Stable Motion Control of a Four Leg Robot

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Abstract: Today, most researchers in the field of robot research has focused on legged robots. The biggest advantage of these robots is capability that can pass through areas have a lot of ups and downs like a man. The aim of this study is simulation motion of four led robot using MATLAB. At the beginning, equations of position, speed and acceleration is achieved in kinematic mode. Then, the mathematical model is formed by obtaining kinetic equations. Then, using the obtained model, transfer function of system can be determined. Using obtained transfer function, behavior of system can be analyzed and also design of controller can be changed for improving behavior of system.

Key words: 4 leg robots, MATLAB, controller, model, system

INTRODUCTION

Robotics is formed of the three main branches: electronics (including robot brain), mechanics (including the physical body of the robot) and software (including the faculty of thinking and decision-making robot).

If a robot be likened to be a man, the man's physical appearance part of robot is constructed by mechanical professionals, electronics specialists of robot's brain are designed with the help of complex electronic circuits and software experts simulate imagination of robot by means of a computer program.

Today, 90% of robots are industrial robots that are applied in factories, laboratories, warehouses, power plants, hospitals and other same industries. In previous years, the majority of industrial robots used in car factories but today only about half of the world's robots are used in car factories. Using of robots in all aspects of life is rapidly spreading. For example, a robot developed for investigating the pile of used to not harm humans, radioactivity. Moving of robot can be studied using our surroundings. For example, considering move of the elephants there are joints in the legs that caused the legs can move backward, forward, left and right are. When this animal moves, divides your weight on his legs. So is it possible to maintain balance and not fall on the ground. The same problem exists in the robots. If one of the robot's legs is in the air it stopped and it is possible to fall on the ground.

Be taken to move of ants. This animal has 6 legs. When moving forward it moves its three legs forward and pusses three her other legs backwards in the same position. Two feet on the one hand and one foot on the other hand always do the same work and this work move ant to the front.

Because of having more legs and more comfortable form of legs, inspects can move easier than four legged animals can maintain theirs balance (Tedeschi and Carbone, 2014; Senthilkumar and Suresh, 2013; Gor *et al.*, 2015; Zhang *et al.*, 2014).

A REVIEW OF RESEARCH DONE

Mr. Daniel J. Pack and colleagues in 1995 examined the effective control of a forward moving of 4 leg robot. In this study, a simplified step forward for fourlegged walking robot was presented. It was shown that for a given position of the center of gravity of the robot, only certain arrangements of the displacement of the legs produced moving. Finally, experimental results confirm the proposed step.

Mr. Ganesh Kumar in 2012 have examined modeling and dynamic simulation of a four legged jumping robot. Page curve and limited steps have been taken to simulate. In order to save energy, passive dynamic agree with legs (linear springs) are used. Different state variables for analysis are obtained. Control measures on dynamic modeling are used to control the speed forward (Kumar and Pathak, 2013).

SELECTING THE SIZE OF THE ROBOT

The body of the robot is considered in the form of an ellipse with a large diameter 2a, small diameter 2b and the diameter 2c about axis z. It is located in the center of the oval axis origin that is as follows:

Its x-axis is along small diagonal of oval, its y-axis oriented in the direction of large diameter, its z-axis is perpendicular to these two axes and its positive direction
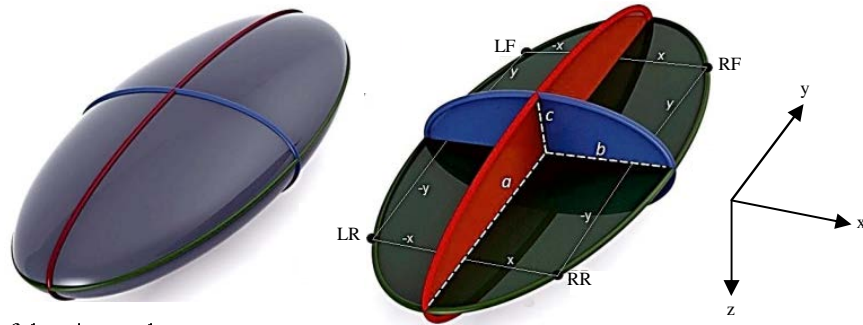
Fig. 1: The body of the given robot

is downward. Figure 1 shows the body of the given robot. One of the assumptions on the basis is that the length of the legs, the length of tibia and radius of attached bulletare equal with each other at all times. In other words, L_2 represents the length of the legs and L_3 represents tibia length and r_4 represent the radius of the shots attached to the tibia at all legs.

The left leg in front of the robot named LF, the right leg in front of the robot named RF, the left leg in rear of robot named LR and right leg of rear of the robot named RR. Each robot leg is composed of four parts.

Hip, thigh, leg and pellet attached to the bottom of leg are named numbers 1, 2, 3 and 4 are named. For example, $(LR)_3$ shows the leg of left rear leg of robot. Coordinates of the hip of each leg is checked respect to the robot's center of mass as follows:

- Coordinates of the hip of left leg in front of the robot in a local coordinate system: $(-x, y, 0)$
- Coordinates of the hip of right leg in front of the robot in a local coordinate system: $(x, y, 0)$
- Coordinates of the hip of left leg in rear of the robot in a local coordinate system: $(-x, -y, 0)$
- Coordinates of the hip of rightleg in rear of the robot in a local coordinate system: $(x, -y, 0)$

Where x and y are optional parameters of the robot. Since, the center of mass of robot is mobile, coordinates systems must be measured relative to a fixed reference. For this purpose, the initial position of the robot in the local coordinate system assume match to the generally coordinate system in first position. Next, how far the robot center of mass travelsevery step can be shown by r. Therefore, hip coordinates of each leg of the general coordinate system is as follows:

- Coordinates of the hip of left leg in front of the robot in a general coordinate system: $(-x+r_x, y+r_y, r_z)$
- Coordinates of the hip of right leg in front of the robot in a general coordinate system: $(x+r_x, y+r_y, r_z)$
- Coordinates of the hip of left leg in rear of the robot in a general coordinate system: $(-x+r_x, -y+r_y, r_z)$

- Coordinates of the hip of rightleg in rear of the robot in a general coordinate system: $(x+r_x, -y+r_y, r_z)$

KINEMATIC EQUATIONS OF THE ROBOT

If the angle of each leg with z-axis is shown by θ and picture of angle between each link in xy-page and x-axis is shown by φ, equations of motion can be achieved parametrically. To obtain the equations of motion of any link, equation of motion of end of link comes down.

