

## Design and Analysis of Full Composite Structure of Lambda Wings

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**Abstract:** In this study, layering of all components of lambda wing including spars, shell and ribs is performed in initial dimensions and sizes using manual calculations. Then, strength analysis and buckling of structure are performed using finite element method. To remove flexural-tensile and flexural-torsional effects in wing structure, the layering is considered in symmetric mode. Moreover, continuity of integrated structures is observed while initial designation. For geometric modeling, Catia Software is used and for stress-strain analysis and types of buckling such as Euler and Crippling buckling, Nastaran Software is used due to boundary conditions. Finally, critical points of the structure against types of analysis are detected and are redesigned if required.

**Key words:** Lambda wing, composite, strength measurement, buckling, safety margin

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### INTRODUCTION

In field of designing traditional aircrafts, a cylindrical body and separate wings and completely separable from each other used to be applied. However, in continue of development of Aviation Industry, design and construction of aircrafts with a body and separate tail was emerged. Such design of aircrafts was began at the first time by English people with specifications such as higher lift and its effects, better wing structural design, improving aerodynamic and larger lift to drag ratio, reducing environmental impact, stealth, less abrupt changes in cross-sectional area and maneuverability (Fearon, 1985). However, Germans became pioneer in military field and presented this project for greater range for their aircrafts. Since, 1980, Americans considered construction of the aircrafts seriously in their agenda due to form of the aircrafts and their stealth. This approach led in fact o design and construct B2 stealth bomber aircraft by Northrop in 1990. Since that time, the aircraft companies of the world began designing such aircrafts across the world seriously.

### MATERIALS AND METHODS

Different methods are presented for arrangement of internal components of such wings in different articles and references and the most important ones are based on classic and finite element methods in conceptual design and details. For example, Locatelli (2012) investigated

strength phenomena using static and buckling analysis and derived configuration of wing for an aircraft with high speed. Polagangu *et al.* (2009) investigated design of a spar and rib of composite wing of light transport aircraft against Brazier load using Finite Element Analysis and could finally optimize it. In this project, 67% of spar cap layers were arranged in one direction mode and in line with loading ( $0^\circ$ ) and 80% of web spar layers were arranged as cloth.

Xu (2012) conducted a study on optimal design of a composite wing structure for a flying-wing aircraft subject to multi-constraint on 2012 in Grandfield University of America. The aircraft had two spars and had used carbon for wing structural components. Moreover, one fourth of layering direction of out shell is  $0^\circ$ , one fourth is  $90^\circ$  and about half of it is  $45^\circ$ .

Kassapoglou (2013) has presented a series of layering considerations such as symmetric layering and its balance because of  $0^\circ$  of tensile bending matrixes and decline of tensile shear matrixes. The researchers of NASA have emphasized use of law of 10% of zero layers, negative and positive  $45$  and  $90^\circ$  and other factors. Moreover, identification of lambda wing structure in current form has been also studied by the scholars separately as statistical analysis (Chamis, 1989).

According to collected studys and references in different publications, in this study, after identification and investigation of relevant theories of rules of designing lambda wings from fiber composite materials,

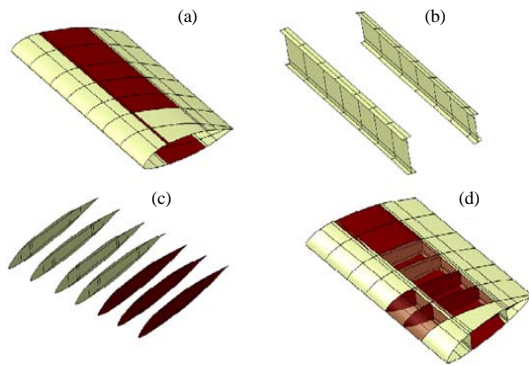


Fig. 1: Geometry of wing components: a) Shell; b) Spars; c) Ribs; d) Assembly of parts in form of lambda wing

layering is firstly performed due to desired considerations and rules. Then, in addition to have an overview on maximum loading applied on outer wing, the applied and applied loads. Then, a series of manual calculations are performed for initial sizing of spar caps webs and ribs. Afterwards, after determining initial sizing, modeling and analysis is done in Nastaran Software and different loadings are applied on it. In last step, all results of structural response are extracted and strength and buckling analyses are performed on wing components and their safety margins are obtained.

