Journal of Engineering and Applied Sciences 11 (4): 851-861, 2016
ISSN: 1816-949X

Simulation of Lamb Waves Propagation and Dispersion Curve Extraction in Plate

[1]Amirreza Pahlevanpour and [2]Mohsen Nalaee
[1] Department of New Science and Modern Technology, University of Tehran, Tehran, Iran
[2] Department of Aerospace Engineering, Amir Kabir University, Tehran, Iran

Abstract: There are different methods for simulating propagation of Lamb waves and also their collision and interaction with discontinues and part boundaries. One of these methods is the finite elements method which includes implicit and explicit approaches. In this research, at first, two methods for simulating the Lamb wave propagation were investigated and compared. Then dispersion curves for an aluminum sheet were extracted for symmetric and asymmetric displacements in a node. Subsequently, the related displacement was imposed on the groups of nodes in order to model real state. The main focus of this research is on extracting the dispersion curve thus, only one mode was stimulated which can be entirely symmetric or asymmetric. Comparing the results with analytical solution shows that error is negligible in one node stimulation while 10% error would be occurred in widespread stimulation.

Key words: Dispersion curve, lamb waves, finite element method, symmetric mode, asymmetric mode

INTRODUCTION

In recent years, using the ultrasonic waves for health monitoring of structures has been one of the main industrial research and development plans. Guided waves is an effective method for structural health monitoring that leads to remarkable cost saving. Lamb waves are used as a nondestructive and health monitoring method to detect the different type of defects in structures made from sheets. When the thickness of structure is not more than three times the emitted wavelength, the assumption of propagation of Lamb waves is correct (Honarvar, 2010). About <90% of surface wave energy is within the depth equal to 2-3 times of the wavelength from the surface. When the wavelength tends to the sheet thickness, surface waves would couple on the upper and lower surfaces and create the Lamb wave.

Lamb waves are dispersive that is one of their important properties. Being dispersive means different frequency components will transmit in different velocities through the solid. In the other words, the transmittal velocity of Lamb waves through the solid depends on its transmittal frequency. Dispersion curve is a beneficial and common method for showing the relation between frequency and group velocity and phase velocity from different modes. That is important in defect detection by Lamb waves because by using these waves according to the structure conditions, the frequency needs for stimulation the intended mode can be extracted (Bray and Stanley, 1996). In addition, the time for arrival of waves can be anticipated using the acquired information from dispersion curve. For example, the group velocity dispersion curve for a steel sheet is shown (Fig. 1).

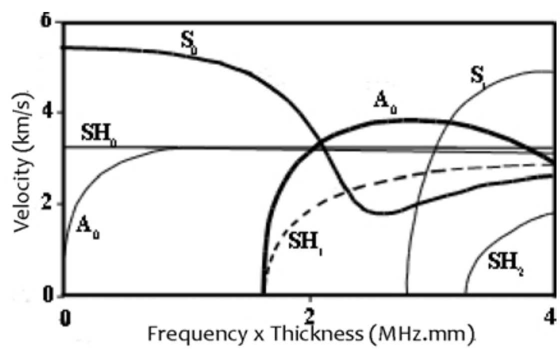

Fig. 1: Dispersion curve of group velocity for steel (Esmaeli, 2011)

As it can be seen from Fig. 1, in each special frequency × thickness, different modes are stimulated in the sheet and each mode is transmitted through the sheet with its individual velocity. This Lamb wave characteristic can cause complication of signal produced from transmitted wave in the structure which makes the interpretation of response signals difficult. On the other hand, numerical methods for analyzing the Lamb wave problems have been used increasingly since 2002 due to the complicated behavior of Lamb waves when they hit

Corresponding Author: Amirreza Pahlevanpour, Department of New Science and Modern Technology, University of Tehran, Tehran, Iran

different defects and also due to mode change phenomena. During 2003-2007, Lee and Staszewski (2003) performed numerical investigation on Lamb waves and mutual effects of wave and crack. In their first study which published in 2003, a new method for numerical modeling of Lamb wave propagation named LISA (Local Interaction Simulation Approach) was introduced. In this study, after introducing and investigating the LISA, wave propagation in one and two dimensions were studied and results shows the capability of LISA for modeling the Lamb wave propagation.

In the second study, mutual effects of Lamb wave and a groove was investigated. In this research, by changing the groove size, parameters such as maximum acquired signal and its time length were studied. In this study, only the effect of groove length change on the acquired wave was studied while width and thickness of groove considered constant. The complication of acquired signal from sensor results in an interpretation that the defect detecting methods based on the amount of signal are not accurate and other indicators such as energy would be more precise (Lee and Staszewski, 2003).

In the third study LISA method is examined more closely and the effect of changes in the length and width of the groove on the Lamb wave has been studied. In this model, they put a layer of air and vacuum elements on a sheet in order to consider the effects of acoustic impedance of air as well. Since, the effect of longitudinal wave dominates the transverse in S_0 mode, a horizontal force was used parallel to the surface to stimulate the S_0 mode. In A_0 mode, two vertically aligned forces outside the sheet were used. In A_0 mode the effect of transverse wave overcomes the longitudinal wave (Lee and Staszewski, 2007a).

After studying the modeling process and interaction of wave with damage, Lee examined the defect finding strategy in sheets, based on the Lamb wave propagation. Two criteria: Peak to peak amplitude and the time for wave to reach to sensor were used for defect finding. Based on the results, he concluded that the numerical modeling can be used as a suitable and reliable tool to study these parameters. Changes in the value of the received signal in the sensor are proportional to the defect size. The first peak value of the received signal from the sensor decreases with increasing the length and width of the groove. In contrast, the second peak value increases for larger groove. The largest peak from the groove happens when the sensor is near the groove. In addition to the signal value, the time for signal to reach the sensor can also be used as a detective tool. The signal arrival time is more sensitive to changes in signal width than its length. In summary, he has shown in his fourth study that to make a proper detection, a set of effective parameters should be evaluated. These parameters include the geometry of the studied system, the position of operator and sensor, selection of wave propagation mode (S_0 or A_0), selection of indicators related to the defect (amount of signal or arrival time), time interval of measuring the response of the sensor and the choice of reached or reflected peak type (Lee and Staszewski, 2007b).