The first case is the hip, the second case is thigh link and the third case leg link. For example, equations of motion of hip, thigh and leg of left leg in front of the robot (LF leg) are obtained in the general coordinate system as shown in Eq. 1-3:

$$\vec{r}_{(LF)_1} = (r_x\text{-}x)\hat{i} + (r_y\text{-}y)\hat{j} + r_z\hat{k} \qquad (1)$$

$$\vec{r}_{(LF)_2} = \vec{r}_{(LF)_1} + \vec{r}_{(LF)_{2/1}} = (r_x\text{-}x + L_2 \sin(\theta_{(LF)_2}) \times$$
$$\cos(\varphi_{(LF)_2}))\hat{i} + (r_y + y + L_2 \sin(\theta_{(LF)_2}) \times \qquad (2)$$
$$\sin(\varphi_{(LF)_2}))\hat{j} + (r_z + L_2 \cos(\theta_{(LF)_2}))\hat{k}$$

$$\vec{r}_{(LF)_3} = \vec{r}(LF)_2 + \vec{r}_{(LF)_{3/2}} =$$
$$(r_x\text{-}x + L_2 \sin(\theta_{(LF)_2}) * \cos(\varphi_{(LF)_2}) +$$
$$L_3 \sin(\theta_{(LF)_3}) * \cos(\varphi_{(LF)_3}))\hat{i} + \qquad (3)$$
$$(r_y + y + L_2 \sin(\theta_{(LF)_2} * \sin(\varphi_{(LF)_2})$$
$$L_3 \sin(\theta_{(LF)_3}) * \sin(\varphi_{(LF)_3}))\hat{j} +$$
$$(r_z + L_2 \cos(\theta_{(LF)_2}) + L_3 \cos(\theta_{(LF)_3}))\hat{k}$$

Velocity equations of hip, thigh and leg of left leg in front of the robot (LF leg) are obtained in the general coordinate system as shown in Eq. 4-6:

$$\vec{V}_{(LF)_1} = \omega_{(LF)_1} \times \hat{r}_{(LF)_1} =$$
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_{x_{(LF)_1}} & \omega_{y_{(LF)_1}} & \omega_{z_{(LF)_1}} \\ r_x\text{-}x & r_y\text{-}y & r_z \end{vmatrix} \qquad (4)$$

$$\vec{V}_{(LF)_1} = \vec{V}_{(LF)_1} + \vec{V}_{(LF)_{2/1}} - \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_{x_{(LF)_1}} & \omega_{y_{(LF)_1}} & \omega_{z_{(LF)_1}} \\ r_x - x & r_y + y & r_z \end{vmatrix} +$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_{x_{(LF)_2}} & \omega_{y_{(LF)_2}} & \omega_{z_{(LF)_2}} \\ L_2 \sin(\theta_{(LF)_2}) * \cos(\varphi_{(LF)_2}) & L_2 \sin(\theta_{(LF)_2}) * \sin(\varphi_{(LF)_2}) & L_2 \cos(\theta_{(LF)_2}) \end{vmatrix} \tag{5}$$

$$\vec{V}_{(LF)_3} = \vec{V}_{(LF)_2} + \vec{V}_{(LF)_{3/2}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_{x_{(LF)_1}} & \omega_{y_{(LF)_1}} & \omega_{z_{(LF)_1}} \\ r_x - x & r_y - y & r_z \end{vmatrix} +$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_{x(LF)_2} & \omega_{y(LF)_2} & \omega_{z(LF)_2} \\ L_2 \sin(\theta_{(LF)_2}) * \cos(\varphi_{(LF)_2}) & L_2 \sin(\theta_{(LF)_2}) * \sin(\varphi_{(LF)_2}) & L_2 \cos(\theta_{(LF)_2}) \end{vmatrix} +$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_{x_{(LF)_3}} & \omega_{y_{(LF)_3}} & \omega_{z_{(LF)_3}} \\ L_3 \sin(\theta_{(LF)_3}) * \cos(\varphi_{(LF)_3}) & L_3 \sin(\theta_{(LF)_3}) * \sin(\varphi_{(LF)_3}) & L_3 \cos(\theta_{(LF)_3}) \end{vmatrix} \tag{6}$$

Acceleration equations of hip, thigh and leg of left leg in front of the robot (LF leg) are obtained in the general coordinate system as shown in Eq. 7-9, respectively:

$$\vec{\alpha}_{(LF)_1} = \vec{\omega}_{(LF)_1} \times \left(\vec{\omega}_{(LF)_1} \times \vec{r}_{(LF)_1} \right) + \vec{\alpha}_{(LF)_1} \times \vec{r}_{(LF)_1} = \left(\omega_{x(LF)_1} \hat{i} + \omega_{y(LF)_1} \hat{j} + \omega_{z(LF)_1} \hat{k} \right) \times$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_{x_{(LF)_1}} & \omega_{y_{(LF)_1}} & \omega_{z_{(LF)_1}} \\ r_x - x & r_y - y & r_z \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_{x_{(LF)_1}} & \omega_{y_{(LF)_1}} & \omega_{z_{(LF)_1}} \\ r_x - x & r_y - y & r_z \end{vmatrix} \tag{7}$$