**System model:** Geometry of studied structure in this study includes different components. Spars are the most important members forming wing and as a longitudinal beam, they have cap and web to bear bending and shear loads. Other component of wing is rib that has different form and dimension for airfoil of each wing. The components transfer the loads applied on wings to spars and shells and another important of the components is preventing buckling of wing shell. Stringers or same longitudinal beams are responsible for lifting buckling of wing shell and as a result, reduced thickness of shell. Finally, the shell can directly take aerodynamic loads and transfer them to other components (spar and rib). In fact, it can form surface shell and outer geometry of wing. Figure 1 illustrates the details mentioned for wing components.

## RESULTS AND DISCUSSION

**Structure design:** The desired wing in the present study includes 2 spars embedded on wing chord. The wing chord is equal to 2734 mm. The front spar is located at a distance of 728 mm from the beginning of the chord and rear spar is located at a distance of 1860mm from beginning of the chord. The height of highest part of wing

in front spar is to 395 mm and rear spar is to 324 mm and wing length is equal to 6 m. Ribs are also arranged in parallel form with longitudinal axis of the aircraft and include 7 ribs with different distances.

To gain features such as low density, strength and high special modulation, high and appropriate mechanical properties, high rigidity to control deformations and lifts, high fatigue strength and finally, low thermal expansion coefficient; majority of fibers used in wing components are carbon fibers and among polymer matrixes, epoxies are appropriate to be combined with carbon fibers. This is because; it has high mechanical properties and it is easy to work with these materials.

### Design of cap spars layering and calculation of their engineering dimensions:

In cap spars, strain of woven and unidirectional fibers is used and unidirectional fibers (with angle of 0) are used to increase strength against bending. On the other hand, to maintain spar integration based on the extracted layering for webs, woven fibers are also used. Hence, unidirectional fibers are embedded in spaces between woven fibers. Therefore, layering is proposed as follows:

$$\text{Cap:} [(0/90), 0_3, (\pm 45), 0_2, (\pm 45), 0]_s \quad (1)$$

To do relevant calculations, it is assumed that total shear force and torque applied on section by web spars and bending torque is beard by cap spars. In fact, in steps of conceptual designation, it is assumed that shell bears no structural loading and spars bear total structural loadings by themselves.

Schematically, in a 2-spar project, the box is formed of two spars and is simplified to perform calculations. In this simplified state, the height of box is considered equal to mean length of front and rear web spars. For engineering calculations, dimensions of cap spars, due to the assumption that bending torque applied on wing section is beard just by cap spars; the cap spar is sized based on bending torque bearable by the section:

$$\sigma = \frac{Mc}{I} \text{ and } c = \frac{h}{2} \quad \sigma < \frac{S}{n} \quad (2)$$

Where:

h = Box height

M = Bending moment

I = Second surface moment

n = Design safety

s = The submission level specified for applied material

Therefore, according to Eq. 2, the calculations would be obtained as follows:

$$\frac{0.7 \times 151962 \times 180 \times 10^{-3}}{I} < \frac{714 \times 10^6}{1.5} \rightarrow \quad (3)$$

$$I > 40.2 \times 10^{-6} \text{ m}^4$$

According to the calculations for front spar, second moment of cross section is calculated just through considering front cap spars:

$$I = 2 \times A_c \times \left(\frac{h}{2}\right)^2 \rightarrow 2 \times A_c \times (180)^2 > \quad (4)$$

$$40.2 \times 10^6 \text{ [mm}^4\text{]}$$

Finally, minimum allowable value for area of front cap spar to bear maximum torque applied on section is obtained:

$$A_c > 620.3 \text{ mm}^2 \quad (5)$$

According to thickness of front cap spar (based on number of composite layers) to 5.46mm, its width ( $b_c$ ) is calculated as follows:

$$A_c = b_c \times t_c \rightarrow b_c > 113.6 \text{ mm} \quad (6)$$

The calculations show that statically, front cap spar with desired layering and thickness of 5.46 mm and minimum width of 113.6 mm can bear the desired bending torque. It should be mentioned that in rear cap spar, because of existence of different flying states and maneuvers, in some cases, larger portion of bending torque may be applied on rear spar and this state is observed mostly in landing conditions. Hence, in this step that the aim is obtaining appropriate early estimation, the same probability of 70% is used and the width of rear spar is also considered to 113.6 mm.

