) an approach to online optimization and systems control for heat exchange has been proposed and reference (Gonzalez *et al.*, 2006) has investigated the online optimization and control of the heat exchanger using MPC's constraint. This study will introduce FTSM_MPC algorithm with auxiliary contractive constraint on the sliding surface to control the heat exchangers system. This algorithm can stabilize system and also is faster than the proposed procedure and difficult constraints (if any) will also meet.

This study has been set: at first, the fast terminal sliding mode is expressed and in the second section, FTSM_MPC algorithm and its steps is presented by indicating auxiliary contractive constraint. The third section discussed the stability and prove it. Fourth section is discussed the system plate heat exchanger dynamics and then next section is about the purposed control method for a discrete sample of system, the simulation results are given in six section and finally the conclusion is offered in seven section.

MATERIALS AND METHODS

Fast terminal sliding mode: Sliding Mode of class terminal is of sliding mode which suggested for the first time by Venkataraman and Gulati (1993), its difference from the ordinary sliding mode technique is that it ensures a finite convergence time to the state variables. If the ordinary sliding mode ensures asymptotic stability in such a case, we have variables converge to their origin but this convergence can be ensured only for infinitive time period. In terminal sliding surface is introduced as follows:

$$s = x_2 + \beta x_1^{q/p} \tag{1}$$

A little later, some researchers (Yu *et al.*, 1999) presented the new sliding surface as:

$$s = x_2 + \alpha x_1 + \beta x_1^{q/p} \tag{2}$$

where, αx_1 ensures faster convergence time so it is called fast terminal. The control law also comes from solving below problem:

$$s(k+1) = 0 \tag{3}$$

$$x(k+1) = f(x(k)) + g(x(k))u(k) \tag{4}$$

So that, x, f(x), g(x)ϵR and $g(x)^{-1}$ are finite and existed. From Eq. 2 the control law to solve above problem will be as follows:

$$u_{eq} = \frac{1}{g(x)\left[-f(x) - \alpha x, \beta \frac{q}{p} x_1^{q-p/p} x_2\right]} \tag{5}$$

It is clearly seen when we enter into the sliding surface, the controller performance will lead x_1 and x_2 to zero.

FTSM_MPC algorithm with contractive constraint for sliding surface: According to Zhou *et al.* (2000), model predictive control combined with sliding mode control and new algorithm presented as follow:

$$p(k,s,N): \arg_u\left(k|k\right), u\left(k|k+1\right) - u\left(k|k+N\right) \min J\left(s(k), u(o)\right) \tag{6}$$

S. t.:

$$x(k+i|k) \in x \; i = 0, 1, ..., N \tag{7}$$

$$u(k+i|k) \in u \; i = 0, 1, ..., N \tag{8}$$

$$3.x(k+i+1|k) = f\left(x(k+i|k), u(k+i|k)\right) \tag{9}$$

Where:

$$J\left(s(k), u(0)\right) = \sum_{1=0}^{N-1}\left\{\left\|s(k+i|k)\right\| + s(k+N|k)\right\|_P^2 \tag{10}$$

And the corresponding weighting matrices Q, P, R are positive definite in this study, a fast terminal sliding surface is used instead of sliding coefficient. Also difficulty of calculation the matrix P of final cost function $\|s(k+N|k)\|_P^2$ for non-linear systems and difficulty the possibility of these responses caused to add the auxiliary constraint so that provide the contraction or reduction state on sliding surface as follows:

$$\left\|s\left((k+N|k)\right\|\le\alpha\left\|s(k|k)\right\|, \alpha\in(0,1)\right. \quad (11)$$

where, α is coefficient of contraction. The steps of FTSM_MPC algorithm can be explained as follows.

Data: Weighting matrices Q, P, R>0, the prediction horizon N, the sampling time T, the coefficient of contraction α∈(0, 1), the u∈X constraints are u∈U, x∈X considered for states and inputs, respectively and initial conditions x_0.

First step: k = 0 is set and zero initial prediction control law is considered:

$$u(i;k)=0, i\in[0,1,2,...,N-1] \quad (12)$$

Second step: The optimization problem (3) is resolved in time t_k. A wave of the control sequence u*(i; k) can be obtained.