$$\vec{\alpha}_{(LF)_2} = \vec{\alpha}_{(LF)_1} + \vec{\alpha}_{(LF)_{2/1}} = \left(\omega_{x_{(LF)_1}} \hat{i} + \omega_{y_{(LF)_1}} \hat{j} + \omega_{z_{(LF)_1}} \hat{k} \right) \times \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_{x_{(LF)_1}} & \omega_{y_{(LF)_1}} & \omega_{z_{(LF)_1}} \\ r_x - x & r_y - y & r_z \end{bmatrix} +$$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha_{x_{(LF)_1}} & \alpha_{y_{(LF)_1}} & \alpha_{z_{(LF)_1}} \\ r_x - x & r_y - y & r_z \end{bmatrix} + \left(\omega_{x_{(LF)_2}} \hat{i} + \omega_{y_{(LF)_2}} \hat{j} + \omega_{x_{(LF)_2}} \hat{k} \right) \times$$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_{x_{(LF)_2}} & \omega_{y_{(LF)_2}} & \omega_{z_{(LF)_2}} \\ L_2 \sin(\theta_{(LF)_2}) * \cos(\varphi_{(LF)_2}) & L_2 \sin(\theta_{(LF)_2}) * \sin(\varphi_{(LF)_2}) & L_2 \cos(\theta_{(LF)_2}) \end{bmatrix} +$$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha_{x_{(LF)_2}} & \alpha_{y_{(LF)_2}} & \alpha_{z_{(LF)_2}} \\ L_2 \sin(\theta_{(LF)_2}) * \cos(\varphi_{(LF)_2}) & L_2 \sin(\theta_{(LF)_2}) * \sin(\varphi_{(LF)_2}) & L_2 \cos(\theta_{(LF)_2}) \end{bmatrix} \tag{8}$$

$$\vec{\alpha}_{(LF)_2} = \vec{\alpha}_{(LF)_2} + \vec{\alpha}_{(LF)3/2} = \left(\omega_{x_{(LF)_1}} \hat{i} + \omega_{y_{(LF)_1}} \hat{j} + \omega_{z_{(LF)_1}} \hat{k} \right) \times \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_{x_{(LF)_1}} & \omega_{y_{(LF)_1}} & \omega_{z_{(LF)_1}} \\ r_x - x & r_y - y & r_z \end{bmatrix} +$$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha_{x_{(LF)_1}} & \alpha_{y_{(LF)_1}} & \alpha_{z_{(LF)_1}} \\ r_x - x & r_y - y & r_z \end{bmatrix} + \left(\omega_{x_{(LF)_2}} \hat{i} + \omega_{y_{(LF)_2}} \hat{j} + \omega_{z_{(LF)_2}} \hat{k} \right) \times$$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_{x_{(LF)_2}} & \omega_{y_{(LF)_2}} & \omega_{z_{(LF)_2}} \\ L_2 \sin(\theta_{(LF)_2}) * \cos(\varphi_{(LF)_2}) & L_2 \sin(\theta_{(LF)_2}) * \sin(\varphi_{(LF)_2}) & L_2 \cos(\theta_{(LF)_2}) \end{bmatrix} + \tag{9}$$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha_{x_{(LF)_2}} & \alpha_{y_{(LF)_2}} & \alpha_{z_{(LF)_2}} \\ L_3 \sin(\theta_{(LF)_2}) * \cos(\varphi_{(LF)_2}) & L_2 \sin(\theta_{(LF)_2}) * \sin(\varphi_{(LF)_2}) & L_2 \cos(\theta_{(LF)_2}) \end{bmatrix} \times \left(\omega_{x_{(LF)_3}} \hat{i} + \omega_{y_{(LF)_3}} \hat{j} + \omega_{x_{(LF)_3}} \hat{k} \right) \times$$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_{x_{(LF)_3}} & \omega_{y_{(LF)_2}} & \omega_{z_{(LF)_2}} \\ L_3 \sin(\theta_{(LF)_3}) * \cos(\varphi_{(LF)_3}) & L_3 \sin(\theta_{(LF)_3}) * \sin(\varphi_{(LF)_3}) & L_3 \cos(\theta_{(LF)_3}) \end{bmatrix} +$$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha_{x_{(LF)_3}} & \alpha_{y_{(LF)_3}} & \alpha_{z_{(LF)_3}} \\ L_3 \sin(\theta_{(LF)_3}) * \cos(\varphi_{(LF)_3}) & L_3 \sin(\theta_{(LF)_3}) * \sin(\varphi_{(LF)_3}) & L_3 \cos(\theta_{(LF)_3}) \end{bmatrix}$$

KINETIC EQUATIONS OF THE ROBOT

In his part, the kinetic equations of is proposed for obtaining transfer function of dynamical behavior of robot. In this robot, the left foot in the front and the right foot in the backward of robot moves together at the same time. Similarly, the right foot in forward and the left foot in backward of robots move at the same time. Thus, due to the symmetrical shape of the robot, the equations are written only for the front legs. The center of gravity is in the middle of the calf's legs but the center of gravity of the thigh due to a pellet attached to one end is not in the center of the thigh.

The hip coordinates relative to the rotational position of the center of mass components in the left leg forward is obtained according to Eq. 10 where geometrical parameters is visible in Fig. 2:

$$y_{(LF)_3} = \frac{\sum y_i v_i}{\sum v_i} = \frac{\frac{L_3}{2} \sin\theta_{(LF)_3} * \pi L_3 (r_3^2 - r_3'^2) + (L_3 \sin\theta_{(LF)_3} + r_4) * \frac{4}{3}\pi r_4^3}{\pi L_3 (r_3^2 - r_3'^2) + \frac{4}{3}\pi r_4^3}$$

$$\overline{z}_{(LF)_3} = \frac{\sum z_i v_i}{\sum z_i} = \frac{\frac{L_3}{2} \cos\theta_{(LF)_3} * \pi L_3 (r_3^2 - r_3'^2) + (L_3 \cos\theta_{(LF)_3} + r_4) * \frac{4}{3}\pi r_4^3}{\pi L_3 (r_3^2 - r_3'^2) + \frac{4}{3}\pi r_4^3} \tag{10}$$