In the last study by Lee about LISA method, based on the studies done in previous articles, he has raised the issue of locating sensors. In this study, the effect of different sensor locations on the defect detecting process is studied and as a result, it is declared that the sensor location has a significant effect on the defect detecting process (Lee and Staszewski, 2007c). In another research, the interaction of the waves with open and closed fatigue crackes is simulated by LISA method. Researchers provided a numerical method based on the time domain spectral element method to analyze the wave propagation in the curved structures. Their innovation is providing the curved shell elements to simulate the wave propagation in the sheet structures. Researchers analyzed the Lamb wave propagation based on the finite element method in the curves elements and showed that for inspecting the structures with waveguide, it is possible to select the appropriate mode and frequency based on the special value analysis (Zhou and Ichchou, 2010).

MATERIALS AND METHODS

Review and compare explicit and implicit methods: One of the Lamb waves modeling methods is plane strain method. In fact, in this method, the changes of parameters in one dimension (width) will be discarded. As a result, these models are not able to simulate the Lamb wave in two dimensions. There are two methods for analyzing transient dynamic problems in ABAQUS: implicit and explicit. In the implicit method, equations of motion integrated implicitly during the time length which leads to the general stiffness matrix for the structure. In the explicit method, integration is done explicitly, resulting in high number of unknown equations needed to be solved. If implicit methods is utilized, piezoelectric elements can be used but in the family of explicit methods, piezoelectric elements are not included and a viable alternative to the piezoelectric effect must be found in an explicit way. Both methods would be reviewed in the following. The

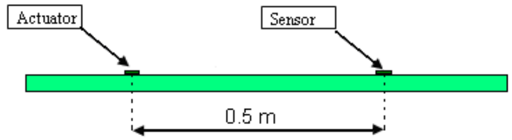

Fig. 2: The intended model for comparing the implicit and explicit methods

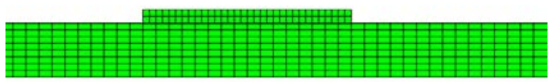

Fig. 3: The piezoelectric operator attached to the aluminum sheet

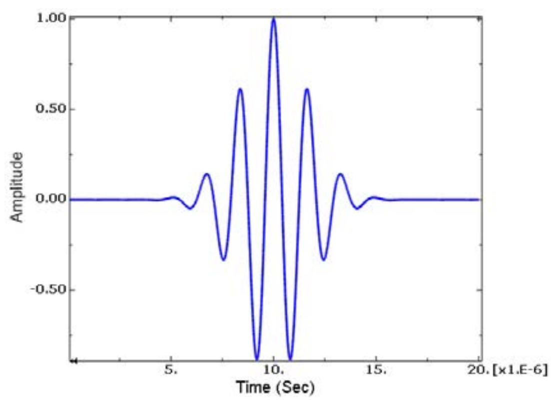

Fig. 4: Stimulating signal

Table 1: Mechanical properties of aluminum

Young modules (GPa)	Poisson ratio	Density (kg m^{-3})
69	0.3	2700

Table 2: Mechanical and electrical properties of piezoelectric material

Variables	Values
Young modules (GPa)	71.4
Poisson Ratio	0.3
Density (kg m^{-3})	7350
Dielectric (F)	1.5e-8
$e_{3\ 11}$ (n/volt.m^2)	19.223
$e_{3\ 22}$ (n/volt.m^2)	-8.2384
$e_{3\ 33}$ (n/volt.m^2)	-8.2384

intended model for an implicit and explicit method is shown in Fig. 2. In this model, an aluminum sheet of 1 m length and 2 mm thickness has been modeled. The operator distance to sensor is set as 0.5 meters and the size of each piezoelectric operator and sensor is 0.5×8 mm. Aluminum properties (Table 1) and piezoelectric material (Table 2) are as follows.

Implicit method: Figure 3 shows the close up view of the operator's location on the sheet in the software model. A

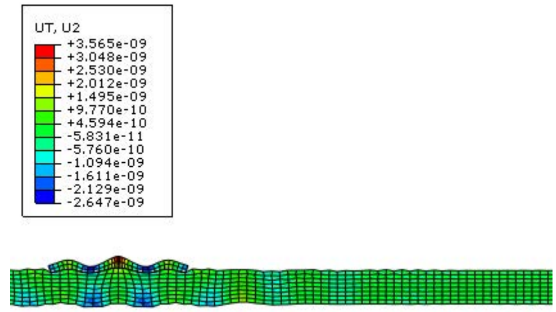

Fig 5: Wave propagation condition near the piezoelectric operator at 15 μ sec

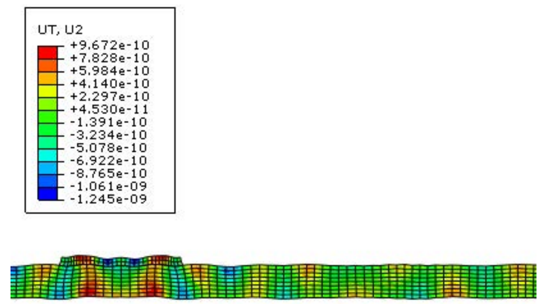

Fig 6: Wave propagation condition near the piezoelectric operator at 24 μ sec

Y-cut piezoelectric transducer is used to stimulate the structure. As a result of the applied voltage to the operator, the deformation of the piezoelectric transducer will be parallel to the sheet surface. The selected signal to stimulate the operator is shown in Fig. 4. The relationship of these signals with time is as follows:

$$V(t) = V_0 \cos(2\pi wt) e^{-\alpha}(t - t_0)^2$$

Where:

v_0 = Peak-to-peak signal frequency
w = Frequency
α = Constant
t = Time
t_0 = Time of the center of signal