**Design of layering web spars and calculations of engineering dimensions for them:** In web spars, woven fibers are used. The woven fibers are mostly with angle of  $\pm 45^\circ$  and are used to bear shear loadings. On the other hand, to maintain integration of web and spar, the layering of spar should be reviewed. Moreover, it should be noted that the  $\pm 45^\circ$  layers should be embedded away from neutral axis as much as possible. As a result, layering is proposed as follows:

$$\text{web: } [(0/90), (\pm 45)_4, (0/90), (\pm 45)_2]_s \quad (7)$$

For engineering calculations of dimensions of web spars, the shear stress created in web spar in one section can be caused either by applying shear force or torque. To facilitate the calculations, first the shear process in

desired section is calculated as showed in Eq 8. According to (q) shear in web spar, the shear stress caused by shear force is approximately calculated as follows:

$$q = \frac{V}{h}, \tau = \frac{q}{t_w} \quad (8)$$

Where:

$V$  = Maximum shear force in desired section

$t_w$  = Thickness

$h$  = Height of web spar

The torque applied on section is considered as force caused by a force coupling that each force can affect each web. The amount of this force is calculated as follows:

$$F = \frac{T}{d} \quad (9)$$

Where:

$T$  = Torque

$d$  = Distance between two front and rear spars

Hence, according to Eq. 8 and 9, the calculations are obtained as follows:

$$q = \frac{63287}{360} = 175.8 \text{ N/mm} \quad (10)$$

The shear stress created by shear loading applied on web spar is calculated as follows:

$$t = \frac{175.8}{5.6} = 31.4 \text{ MPa} \quad (11)$$

The shear stress caused by torque is obtained as follows: firstly, shear force caused by torque using rule of force coupling in wing box with two spars is as follows:

$$F = \frac{33118.5 \times 10^3}{1132} = 29256.6 \text{ N} \quad (12)$$

Then, using shear force obtained,  $q$  and  $t$ -values are obtained as follows:

$$q = \frac{29256.6}{360} = 81.3 \text{ N/mm} \quad (13)$$

$$t = \frac{81.3}{5.6} = 14.51 \text{ MPa}$$

Through comparing Eq. 11 and 13, maximum shear stress is obtained to 31.4 MPa. Moreover, the confidence coefficient is obtained as follows:

$$n = \frac{S_s}{\tau} = \frac{59.6 \text{ MPa}}{31.4 \text{ MPa}} = 1.9 \quad (14)$$

**Design of rib layering:** The effect of rib is low in lift and its most important role is conducting shear flow and preventing buckling in shells. From perspective of structural strength, number of ribs is calculated due to minimum distance of ribs based on requirements of shell buckling. Forming a box with its limit in rib, wing shell and spar, adequate strength and distribution of load would be applied on structure and rib plays important role in formation of this box. Layering of ribs due to important point of loading capacity of  $\pm 45^\circ$  cloth layers in shear and idea of similar aircrafts is as follows:

$$\text{Rib:} [(0/90), (\pm 45)_3]_s \quad (15)$$

**Design of wing shell layering:** Shell takes aerodynamic loads directly and transfers it to all components of wing. In fact, shell can form surface and outer geometry of wing. In aircraft wing, distance of ribs from each other can play role in buckling capacity of wing shell. As shell panels between two spars are intensely exposed to buckling, through calculating minimum effective length for the panels, minimum number of ribs to prevent shell buckling is calculated. According to experiences, bending torque and compressive stresses have more portion than other loads to create buckling in planes of aircraft wing:

$$\text{Skin:} [(0/90), (\pm 45)_3]_s \quad (16)$$

**Strength analysis of structure:** The forces in loading points are applied according to USAR standard paragraph 301 (Valavanis and Vachtsevanos, 2014) and with applying coefficient of 1.5 to the model. After completing the FEM model, model is analyzed and results are obtained. In Fig. 2 and 3, two examples of total model analysis are illustrated and other results are used due to their use in calculations.

For failure of isotropic materials, criteria such as the maximum axial stress (Rankin), the maximum shear stress (Trska) or distortion energy criteria (von Mises) are existed (Fatemi and Socie, 1989). Hence, different criteria are also defined for composite materials based on their orthotropic structure. To ensure about safety of composite structure components to bear applied loads, this study has used an interactive criterion. Interactive criteria consider interaction of different stresses in failure of a part. In interactive perspective, failure is on this basis that it is assumed that the failure of material varies under multidirectional loadings.