The total weight of the robot which includes a robot body weight and weight per foot splits on the legs. Since, the LF leg and RR moves together and also RF and LR moves together, normal force of each leg is equal to half of robot weight as shown in Eq. 11:

$$N = \left[\frac{m}{2} + 2(m_2 + m_3 + m_4) \right] * g \tag{11}$$

If the linear acceleration of the robot's center of mass to be shown by ÿ, linear acceleration of the center of mass

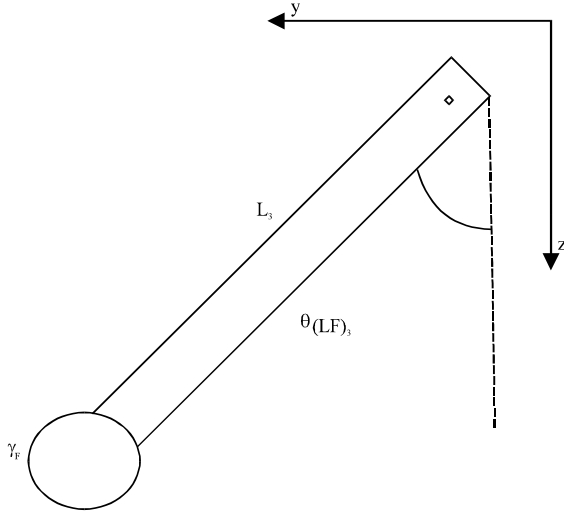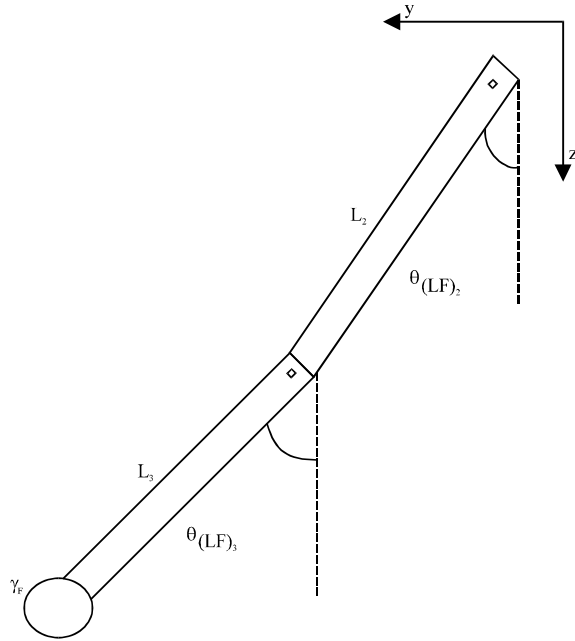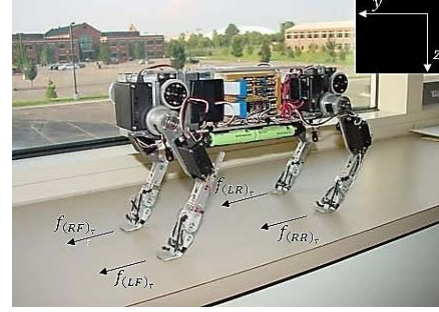Fig. 2: Center of mass of legs and bullet attached to it

Fig. 4: Free-body diagram of robot

Using the total free-body diagram (Fig. 4), the relationship between the forces of friction and linear acceleration of the robot is obtained in accordance with Eq. 13. Because of the symmetry, the friction force of RR leg is equal to friction force of LF leg and also the friction force of LR leg is equal to friction force of RF leg:

$$\sum F_y = (m + 4(m_2 + m_3 + m_4))\alpha_y \Rightarrow 2f_{(LF)_3} + 2f_{(RF)_3}$$
$$= (m + 4(m_2 + m_3 + m_4))\ddot{y} \Rightarrow f_{(LF)_3} + f_{(RF)_3} \quad (13)$$
$$= \left(\frac{m}{2} + 2(m_2 + m_3 + m_4)\right)\ddot{y}$$

Using free-body diagram for the LF leg (Fig. 4-6) and apply Newton's second law for leg and tibia, Eq. 14-15 were obtained:

$$\frac{T_{(LF)_3}}{2} - ((m_3 + m_4)g - N) * \overline{y}_{(LF)_3} +$$
$$\left(f_{(LF)_3} - (m_3 + m_4)\left(\ddot{y} + L_2\ddot{\theta}_{(LF)_2} + \left(\frac{\overline{z}_{(LF)_3}}{\cos\theta_{(LF)_3}}\right)\ddot{\theta}_{(LF)_3}\right)\right) *$$
$$\overline{z}_{(LF)_3} + m_3 g * \left(\overline{y}_{(LF)_3} - \frac{L_3}{2}\sin\theta_{(LF)_3}\right) + f_{(LF)_3} *$$
$$(L_3 \cos\theta_{(LF)_3}) + r_4 - \overline{z}_{(LF)_3}) + (N - m_4 g)$$
$$(L_3 \sin\theta_{(LF)_3}) + r_4 - \overline{y}_{(LF)_3}) + \overline{I}_{(LF)_3} \ddot{\theta}_{(LF)_3}$$

$$(14)$$

$$\frac{T_{(LF)_2}}{2} - ((m_3 + m_4)g - N + m_2 g) * \frac{L_2}{2}\sin\theta_{(LF)_2} +$$
$$\left(f_{(LF)_3} - (m_3 + m_4)\left(\ddot{y} + L_2\ddot{\theta}_{(LF)_2} + \left(\frac{\overline{z}_{(LF)_3}}{\cos\theta_{(LF)_3}}\right)\ddot{\theta}_{(LF)_3}\right) -$$
$$m_2\left(\ddot{y} + \frac{L_2}{2}\ddot{\theta}_{(LF)_2}\right)\right) * \frac{L_2}{2}\cos\theta_{(LF)_2} +$$
$$\left(f_{(LF)_3} - (m_3 + m_4)\left(\ddot{y} + L_2\ddot{\theta}_{(LF)_2} + \left(\frac{\overline{z}_{(LF)_3}}{\cos\theta_{(LF)_3}}\right)\ddot{\theta}_{(LF)_3}\right)\right) *$$

$$(15)$$

$$\frac{L_2}{2}\cos\theta_{(LF)_2} - ((m_3 + m_4)g - N) * \frac{L_2}{2}\sin\theta_{(LF)_2} = \overline{I}_{(LF)_2} \ddot{\theta}_{(LF)_2}$$