This signal has been used as a triggering signal due to the high energy density in numerous references. The frequency of signal shown in Fig. 4 is selected to be 600 kH_z. After stimulate the operator, the wave is created and propagated in the sheet. Figure 5-8 shows the wave propagation at different times. Farther away from the operator, the two modes A_0 and S_0 can be observed which illustrated in Fig. 6. As seen in this Fig. S_0 mode propagates more quickly than the A_0 mode. Image magnifies for 5000000 (5 million) times. Since, sensors

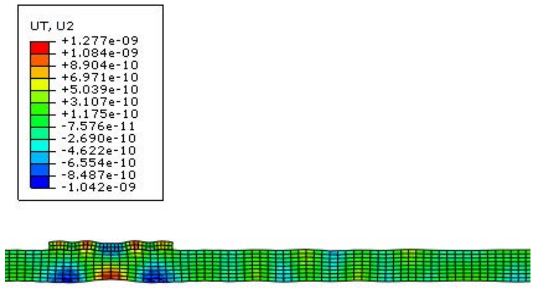

Fig 7: Wave propagation condition near the piezoelectric operator at 30 μ sec

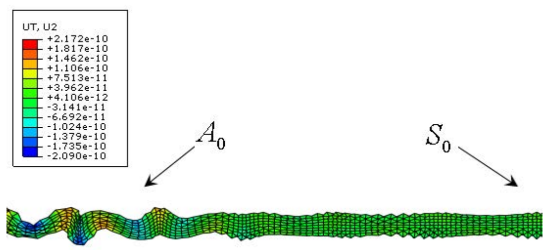

Fig. 8: Propagation of modes A_0 and S_0 in the sheet as a result of piezoelectric operator

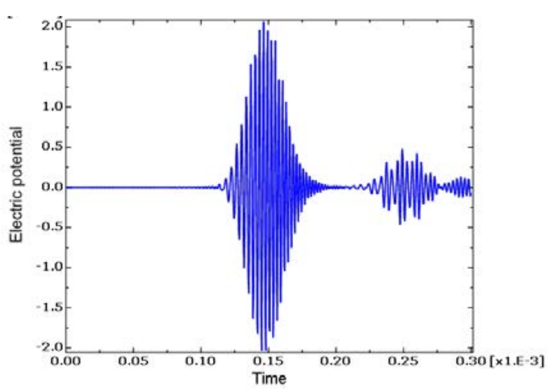

Fig. 9: The acquired response from piezoelectric sensor in implicit model

are also meshed by various elements, the mean voltage of upper surface elements is used to acquire the response from the sensor. Figure 9 shows the response of the sensor.

Based on the arrival time of waves to sensor, the wave propagates at a speed of 3571 m sec^{-1}. While the propagation speed in mode A_0 and S_0 in the frequency of 600 kH$_z$ in sheet thickness of 2 mm are 3047 and 4894.9 ms^{-1} respectively. Another observed peak in signal is caused by the wave reflection of the right border. As can be seen, implicit method does not lead to accurate results. Explicit finite element method is suitable for wave propagation problems. Of course, in some of the sensor nodes, acquired response from the structure is suitable but in practice because the sensor voltage is collected from the surface, performing an averaging process is necessary.

Explicit method: As noted earlier, in addition to the implicit method for modeling wave propagation phenomena, explicit method also can be used in ABAQUS. In this method, the equations of motion are integrated explicitly in time. Unlike implicit method which in any time development, [M][A]+[C][V]+[K][X] = [F] equation will be resolved, in explicit method to find displacement, at first, acceleration of a node in a half time development is obtained and then by integrating with time, velocity and displacement are obtained. The main obstacle against using explicit method in ABAQUS Software is the absence of piezoelectric elements in this method, thus, it should be a good alternative for sensor and operator. Pin-force model of piezoelectric materials is used for piezoelectric operator and a horizontal force is utilized instead of piezoelectric operator. In this model, the equivalent force is obtained from the following equation:

$$F_\alpha = \frac{\left(E_\alpha w_\alpha t_\alpha\right) Y}{Y+6}\Delta$$

$$Y = \frac{E_\alpha w_\alpha t_\alpha}{E_p w_p t_p}, \Delta = \frac{d_3 l_\alpha}{t_\alpha} V$$

Where:

α = Relevant to operator
p = Represents the sheet
E = Modulus of elasticity
F = Equivalent force
w = Width
w = Thickness
V = Applied voltage

For properties in Table 1 and 2, the equivalent force is calculated as 4.8 N. Also, since the finite element method is used based on the displacement, the displacement in the Y direction is considered as the response of the sensor. This has been suggested in various papers. Horizontal force acts as a stimulator, results in both A_0 and S_0 modes to stimulate. Also, Fig. 10 and 11shows the wave propagation condition at different time steps after forcing the operator. In areas farther from the operator, the separation between modes S_0 and A_0 is evident. As it can be seen because the propagation velocity of the A_0 mode is more than S_0, this mode go farther from A_0 as the time passes.

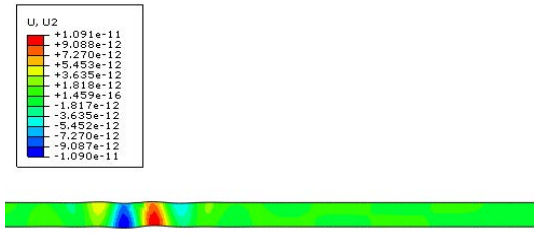

Fig. 10: Condition of force propagation when stimulating near the operator at 48 μ sec in explicit model

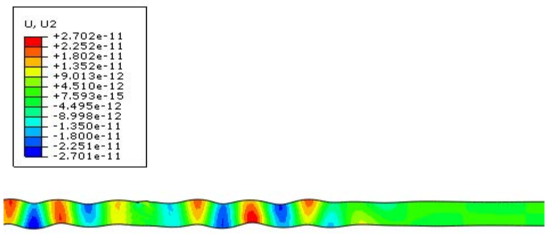

Fig. 11: Condition of force propagation when stimulating near the operator at 52 μ sec in explicit model