Third step: The first rule u*(1; k) is achieved using control sequence and enters into the dynamics of system in the time t_k so that determine the state values at the time of $t_k+1 = t_k+N$.

Fourth step: The value of (t_k) is calculated.

Five step: k = k+1 is placed, if $\|s(t_k)\| = 0$ (states are placed on the sliding surface) algorithm continues, otherwise the algorithm goes back to the second step.

Six step: The system dynamics u_{eq} has used instead of control input to determine the state of system dynamics in next sample. Then algorithm returned to the second step.

Stability: Before verification of algorithm stability is discussed, three assumptions must be determined. Moving on the slide area is stable, this means when the system states were on the sliding surface, this surface keeps these, u_{eq} using on itself and the system states tend to be asymptotic to their origin (Bandyopadhyay *et al.*, 2009).

For k∈(k_j, k_j+1), there is constant parameter φ∈(0, ∞) so that transient sliding coefficient s (k) satisfies the following inequality:

$$\left\|s(k)\right\|p\le\varphi\left\|s(k_j)\right\|p \quad (13)$$

The constant parameter is as σ∈(0, ∞) such that for all $x(k_j)\in B_\sigma = \{x\in R^n \|x\|\le\sigma\}$ limited by the constraints (Michalska and Mayne, 1993; Keerthi and Gilbert, 1988), optimization problem in Eq. 3 is possible for all.

Theorem: If the assumption 1 is true, the contraction coefficient α∈(0, 1), φ∈(0, ∞) and σ∈(0, ∞) is determined and the assumptions 2 and 3 established. Then for all $x_0\in B_\sigma$ if the following inequality is true for all the movement of the sliding factors, the system will be stable:

$$\left\|s(k)\right\|_p\le\varphi\left\|s(0)\right\|_p e^{-(1-\alpha)\left(\frac{k}{N}-1\right)} \quad (14)$$

Proof:

$$\left\|s(k_j)\right\|_p\le\alpha\left\|s(k_{j-1})\right\|_p\le...\le\alpha^j\left\|s(0)\right\|_p \quad (15)$$

$$\left\|s(k)\right\|_p\le\varphi\left\|s(k_j)\right\|_p\le\varphi\alpha^j\left\|s(0)\right\|_p \quad (16)$$

where, j = 1, 2, ... Due to α∈(0, 1) and $k\in z^+$, we have:

$$e^{(\alpha-1)}-\alpha\ge0\Leftrightarrow\alpha^k\le e^{-(1-\alpha)k} \quad (17)$$

Then:

$$\left\|s(k_j)\right\|_p\le\left\|s(0)\right\|_p e^{-(1-\alpha)j} \quad (18)$$

$$\left\|s(k_j)\right\|_p\le\varphi\left\|s(0)\right\|_p e^{-(1-\alpha)j} \quad (19)$$

For $k\in z^+$ from Eq. 19, we can get:

$$\left\|s(k_j)\right\|_p\le\varphi\left\|s(0)\right\|_p e^{-(1-\alpha)\mathrm{int}\left(\frac{k}{N}\right)} \quad (20)$$

Also, it is clear that:

$$\varphi\left\|s(0)\right\|_p e^{-(1-\alpha)\mathrm{int}\left(\frac{k}{N}\right)}\le\varphi\left\|s(0)\right\|_p e^{-(1-\alpha)\left(\frac{k}{N}-1\right)} \quad (21)$$

According to Eq. 20 and 21, we reach to Eq. 14 so the presented algorithm keep system stable.