Fig. 3: Left foot in front of the robot

of every link in the robot can be wrote center according to acceleration of center of mass and angular acceleration. Linear acceleration of the center of mass of the thigh and leg of left foot are in accordance with the relationship 12 where geometrical parameters are visible in Fig. 3:

$$\ddot{y}(LF)_2 = \ddot{y} + \frac{L_2}{2}\ddot{\theta}_{(LF)_2}$$
$$\ddot{y}(LF)_3 = \ddot{y} + L_2\ddot{\theta}_{(LF)_2} + (\frac{\overline{z}_{(LF)_3}}{\cos\theta_{(LF)_3}})\ddot{\theta}_{(LF)_3}$$
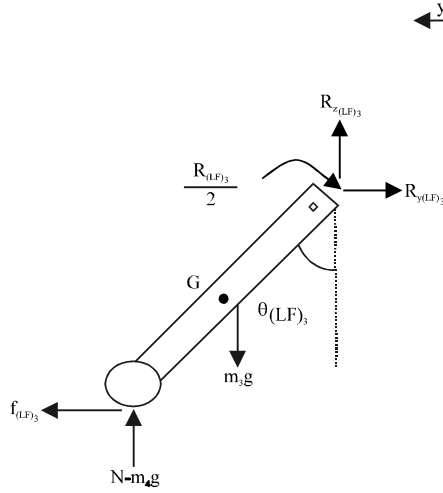
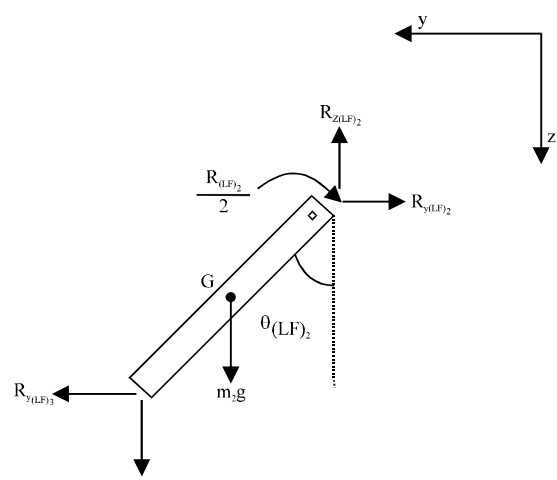$$(12)$$

Fig. 5: Free body diagram of left leg

Fig. 6: Free body diagram of left tight

A subsidiary relationship can be derived using geometry problem. The first relationship comes from the left leg and the second relation comes from leg of the right according to Eq. 16:

$$\begin{cases} y = L_2 \sin\theta_{(LF)_2} + L_3 \sin\theta_{(LF)_3} \\ y = L_2 \sin\theta_{(RF)_2} + L_3 \sin\theta_{(RF)_3} \end{cases}$$
$$\Rightarrow L_2 \sin\theta_{(LF)_2} + L_3 \sin\theta_{(LF)_3} =$$
$$L_2 \sin\theta_{(RF)_2} + L_3 \sin\theta_{(RF)_3} \tag{16}$$

The linearization of the equations is performed in this part. That assumes that the time interval angle of each link is so small that it can be replaced $\sin(\theta)$ with θ and $\cos(\theta)$ with 1. At the same time equations can be written about angle and angular acceleration of links as shown in Eq. 17 and 18.

$$L_z \overline{z}_{(LF)_3}(m_3 + m_4)\ddot{\theta}_{(LF)_2} + ((\overline{z}_{(LF)_3})^2(m_3 + m_4) + \overline{I}_{(LF)_3})\ddot{\theta}_{(LF)_3} + \left((\frac{m_3 g}{2} + m_4 g - N)L_3\right)$$

$$\theta_{(LF)_3} = \frac{T_{(LF)_3}}{2} + (L_3 + r_4)f_{(LF)_3} - \overline{z}_{(LF)_3}(m_3 + m_4)\ddot{y} + (N - m_4 g)r_4$$

$$\left((m_3 + m_4)L_2^2 + \frac{m_2 L_2^2}{4} + \overline{I}_{(LF)_2}\right)\ddot{\theta}_{(LF)_2} + ((m_3 + m_4)L_z \overline{z}_{(LF)_3}\ddot{\theta}_{(LF)_3} - \left(\left(\frac{m}{2} + m_4\right)g\right)$$

$$\theta_{(LF)_2} = \frac{T_{(LF)_2}}{2}L_2 f_{(LF)_3} - \frac{L_2}{2}(m_2 + 2(m_3 + m_4))\ddot{y} \tag{17}$$

$$L_z \overline{z}_{(RF)_3}(m_3 + m_4)\ddot{\theta}_{(RF)_2} + ((\overline{z}_{(LF)_3})^2(m_3 + m_4) + \overline{I}_{(RF)_3} + \left(\left(\frac{m_3 g}{2} + m_4 g - N\right)L_3\right)$$

$$\theta_{(RF)_3} = \frac{T_{(RF)_3}}{2}(L_3 + r_4)f_{(RF)_3} - \overline{z}_{(RF)_3}(m_3 + m_4)\ddot{y} + (N - m_4 g)r_4$$

$$\left((m_3 + m_4)L_2^2 + \frac{m_2 L_2^2}{4} + \overline{I}_{(RF)_2}\right)\ddot{\theta}_{(RF)_2} + (m_3 + m_4)L_z \overline{z}_{(RF)_3}\ddot{\theta}_{(RF)_3} - \left((\frac{m}{2} + m_4)g\right)$$

$$\theta_{(RF)_2} = \frac{T_{(RF)_2}}{2}L_2 f_{(RF)_2} - \frac{L_2}{2}(m_2 + 2(m_3 + m_4))\ddot{y}$$

$$L_2 \theta_{(LF)_2} + L_3 \theta_{(LF)_3} = L_2 \theta_{(LF)_2} + L_3 \theta_{(LF)_3} \tag{18}$$