Tsai-Wu failure criterion (Groenwold and Haftka, 2006) known as Tansuri failure criterion has more phrases and main and general parameters than other criteria and also considers difference between compressive and tensile strengths of a layer. Tsai-Wu (Groenwold and Haftka, 2006) applies failure theory on a layer in plane stress. Hence, a layer is failed, if following equation is violated:

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \quad (17)$$

Components  $H_1$ ,  $H_2$ ,  $H_6$ ,  $H_{11}$ ,  $H_{22}$  and  $H_{66}$  are coefficients of Tsai-Wu failure criterion for orthotropic materials. To calculate Tsai-Wu value for each layer, due to program analysis, its outputs as local stresses of each layer in directions 1 and 2 ( $\sigma_1$ ,  $\sigma_2$ ) and  $\tau_{12}$  shear stress are used as input. According to calculation of constants of Tsai-Wu criterion, strength of each layer against applied loads and failure could be obtained.

**Validation of programs:** In the book "Mechanic of Composites", Kaw (2005) has exposed a multilayer composite with graffiti/epoxy properties under loading and after obtaining matrixes of (A), (B) and (D), strains and curvatures of the middle plates, overall and local stresses in each layer are presented in form of tables and the programs presented in this study are absolutely based on numbers in tables.

**Buckling analysis:** Upper plate of lambda wing is formed of 7 buckling panels ended to front and rear spars from width and is ended to ribs from length. To obtain critical force of buckling in lambda wing plate, hybrid compressive and shear formulations are used. In Fig. 4, example of composite panel is illustrated that is limited between two values of rib and front-rear spars.

Through deriving output forces of Nastaran software obtained from wing panels and using the information, firstly the shells are more critical near to wing root and secondly, shells are more sensitive to compressive loadings compared to shear loads. Hence, first the buckling safety margin is calculated for 3 panels as follows.

As it is obvious in Table 1 wing shell is unstable in terms of buckling and is failed. According to output loads of software, different methods are existed to increase critical buckling force as follows Reducing the width of the panels or in other words Stringer planting along the length of the wing increasing number of ribs increased flexural stiffness that results in increased  $D_{11}D_{12}D_{22}D_{66}$  and increased buckling force d) in many wing shells, to