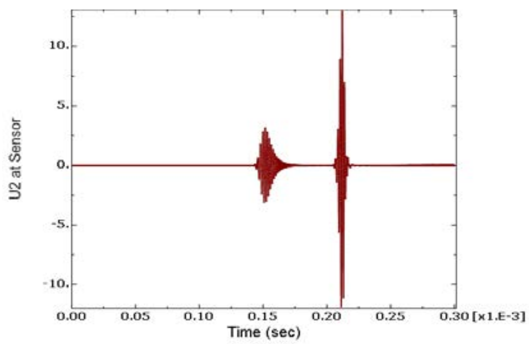

Fig. 12: Response of sensor to the force stimulation

Figure 12 shows the response of the sensor. As is observed, response of the sensor has two peaks. The first peak is due to S_0 mode and the second peak is due to A_0 mode. Speed of 4914 m sec^{-1} is obtained for S_0 mode and the speed of mode A_0 is 3083 m sec^{-1}. Speed obtained from the analytical approach for the two modes S_0 and A_0 are 4894.9 and 3047 m sec^{-1}, respectively. Thus, the results of the horizontal force stimulation are appropriate. The point here is that the stimulation of the horizontal force leads to two modes. If this type of stimulation is used for diagnostic purposes in the structure with a defect, mode change phenomenon in the discontinuity results in two other modes from each mode which adds to the complexity of wave propagation. As has been mentioned, the suitable option is to stimulate only one mode for detecting the structures defects. It could be concluded that the explicit method for simulating the lamb wave propagation leads to more accurate results.

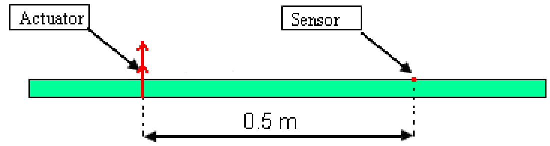

Fig. 13: Same direction displacement application for A_0 single point stimulation mode

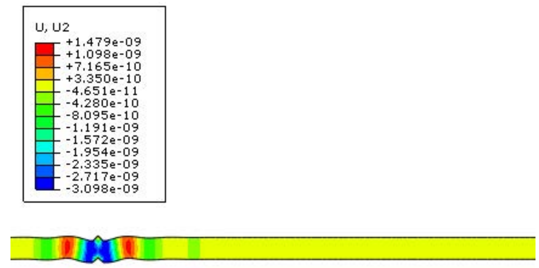

Fig. 14: Near operator wave propagation condition for A_0 stimulation mode by displacement at frequency of 600 kH$_z$ at 48 μ sec

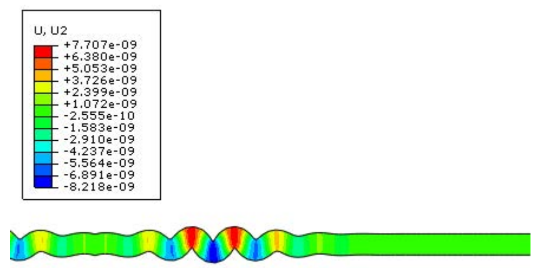

Fig. 15: Near operator wave propagation condition for A_0 stimulation mode by displacement at frequency of 600 KHz at 54 μ sec

RESULTS AND DISCUSSION

Dispersion curve extraction by single point stimulation: Dispersion curve extraction is the next step in investigating the dispersion phenomenon. This curve would be extracted based on the explicit simulations. But as mentioned earlier, lack of piezoelectric elements is a limitation in using the explicit solver in ABAQUS. Thus, for modes stimulations, displacement applications are used. The noticing point is that force application is not used due to both symmetric and asymmetric stimulation by force applications. Figure 13 shows the displacement application of a point for A_0 stimulation mode. Wave propagation condition for frequency of 600 kH$_z$ in different time steps is shown in Fig. 14-16.

As could be observed, for asymmetric stimulation just the A_0 mode is developed. Simulations have been

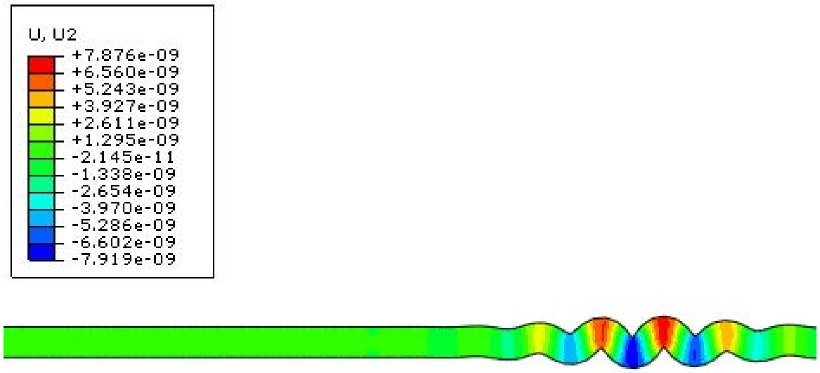

Fig. 16: Near operator wave propagation condition for A_0 stimulation mode by displacement at frequency of 600 kH_z at 61.5 μ sec