DETERMINING THE TRANSFER FUNCTION:

Due to the complex solve of Eq. 17, it can be solved using transfer functions. To obtain the transfer function, it can use Laplace transform these equations assuming zero initial conditions that it obtained by Eq. 19:

$$\left(L_2 \overline{z}_{(LF)_3}(m_3+m_4)s^2\right)\theta_{s(LF)_2} + \left(\left((\overline{z}_{(LF)_3})^2 (m_3+m_4)+\overline{I}_{(LF)_3}\right)s^2 + \left(\frac{m_3 g}{2}+m_4 g\text{-}N\right)L_3\right)$$

$$\theta_{s(LF)_3} = \frac{(L_3+r_4)f_{(LF)_3}+(N\text{-}m_4 g)r_4}{S} \text{-} \overline{z}(LF)_3 (m_3+m_4)s^2 y_s + \frac{T_{s(LF)_3}}{2}\left(\left(m_3+m_4\right)L_2^2 + \frac{m_2 L_2^2}{4}\overline{I}_{(LF)_2}\right)s^2 \text{-}$$

$$\left(\left(\frac{m}{2}+m_4\right)g\right)\frac{L_2}{2}\theta_{s(LF)_2} + \frac{T_{s(LF)_2}}{2}\text{-}(\frac{L_2}{m}m_2 + 2(m_3+m_4))s^2)y_s + \frac{L_2 f_{(LF)_3}}{s}\left(L_2 \overline{z}_{(RF)_3}(m_3+m_4)s^2\right)\theta_{s(RF)_2} +$$

$$\left(\left((\overline{z}_{(RF)_3})^2 (m_3+m_4)+\overline{I}_{(RF)_2}\right)s^2 + \left(\frac{m_3 g}{2}+m_4 g\text{-}N\right)L_3\right)$$

$$\theta_{s(RF)_3} = \frac{(L_3+r_4)f_{(RF)_3}+(N\text{-}m_4 g)r_4}{S} \text{-} \overline{z}_{(RF)_3}(m_3+m_4)s^2 y_s + \frac{T_{s(RF)_3}}{2}$$

$$\left(\left(m_3+m_4\right)L_2^2 + \frac{m_2 L_2^2}{4}\overline{I}_{(RF)_2}\right)s^2 \text{-}\left((\frac{m}{2}+m_4)g)\frac{L_2}{2}\right)\theta_{s(RF)_2} +$$

$$\left(((m_3+m_4)L_2 \overline{z}_{(RF)_3})s^2\right)\theta_{s(LF)_3} + \frac{T_{s(RF)_2}}{2}\text{-}(\frac{L_2}{m}m_2 + 2(m_3+m_4)s^2)y_s + \frac{L_2 f_{(RF)_3}}{s} \tag{19}$$

After solving the equations using Cramer and simplifying Eq. 20 is obtained:

$$\theta_{s(LF)_2} = \frac{-2}{S}*\frac{\#i}{\#ii}, \theta_{s(LF)_3} = \frac{-2}{S}*\frac{\#iii}{\#ii}, \theta_{s(RF)_2} = \frac{1}{S}*\frac{\#iv}{\#v}, \theta_{s(LF)_3} = \frac{1}{S}*\frac{\#vi}{\#v} \tag{20}$$

Parameters that are used in these equation can be written as Eq. 21 and 22:

$$\#i = (2sT_{(LF)_2}+4f_{(LF)_3}L_2)(NL_3 \text{-}\overline{I}_{(LF)_3}s^2)\text{-}$$

$$\begin{pmatrix} 4f_{(LF)_3}L_2 s^2 (\overline{z}_{(LF)_3})^2+2s^3 T_{(LF)_2}(\overline{z}_{(LF)_3})^2 \text{-} \\ 2L_2 s^3 T_{(LF)_3}\overline{z}_{(LF)_3}\text{-}4NL_2 T_4 s^2{}_{(LF)_3}\text{-}4f_{(LF)_3}L_2 L_3 s^2 \overline{z}\text{-}_{(LF)_3} \\ L_2 L_3 T_4 s^2 \overline{z}_{(LF)_3} \text{-}4f_{(LF)_3}\text{-}L_2 r_4 s^2 \overline{z}_{(LF)_3}+2L_2 m_2 s^5 (\overline{z}_{(LF)_3})^2 \end{pmatrix}$$

$$(m_3+m_4)\text{-}(gL_3 sT_{(LF)_2}+2f_{(LF)_3}gL_2 L_3)(m_3+2 m_4)+$$

$$(2\overline{I}_{(LF)_3}L_3 s^3 y\text{-}2NL_2 L_3 s^3 y)(m_2+2m_3+2m_4)+$$

$$(2gL_2 L_3 s^3 y)(m_3^2+2m_4^2)+ (gL_2 s^2)\begin{pmatrix} (L_3 sy)(m_2 m_3+2m_2 m_4+ \\ 6m_3 m_4)\text{-}4m_3 m_4 r_4 \overline{z}_{(LF)_3}\text{-} \\ 4m_4^2 r_4 \overline{z}_{(LF)_3} \end{pmatrix} \tag{21}$$

$$\#ii = (\text{-}2L_3 g^2 L_2 m_4)(m+m_4)\text{-}L_3 mg^2 L_2 m_3 \text{-}4L_3 mg^2 L_2 m_3 \text{-}$$

$$4L_3 g^2 L_2 m_4^2+(4L_3 g^2 L_2^2 m_3 s^2)(m_3+3m_4)+L_3 m_2 gL_2^2 m_3 s^2 +$$

$$(2L_3 gL_2^2 m_4 s^2)(m_3+4m_4)\text{-}\begin{pmatrix} (4gL_2 m_4 gs^2)(\overline{z}_{(LF)_3})^2+2mgL_2 s^2 (\overline{z}_{(LF)_3})^2 \text{-} \\ 8\overline{I}_{(LF)_3}s^4 (\overline{z}_{(LF)_3})^2 \end{pmatrix}$$