Plate heat exchanger dynamic: The plate heat exchanger dynamic model can be written as follows (Hangos *et al.*, 2004):

$$T_{co}(t)=-k_1\left(T_{co}(t)-T_{ho}(t)\right)+\frac{U_c}{V_c}\left(T_{c1}-T_{co}(t)\right) \quad (22)$$

And the output is:

$$T_{ho}(t) = -k_2\left(T_{ho}(t) - T_{co}(t)\right) + \frac{1}{V_h}\left(T_{h1} - T_{ho}(t)\right)u(t) \quad (23)$$

$$y(k) = T_{co}(k) \quad (24)$$

Where:

$$k_1 = \frac{UA}{c_{p,c}\rho_c v_c}, k_2 = \frac{UA}{c_{p,h}\rho_h v_h}$$

The system has two state variables: Outlet temperature of the cold water and hot water. The output of the system is taken to be first state and control input is cold water flow rate. As well as other parameters above, are indicated the dynamic characteristics of the system. The system will be discretize with T sampling time as follows:

$$T_{co}(k+1) = T_{co}(k) + T\left\{-k_1\left(T_{co}(t) - T_{ho}(t)\right) + u_c/v_c\left(T_{ci} - T_{co}(t)\right)\right\} \quad (25)$$

$$T_{ho}(k+1) = T_{ho}(k) + T\left\{-k_2\left(T_{ho}(t) - T_{co}(t)\right) + 1/v_h\left(T_{hi} - T_{ho}(t)\right)u(t)\right\} \quad (26)$$

The objective of the control scheme is to regulate the output to a desired constant value T_{cr}. According to the Eq. 22 and 23 can be seen clearly if $y(k) = T_{co}(k) = T_{cr}$, the output value of hot water will push toward $U_c/k_1v_c(T_{cr}-T_{c1})+T_{cr}$ so, x_2 and x_1 variable are defined as:

$$x_1(k) = T_{co}(k) - T_{cr} \quad (27)$$

$$x_2(k) = k_1T_{ho}(k) - k_1T_{cr} - \frac{U_c}{V_c}\left(T_{co}(k) - T_{c1}\right) \quad (28)$$

So, the dynamics of the heat exchanger system in each sample can be rewritten in a compact form as:

$$x_1(k+1) = x_2(k) \quad (29)$$

$$x_2(k+1) = f\left(x(k)\right) + g\left(x(k)\right)u(k) \quad (30)$$

Where:

$$g(k) = \frac{k_1}{v_h}\left[v_{h1} - \left(1 + \frac{u_c}{k_1v_c}\right)T_{cr} + \frac{u_c}{k_1v_c}T_{c1}\right] - \frac{k_1}{v_h}\left[1 + \frac{u_c}{k_1v_c}\right]x_1(k) - \frac{k_1}{v_h}x_2(k) \quad (31)$$

$$f(k) = \frac{k_2u_c}{v_c}\left(T_{c1} - T_{cr}\right) - \frac{k_2u_c}{v_c}x_1(k) - \left[k_2 + \frac{u_c}{v_c}\right]x_2(k) \quad (32)$$

RESULTS AND DISCUSSION

In this study, the heat exchangers plate parameters of the previous section are placed in the Eq. 4 and control method is provided for the implementation. The value of heat exchanger parameters are shown in Table 1.

In all the simulations, the sliding surface in Eq. 2 is used with components, $\alpha = 0.5$, $\beta = 0.1$, $p = 9$ and $q = 7$. Incoming cold water temperature $T_{ci} = 20°C$, input hot water $T_{hi} = 80°C$ and the desire temperature is $T_{cr} = 40°C$. For algorithm is considered the weighting matrices $Q = P = R = 1$, the sampling time $T = 0.5$, the initial conditions $x_0 = [-20\ 0.53]$ and the variable constraint for system state as $x\epsilon(-20, 5)*(-10, 10)$ and input $u\epsilon(-1.5, 1.5)$.

Figure 1-4 the shows trajectory of constraint variables were shown by applying algorithms and is shown in the presence or absence of contractive constraint on it. Due to above consideration, increased prediction horizon can meet constraints in regard to $x\epsilon X$ and also the variable speed is increased to reach the reference which means the output system temperature reach to desired temperature and by imposing contractive constraint $\alpha = 0.8$ for the sliding surface, their performance will improve. According to Fig. 3-8 can be seen that for N = 2, the states can reach the reference level in 24 sec while contractive constraint has been reduced this time to 8 sec. For N = 4, the effect of reducing the time of applying constraint decreased to 7-19 sec.