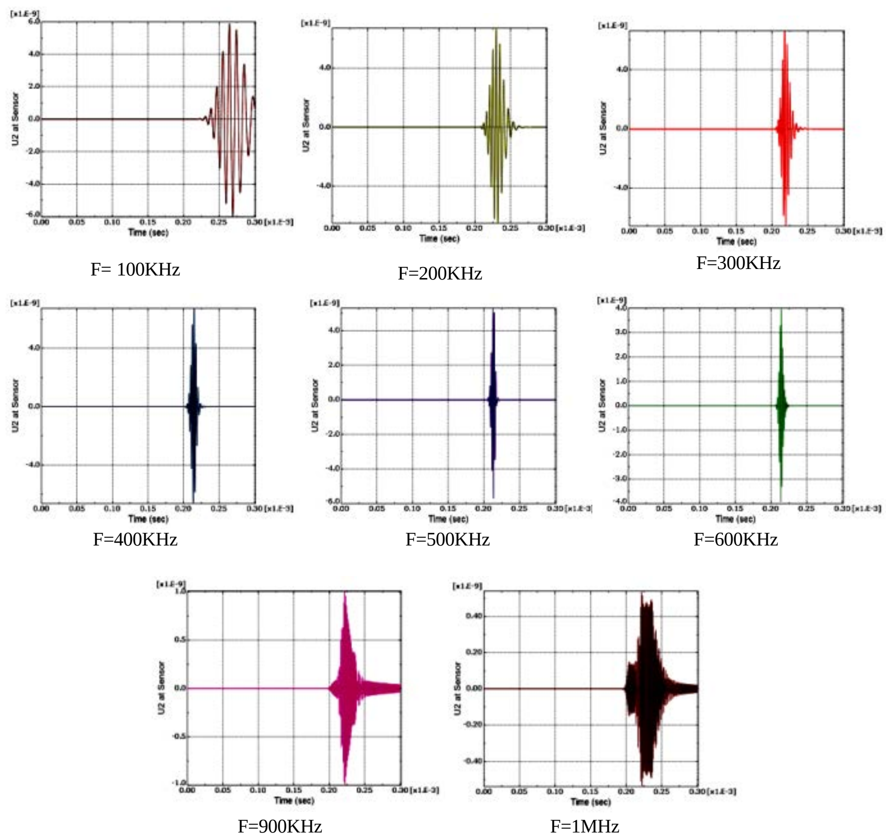

Fig. 17: Sensor response to displacement stimulation for A_0 mode development in various frequencies

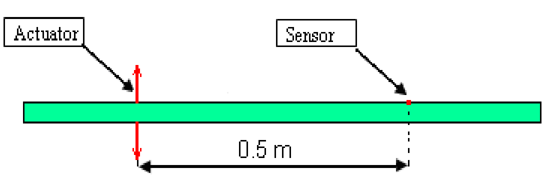

Fig. 18: Application of two symmetric displacements for S_0 mode stimulation at a point

carried out for frequencies from 100-1 MHz. The results are shown in Fig. 17. As observed, by increasing the frequency the wave arrival time to the sensor decreases which shows the frequency dependency to the speed. Displacement application method is also used for S_0 mode development. To do this, two perpendicular displacements normal to the plane in opposite directions are applied to upper and lower of the operator position. Figure 18 shows the displacement application as S_0 mode

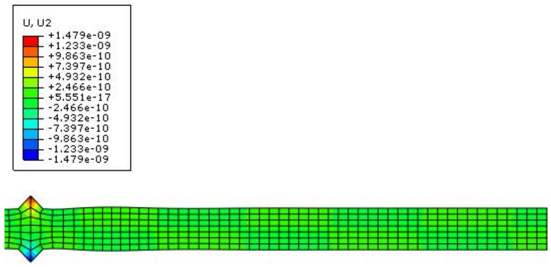

Fig. 19: Near operator wave propagation condition for S_0 stimulation mode by displacement at frequency of 600 kH_z at 48 μ sec

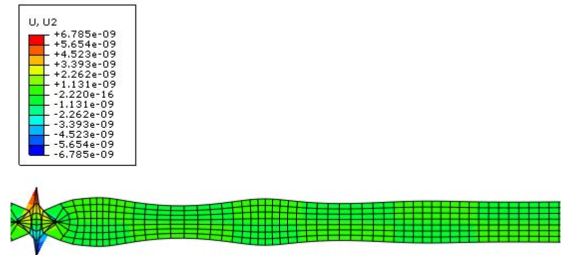

Fig. 20: Near operator wave propagation condition for S_0 stimulation mode by displacement at frequency of 600 kH_z at 51 μ sec

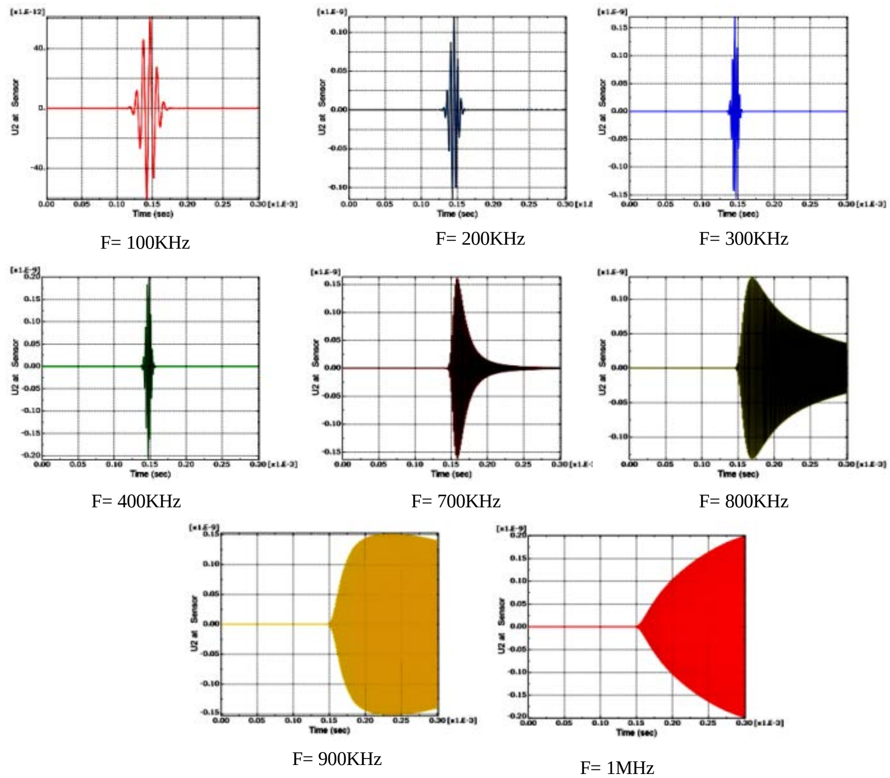

Fig. 21: Sensor response to displacement stimulation for S_0 mode development at various frequencies

stimulator. For S_0 mode the stimulation signal has the general form as shown in Fig. 4 and its frequency is altered from 100 kH_z to 1 MHz. The wave propagation condition for frequency of 600 kH_z is shown in Fig. 19 and 20. Picture magnification in this condition is 4×10^5. In this condition, S_0 mode is the only stimulated mode and there is no indication of A_0 mode. Particle movement path in this condition is elliptic on surface.