$$(m_3+m_4)+ (2NL_2 gL_2 s^2 \text{-}2\overline{I}_{(LF)_3}gL_2 s^2)(m+2m_4)+(4\overline{I}_{(LF)_3}L_3 gs^2)(m_3+2m_4)+$$

$$(2\overline{I}_{(LF)_3}L_2^2 s^4 \text{-}2NL_3 L_2^2 s^2 y)(m_2+4m_3+4m_4)+(8\overline{I}_{(LF)_2}s^2)(\overline{I}_{(LF)_2}s^2 \text{-}NL_3)$$

$$\#iii = (4\bar{I}_{(LF)_2}s^2 + L_2^2m_2s^2)(sT_{(LF)_3} + 2Nr_4) + (2f_{(LF)_2}L_2^2m_2s^2 + 8f_{(LF)_2}\bar{I}_{(LF)_2}s^2)(L_3 + r_4) -$$

$$(2f_{(LF)_3}gL_2(L_3 + r_4) + gL_2sT_{(LF)_3} + 2NgL_2mr_4 - 2g^2L_2m_4r_4)(m + 2m_4) - 8gL_2^2m_4^2r_2s^2 -$$

$$8g\bar{I}_{(LF)_2}m_4r_4S^2 - \begin{pmatrix} 8\bar{I}_{(LF)_2}S^5y\bar{z}_{(LF)_2} + 4L_2s^3T_{(LF)_2}\bar{z}_{(LF)_3} + 8f_{(LF)_3}L_2^2s^2\bar{z}_{(LF)_3} + \\ 8f_{(LF)_3}L_2^2r_4s^2 - 8NL_2^2r_4s^2 - 4L_2^2s^3T_{(LF)_3} - (2L_2^2m_2s^5y\bar{z}_{(LF)_2}) \end{pmatrix}$$

$$(m_3 + m_4) - (2gL_2^2m_4r_4s^2)(m_2 + 4m_4) - (2gL_2^2m_4r_4s^2)(m_2 + 4m_4) +$$

$$(2gL_2S^3y\bar{z}_{(LF)_3})(2m_4^2mm_3 + mm_4 + 2m_3m_4)$$

$$\#iv = (2sT_{(RF)_2} + 4f_{(RF)_3}L_2)(NL_3 - \bar{I}_{(RF)_3}s^2) -$$

$$\begin{pmatrix} 4f_{(RF)_3}L_2S^2(\bar{z}_{(RF)_2})^2 + 2s^3T_{(RF)_2}(\bar{z}_{(RF)_2})^2 - \\ 2L_2s^3T_{(RF)_2}(\bar{z}_{(RF)_2})^2 - 4NL_2r_4s^2\bar{z}_{(RF)_3} - 4f_{(RF)_3}L_2L_3s^2\bar{z}_{(RF)_3} - 4f_{(RF)_3} \\ L_2r_4s^2\bar{z}_{(RF)_3} + 2L_2m_2s^5(\bar{z}_{(RF)_3})^2 \end{pmatrix}$$

$$(m_3 + m_4) - (gL_3ST_{(RF)_2} + 2f_{(RF)_3}gL_2L_3)(m_3 + 2m_4) +$$

$$(2\bar{I}_{(RF)_3}L_2s^5y - 2NL_2L_3S^3y)(m_2 + 2m_3 + 2m_4) +$$

$$(2gL_2L_3s^3y)(m_3^2 + 2m_4^2) + (gL_2s^2)\begin{pmatrix} (L_3sy)(m_2m_3 + 2m_2m_4 + \\ 6m_3m_4) - 4m_3m_4r_4\bar{z}_{(RF)_3} - \\ 4m_4^2r_4\bar{z}_{(RF)_3} \end{pmatrix} \quad (22)$$

$$\#v = (-2L_3g^2L_2m_4)(m + m_3) - L_3mg^2L_2m_3 - 4L_3g^2L_2m_4^2 +$$

$$(4L_3gL_2^2m_3S^2)(m_3 + 3m_4) + L_3m_2gL_2^2m_3s^2 +$$

$$(2L_3gL_2^2m_4s^2)(m_2 + 4m_4) - \begin{pmatrix} (4gL_2m_4s^2)(\bar{z}_{(RF)_3})^2 + 2mgL_2s^2(\bar{z}_{(RF)_3})^2 - \\ 8\bar{I}_{(RF)_3}s^4(\bar{z}_{(RF)_3})^2 + 2m_2L_2^2s^4(\bar{z}_{(RF)_3})^2 \end{pmatrix}$$

$$(m_3 + m_4) + (2NL_3gL_2s^2 - 2\bar{I}_{(RF)_3}gL_2s^2)(m + 2m_4) + (4\bar{I}_{(RF)_3}L_3gs^2)(m_3 + 2m_4) +$$

$$(2\bar{I}_{(RF)_3}L_2^2s^4 - 2NL_3L_2^2s^2y)(m_2 + 4m_3 + 4m_4) + (8\bar{I}_{(RF)_2}s^2)(\bar{I}_{(RF)_2}s^2 - NL_3)$$

$$\#vi = (4\bar{I}_{(RF)_2}s^2 + L_2^2m_2s^2)(ST_{(RF)_3} + 2Nr_4) + (2f_{(RF)_2}L_2^2m_2s^2 + 8f_{(RF)_2}\bar{I}_{(RF)_2}s^2)(L_3 + r_4) -$$

$$(2f_{(RF)_3}gL_2(L_3 + r_4) + gL_2sT_{(RF)_3} + 2NgL_2mr_4 - 2g^2L_2m_4r_4)(m + 2m_4) - 8gL_2^2m_4^2r_2s^2 -$$