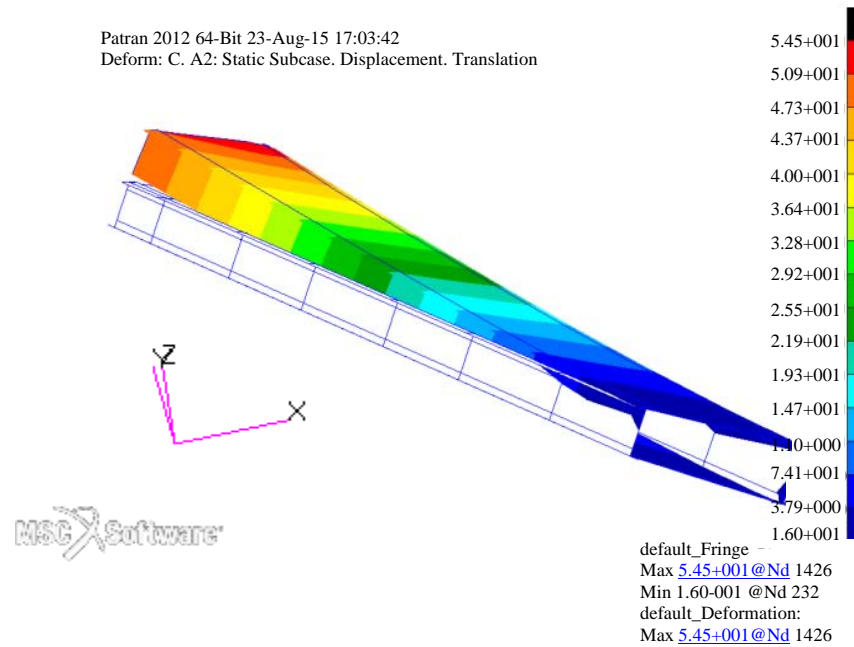


Fig. 2: Maximum drift counter in wing

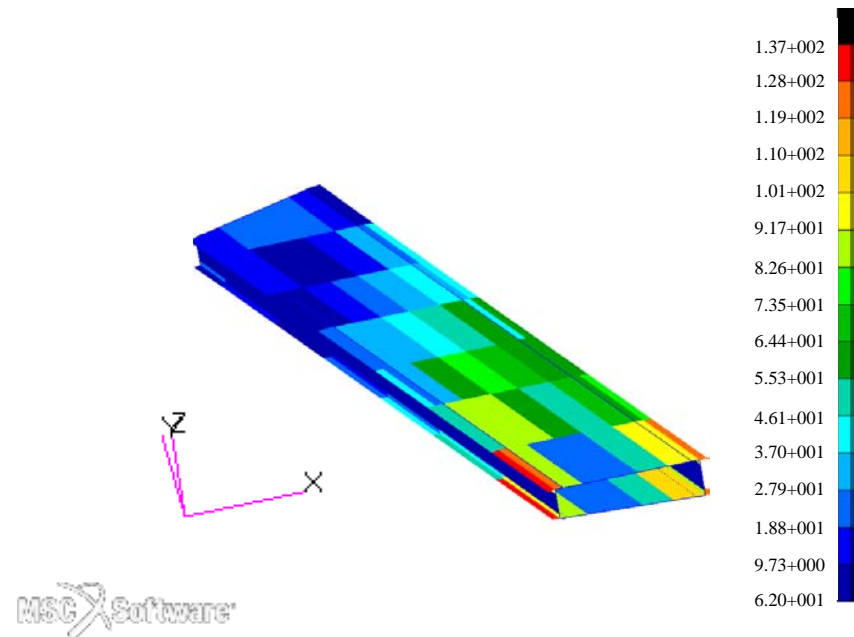


Fig. 3: Stress counter in wing

increase critical buckling loading, panel sandwich structures are also used. By Niu (1992), a plate is considered under effect of compressive loading and on simple support. Length of plate is 3 times more than its width ( $a = 3b$ ). To increase critical stress of plate, 3

states are considered and are compared: Add two lateral beams (rib), add five lateral beams (rib) and adding a longitudinal beam (stringer).

After the calculations, it was observed that in first state, critical stress is not added, in second state with

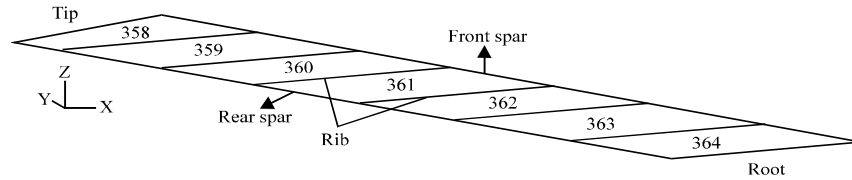


Fig. 4: Example of composite panel limited between rib and spar

Table 1: calculating safety margin of upper shell in regions

Panel No.	a	b	Nx	AR	m	Nxcrit	Rc	MS
1 (364)	0.656	1.1339	154232	0.578534	5.77E-01	1.85E+03	8.35E+01	-0.98802
2 (363)	0.739	1.1339	93944	0.651733	6.51E-01	1.85E+03	5.09E+01	-0.98034
3 (362)	0.612	1.1339	68859	0.53973	5.39E-01	1.85E+03	3.73E+01	-0.97317

adding 5 beams, critical stress is increased just to 56%; although in third state, only one longitudinal beam is added and critical stress is increased 4 times.

In another method, panel sandwich structures can be applied. These sandwich structures act similar to I-shaped sections that original bending force is beard by wings of this section and the performance is shear and is used for jointing wings. In such structures, integrated function of sandwich plates needs appropriate joint between plates and core. It means that the issue of technology and use of skilled human forces to make such structures is very important and in case of lack of observance of technology, the plates can't be effective. Sandwich plates have more complicated calculations than reinforced plates. Hence, the discussion of accuracy and speed in these structures is very important. Investigations have shown that for aircrafts with smaller bodies, the cost of using sandwich plates is significantly lower than reinforced plates with stiffeners. Hence, in large size aircrafts, the result is reverse and plates reinforced with stiffeners are more economic and cost-effective than sandwich plates (Allen, 2013). It should be mentioned that the studied aircraft is in class of large aircrafts.