The sensor received signal for different frequencies is shown in Fig. 21. For S_0 mode, the peak arrival time to sensor is increased by increasing frequency and this means wave propagation speed is decreased by increasing frequency in S_0 mode. After conducting analysis by using single point displacement as stimulation signal of operator, it is possible to get dispersion curve for 2 mm thick aluminum plate. This curve is shown in Fig. 21. As seen in Fig. 22, using explicit method for wave propagation modelling leads to reasonable results and good matching is observed between numerical modelling and analytical solution. It would be noted that due to vast theoretic calculations, details of this calculations are not covered in this study and just the results are presented.

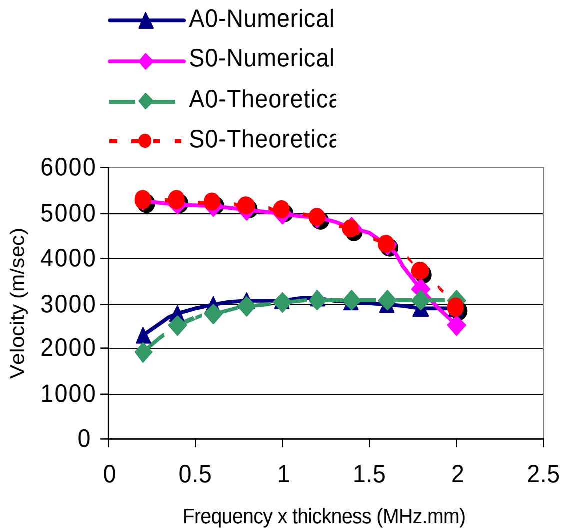

Fig. 22: Achieved dispersion curve for aluminum plate by single point stimulation

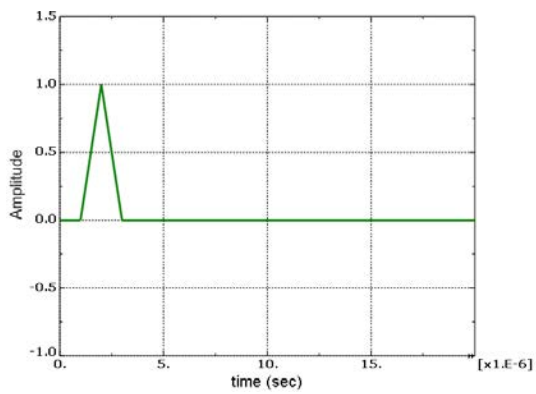

Fig. 23: Triangular pulse as a stimulation signal

Fig. 24: Wave propagation condition of pulse stimulation in plane strain model at 1.5 μ sec

To show the dispersion phenomenon in different way, a signal in triangular pulse is applied in operator and the results are investigated. This pulse is shown in Fig. 23. The results for normal displacement application of this pulse are shown in Fig. 24-26. The received signal of sensor is shown in Fig. 27. As observed in sensor

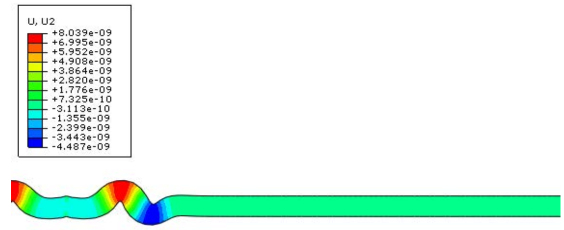

Fig. 25: Wave propagation condition of pulse stimulation in plane strain model at 4.5 μ sec

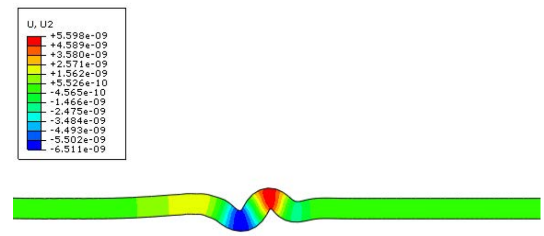

Fig. 26: Wave propagation condition of pulse stimulation in plane strain model at 9 μ sec

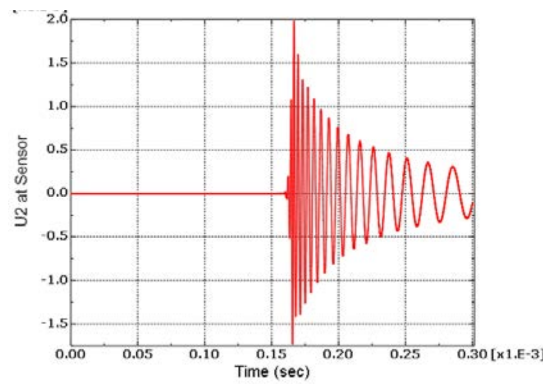

Fig. 27: Received signal of the sensor by pulse stimulation

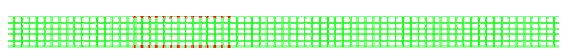

Fig. 28: Applied symmetric and asymmetric load nodes as lamb wave stimulation in wide stimulation model

response, wave with higher frequency has arrived to sensor sooner and then waves with lower frequencies affect the sensor. Curve widening at longer times indicates this point. It could be claimed that if the applied signal to the operator is a wide band frequency signal, dispersion characteristic which means wave propagation with different speeds will be observed.