Table 1: Value of parameter

Dynamic	Sign	Values
Constant heat transfer coefficient	U	300 w/mz°C
Constant area	A	0.0672 m^2
Volumetric flow rate of 2 side	V_c, V_h	5.37×10^{-4} m^3
Density	ρ_c, ρ_h	1000 kg/m^3
Specific heat of 2 side liquid	c_p, h, c_p, c	4180 J/kg°C

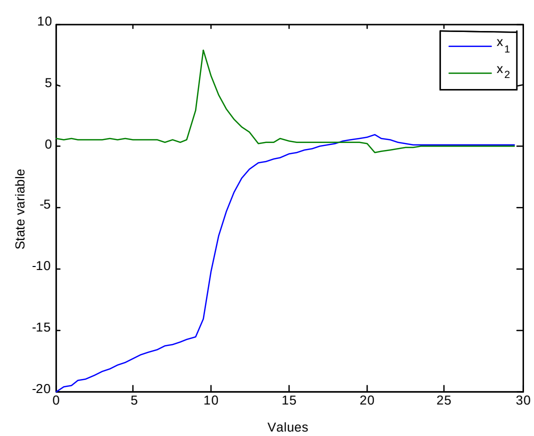

Fig. 1: States trajectory with FTSM_MPC and N = 2

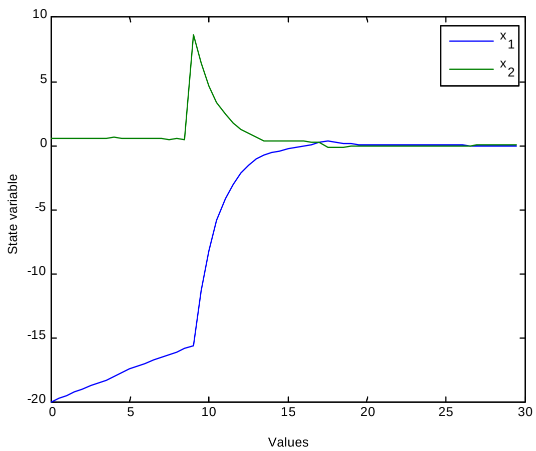

Fig. 2: States trajectory with FTSM_MPC and N = 4

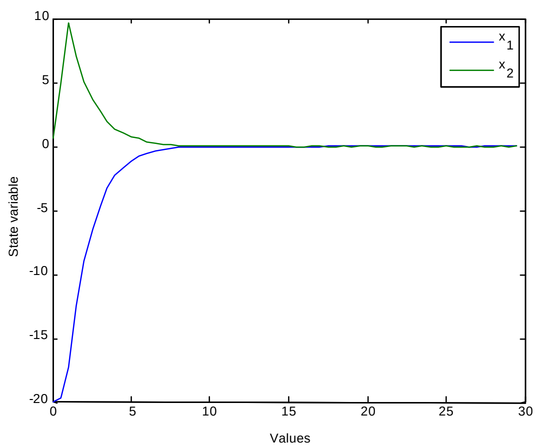

Fig. 3: States trajectory with contractive FTSM_MPC and N = 2

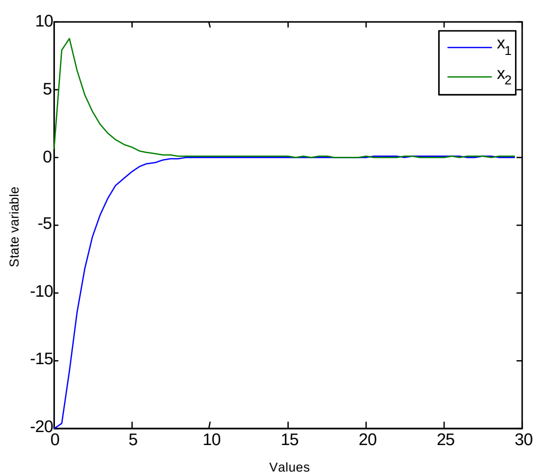

Fig. 4: States trajectory with contractive FTSM_MPC and N = 4

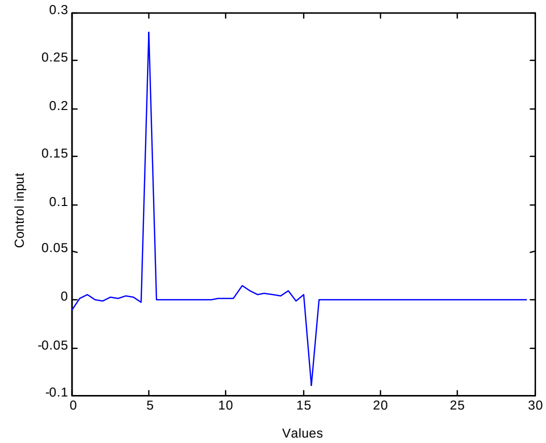

Fig. 5: Control effort with FTSM_MPC and N = 2

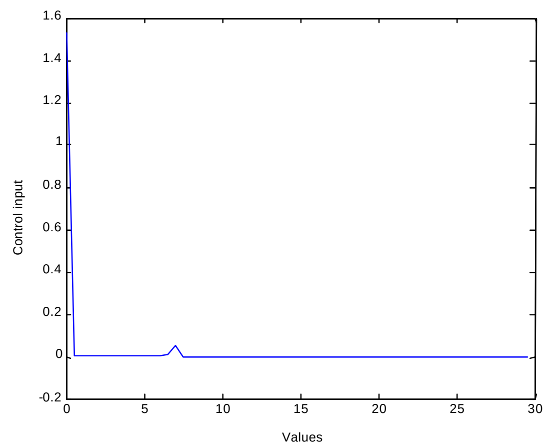

Fig. 6: Control effort with FTSM_MPC and N = 4

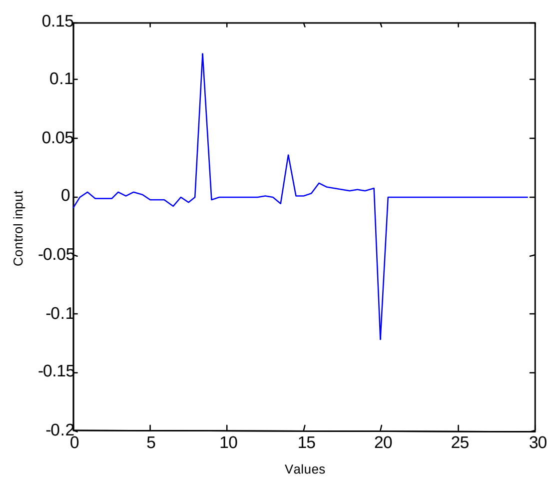

Fig. 7: Control effort with contractive FTSM_MPC and N = 2

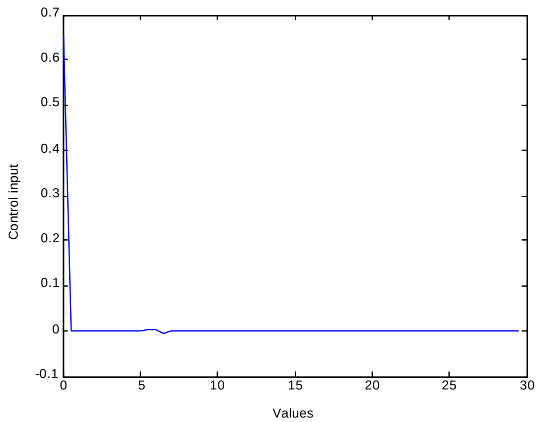

Fig. 8: Control effort with contractive FTSM_MPC and N = 4

The impact of the prediction horizon in high speed modes is evident by comparing Fig. 1 and 2. Also, the control input of Fig. 5-8 shown the effect of contractive constraint to reduce the control effort which is the reduction of cold water flow rate. Due to these considerations, for N = 2 with constraint imposing, the overshoot value is decreased to 0.13-0.28 and for N = 4 from 1.35-0.5

CONCLUSION

In this study, first, FTSM_MPC algorithm which use model predictive control in fast terminal sliding mode of the system were introduced, then applied it for the plate heat exchangers constrained nonlinear systems. After that, adding an auxiliary contractive constraint was to ensure the stability algorithm of it. The simulation results indicate that the contractive constraints imposed to satisfy system's constraints, in addition to improve the control performance of the plate heat exchangers nonlinear systems.

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