In all Boeing 787 models containing 35 ton carbon fibers with reinforced polymer (usually epoxy), 23ton is composed of fiber (Toensmeier, 2005).

In wings of Boeing 787 Aircrafts, reinforced composite plates are used. Using panel sandwich plates can make limitation for internal volume of large wings to use fuel and equipment. In panel sandwich plates, corrosion in structure happens after a while and especially in humid regions more than reinforced structures. Repairing and maintaining panel sandwich plates is more complicated than reinforced plates.

Therefore, for lambda wing shell, due to obtained safety margins obtained from calculations above, two methods of increasing stringer and increasing bending stiffness are used.

## CONCLUSION

In this study, first the complementary process of wings and the way of reaching to advanced lambda wings is investigated and in addition to assessing advantages and challenges with these wings, technical comparisons are conducted between lambda-shaped and traditional wings. After assessment of structures of lambda wings of the current age, the rules of designation and arrangement of wing structures is reviewed. Afterwards, architectural engineering of designing wing components in practice and using composite and metal materials are discussed. In this engineering design, advantages and disadvantages of 2 and 3-spar wings, design of ribs, stringers and shells are discussed. Then, design of full composite structures, the manner of choosing composite materials, advantages and disadvantages of composites are explained. After determining spatial information of spars and ribs in wing, relevant considerations of design of layering, layering wings of several aircrafts, choosing materials for desired wing and their information, deriving maximum loads on wing structure, cap spar layering, web spar layering, rib and shell layering and manual engineering calculations and finally, early sizing of all wing components are obtained. For purpose of analysis of lambda wing and derive forces, modeling is done firstly in Catia software for all parts of wing like shell, spars and ribs and the location of loading is also specified. Moreover, the surfaces are decorated for continuity of meshes. Then, the prepared model is entered to Nastaran software following import order and meshing is done using isomesh method to have exact results. After unification of all nodes, boundary conditions and loading are applied and coefficient of 1.5 is applied on it. Then, the selected materials are defined and layering is performed in software and the results are derived after the analysis. For strength analysis of layers against applied loads, after comparing criteria of composites, Tsai-Wu failure criterion is used. According

to need to calculate matrixes (A), (B) and (D), encoding is performed to obtain applied stresses on layers and the results of analysis are considered as input of the program. The program was written for all wing structures separately and after obtaining stresses and strains, Tsai-Wu is obtained numerically. Then, buckling considerations are investigated in full composite lambda wing. Composite plates are sensitive to compressive and shear loads and buckling failure happens in them. experiences have shown that torque and compressive loadings have more portion in causing buckling in composite plates of aircraft wing compared to other types of loading. In buckling analysis, determining type of boundary conditions is very important. Boundary conditions around a plate are determined using type of support limited boundaries. The supports are in simple, clamped and free types. In wing structures, shells and web spars have usually a behavior between simple and clamped support. The calculations are usually done conservatively. Hence, the support would be usually selected in a type to bear lower buckling load. Hence, simple support is selected as boundary condition. Moreover, all boundary conditions are compared using diagrams and finally, the equation for shell, web spars and ribs is derived as used in each section. For calculations of composite plate buckling (after adjusting boundary conditions) against applied loadings on composite structure, matrix (D), length (a) and width (b) of leaded panels, length to width ratio of  $AR = a/b$  plate and finally, the most critical buckling or  $m = AR(D_{22}/D_{11})^{1/4}$  should be calculated. To calculate the above mentioned items, outputs of first program are used and in third program, according to boundary conditions, critical force of relevant panel is obtained based on inputs. Then, in same program, output of forces analyzed in Nastaran software are entered and due to critical force of relevant panel, if  $R_c$  is compressive force of relevant panel and if  $R_s$  is the shear load of relevant panel, both of them would be obtained and finally, safety margin of relevant panel or MS is determined. The third program has the capability to calculate safety margin for all panels simultaneously (shells web spars and ribs). Finally, the analysis results are presented as follows:

- All structural parts have positive safety margin and no failure happens in wing structure
- The structure is in high level in terms of strength
- In terms of buckling, webs and ribs are good
- Crippling buckling in cap spars is better than Euler buckling
- Wing shell has reached steady state with the increase in stringers and increase in bending stiffness

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