Dispersion curve extraction by wide stimulation: In practice, there is no transducer that develops wave in structure as a point and of course transducer has a finite width. To consider the transducer width, stimulation is done on a group of nodes in the software. Figure 28

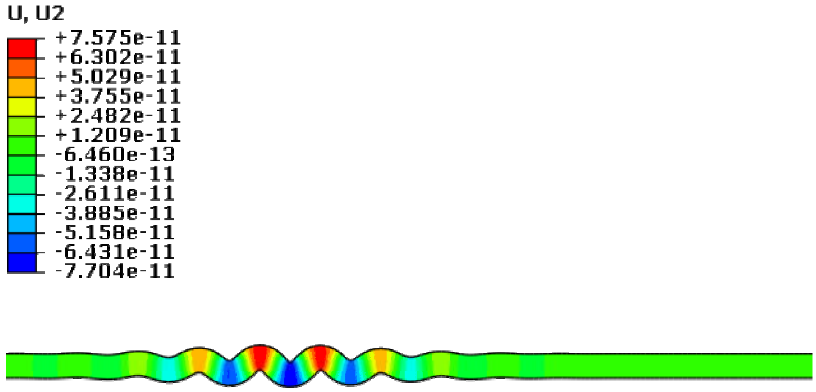

Fig. 29: A_0 propagation mode in wide stimulation

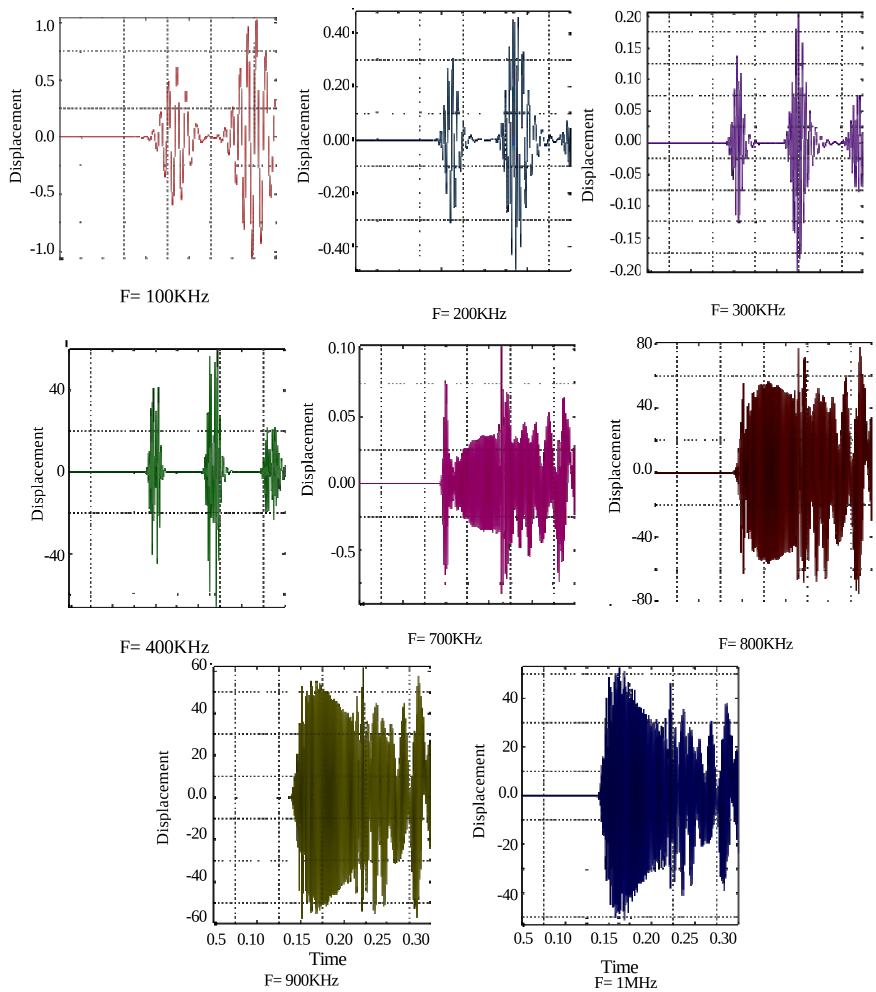

Fig. 30: Sensor response to wide displacement stimulation for A_0 mode development at various frequencies

shows the nodes where the load is applied. Center to center distance of sensor and operator in wide stimulation model was 30 cm and peak stimulation signal time was 5×10^{-5} sec. The results of asymmetric load application for