$$8g\bar{I}_{(RF)_2}m_4r_4s^2 - \begin{pmatrix} 8\bar{I}_{(RF)_2}s^5y\bar{z}_{(RF)_2} + 4L_2s^3T_{(RF)_2}\bar{z}_{(RF)_3} + 8f_{(RF)_3}L_2^2s^2\bar{z}_{(RF)_3} + \\ 8f_{(RF)_3}L_2^2r_4s^2 - 8NL_2^2r_4s^2 - 4L_2^2s^3T_{(RF)_3} - (2L_2^2m_2s^5y\bar{z}_{(RF)_2}) \end{pmatrix}$$

$$(m_3 + m_4) - (2gL_2^2m_4r_4s^2)(m_2 + 4m_4) - (2gL_2^2m_4r_4s^2)(m_2 + 4m_4) +$$

$$(2gL_2s^3y\bar{z}_{(RF)_3})(2m_4^2mm_3 + mm_4 + 2m_3m_4)$$

Using the Laplace transform in Eq. 13-18 and 23-24 are obtained:

$$\frac{f_{(LF)_3} + f_{(RF)_3}}{S} = (\frac{m}{2} + 2(m_2 + m_3 + m_4))s^2y_s$$

$$\Rightarrow f_{(LF)_3} + f_{(RF)_3} = (\frac{m}{2} + 2(m_2 + m_3 + m_4))s^3y_s \quad (23)$$

$$L_2\theta_{S(LF)_2} + L_3\theta_{S(LF)_3} = L_2\theta_{S(RF)_2} + L_3\theta_{S(RF)_3} \quad (24)$$

Using Eq. 21-23 and 25 is obtained.

$$L_2 * \left(\frac{-2}{S} * \frac{\#i}{\#ii}\right) + L_3 * \left(\frac{-2}{S} * \frac{\#iii}{\#ii}\right) = L_2 * \left(\frac{1}{S} * \frac{\#iv}{\#v}\right) +$$

$$L_3 * \left(\frac{1}{S} * \frac{\#vi}{\#v}\right) \Rightarrow \left(\frac{2L_2 * (\#i) + 2L_3 * (\#iii)}{\#ii}\right) + \quad (25)$$

$$\left(\frac{L_2 * (\#iv) + L_3 * (\#vi)}{\#v}\right) = 0$$

The string (#ii) and (#v) are equal to each other because the dimensions of legs are equal. The result is equality between $\bar{z}_{(LF)_3}$ and $\bar{z}_{(RF)_3}$, between $\bar{I}_{(LF)_2}$ and $\bar{I}_{(RF)_2}$ and between $\bar{I}_{(RF)_3}$ and $\bar{I}_{(LF)_3}$. As a result and after simplification Eq. 26 is achieved:

Table 1: Numerical values of the robot parameters

Values	Length (m)	Mass (kg)	Inertia (kg.m²)	Radius (m)
Body	-	3	-	-
Leg	0.1	0.1854	1.6×10^{-4}	0.01
Tigth	0.1	0.1854	1.6×10^{-4}	0.01

$$(\#vii)y + (\#viii)(f_{(LF)_3} + f_{(RF)_3}) + (\#ix)$$
$$(T_{(LF)_3} + T_{(RF)_3}) + (\#x)(T_{(LF)_3} + T_{(RF)_3}) + (\#ix) = 0 \quad (26)$$

By substituting Eq. 23 in Eq. 26, the only remaining unknown parameter, friction force is also deleted from the equation. And the relationship 27 is obtained where it is the final transfer:

$$(\#xii)y + (\#xiii)(T_{(LF)_2} + T_{(RF)_2}) + (\#xiv)$$
$$(T_{(LF)_3} + T_{(RF)_3}) + (\#v) = 0 \quad (27)$$

Robot parameters such as mass, length, etc. are assumed with data given in Table 1 to obtain the system transfer function.

Some other parameters are defined as = 9.81 m/sec², N = 21.88 (N), $z_{(LF)_3} = 0.05$ m and $z_{(RF)_3} = 0.05$ m. Body of robot is assumed to be steel by density 7.85 g/cm³ that the mass and radius of the ball is negligible compared to other dimensions. By substituting and simplifying, transfer function is obtained as Eq. 28:

$$(7.69\times10^4 s + 11.07s^3)(T_{s(LF)_2} + T_{s(RF)_2}) +$$
$$(2.68\times10^4 s - 56.55s^3)(T_{s(LF)_2} + T_{s(RF)_2}) + \quad (28)$$
$$(2.26\times10^6 - 8.29\times10^3 s^2 3.69\times10^4 s^3 + s^4 - 19.52s^5) y_s = 0$$

The transfer function of the input torque of tight and input torque of the leg are achieved as Eq. 29 and 30, respectively:

$$\frac{y_s}{(T_{s(LF)_2} + T_{s(RF)_2})} = \frac{11.07s^3 + 7.69\times10^4 s}{19.52s^5 - s^4 - 3.69\times10^4 s^3 + 8.29\times10^3 s^2 - 2.26\times10^6} \quad (29)$$

$$\frac{y_s}{(T_{s(LF)_2} + T_{s(RF)_2})} = \frac{-56.55s^3 + 2.68\times10^4 s}{19.52s^5 - s^4 - 3.69\times10^4 s^3 + 8.29\times10^3 s^2 - 2.26\times10^6} \quad (30)$$

CONCLUSION

Transfer function shows relationship between output function, move the center of gravity with the input function, torque applied by the motor. Using transfer function determines the behavior of the system and its improvement is possible. Therefore, the continuation of the project can be added controller designed to improve system performance.

Since, the transfer function of the system is calculated, a Proportional-Integral-Derivative (PID) controller can be a designed using Ziegler-Nichols method. For this purpose, after calculating the starting point this point will be enough to change parameters such as maximum overshoot, the peak time and so affordable.

Coordinate system used in the project, Cartesian coordinate system is not recommended. It is suggested that the coordinate system used Dnavyt-Hartnbrg.

Linearization of nonlinear equation was assuming at small angles. This assumption is acceptable in a small range of angle. So, another solution must be offered. It is suggested that the mass and radius of ball connected to the leg be negligible.

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