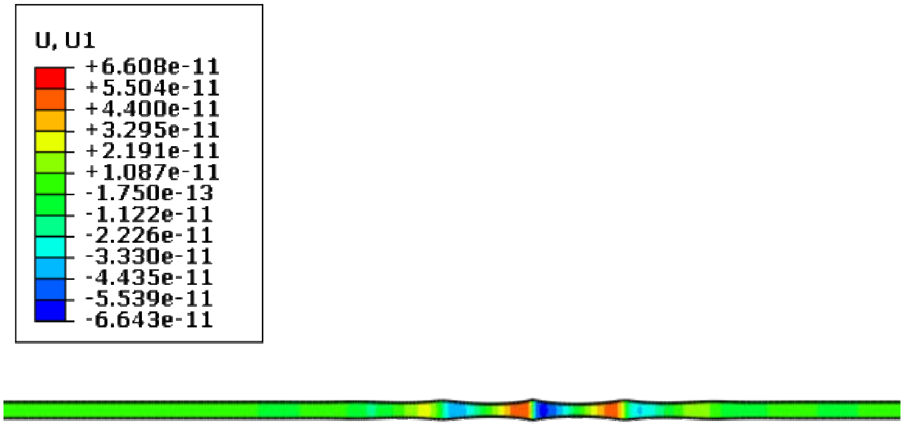

Fig. 31: S_0 mode propagation in wide stimulation

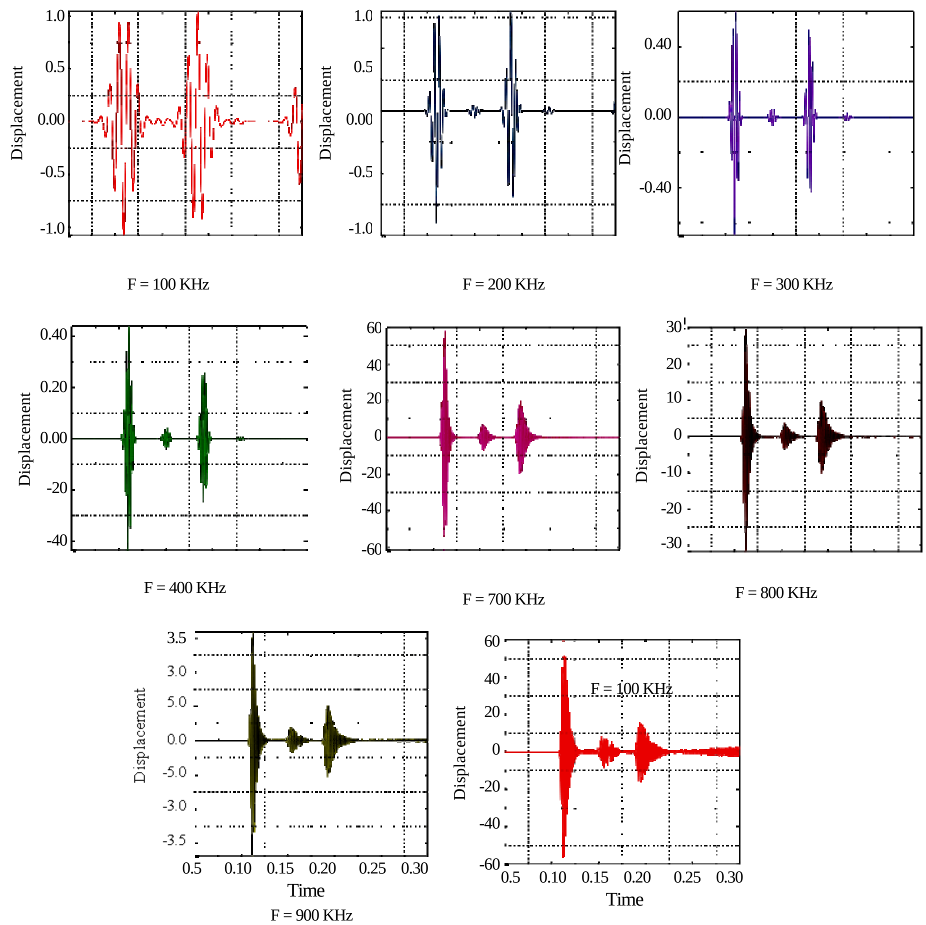

Fig. 32: Sensor response to wide displacement stimulation of S_0 mode at various frequencies

A_0 mode stimulation in shown in Fig. 29. Resulted sensor signals are shown in Fig. 30 and 31. Sensor signals results are shown in Fig. 32. The noticing point is that according to predominance of longitudinal wave to shear wave in S_0 mode, horizontal displacement is considered as the sensor signal. Dispersion curve obtained from wide stimulation is shown in Fig. 33. The results indicate that in wide stimulation, higher errors up to 10% are observed.

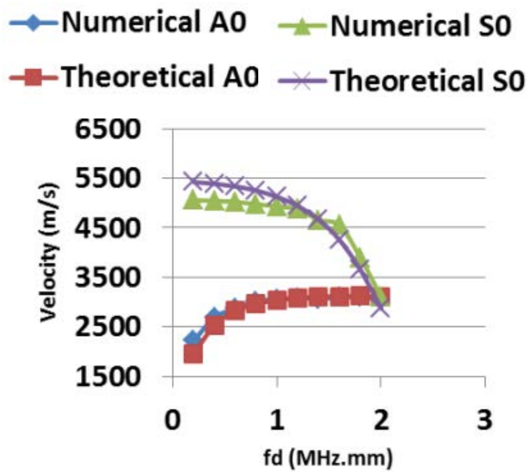

Fig. 33: Wide stimulation dispersion curve

CONCLUSION

In this study wave propagation in plane as a sample structure was investigated. Primary focus of this study was on describing the simulation process and verification of wave propagation in structures. Based on conducted simulations, extraction of dispersion curve for sheet was performed. To evaluate performance and verification of simulation process, the obtained results were compared with theoretical results. This comparison confirmed the performance accuracy of utilized simulation method.

REFERENCES

Bray, D.E. and R.K. Stanley, 1996. Nondestructive Evaluation: A Tool in Design, Manufacturing and Service. CRC Press, Boca Raton, New Yor, London, Tokyo, pp: 62-74.

Honarvar, F., 2010. Non-Destructire Test-Ultra Sonic Test. Norpardazan, Tehran pp: 20-37.

Lee, B. and W. Staszewski, 2003. Modelling of lamb waves for damage detection in metallic structures: Part I. Wave propagation. Smart Mater. Struct. Vol. 12.

Lee, B.C. and W.J. Staszewski, 2007a. Lamb wave propagation modelling for damage detection: I. Two-dimensional analysis. Smart Mater. Struct., Vol. 16.

Lee, B.C. and W.J. Staszewski, 2007b. Lamb wave propagation modelling for damage detection: II. Damage monitoring strategy. Smart Mater. Struct., Vol. 16.

Lee, B.C. and W.J. Staszewski, 2007c. Sensor location studies for damage detection with Lamb waves. Smart Mater. Struct., Vol. 16.

Zhou, W.J. and M.N. Ichchou, 2010. Wave propagation in mechanical waveguide with curved members using wave finite element solution. Comput. Meth Applied Mech. Eng., 199: 2099-2109.