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Existence of Solutions of Impulsive Quantum Stochastic Differential Inclusion

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Abstract: In this study, we establish existence of solutions of impulsive Quantum Stochastic Differential Inclusion (QSDI). Our technique is based on the fixed point approach for multivalued maps.

Key words: Impulsive inclusion, multivalued maps, non-commutative stochastic processes, fixed point theorems, existence

INTRODUCTION

Existence of continuous selections of multifunctions associated with the sets of solutions of Lipschitzian Quantum Stochastic Differential Inclusions (QSDIs) have been considered by Ayoola (2008) while the existence of solution of quantum stochastic evolution arising from hypermaximal monotone coefficients was established by Ekhaguere (1992). Results concerning the topological properties of solution sets of Lipschitzian QSDIs were also considered by Ayoola (2008). In order to generalize the results in the literature concerning QSDIs, existence of continuous selections of solutions sets of non-lipschitzian quantum stochastic differential inclusions and existence of continuous selection of multifunctions associated with quantum stochastic evolution inclusions under a general Lipschitz condition were considered by Bishop and Anake (2013), respectively.

In the case of classical differential equations, intensive research have been done concerning the existence of solutions of impulsive differential equations and inclusions of several types (Fan and Li, 2010; Federson and Schwabik, 2006; Pan, 2010; Ji and Li, 2011). For the importance and applications of impulsive differential inclusions (Bishop and Agboola, 2014; Ogundiran, 2013). The role of impulsive differential inclusions in the theoretical and analytical study of differential equations as outlined in the above references is a motivation for studying this class of inclusions. In "QSDE" not much has been done concerning impulsive QSDEs. However, some results concerning impulsive QSDEs have been established by Bishop and Agboola (2014) and Ogundiran (2013). This research is therefore, concerned with similar results established by Benchohra *et al*. (2006). Therefore, the results obtained here are generalizations of analogous results due to the references (Benchohra *et al*., 2006) concerning classical differential inclusions to the non-commutative quantum setting.

In what follows (Ayoola, 2008; Ekhaguere, 1992; Ogundiran, 2013), we adopt the definitions and notations of the following spaces; PC(I, Ã), PC'(I, Ã), PC(I, sesq(D⊗E)), PC'(I, sesq(D⊗E)), clos(N), clos(Ã), Ad(Ã), Ad(Ã)$_{wac}$, $L^p_{loc}(Ã)$, $L^p_{loc}(Ã)_{mus}$ and the Hausdorff topology on clos(Ã). The Hausdorff distance, ρ(A, B) is defined as:

$$\rho(A, B) = \max(\delta(A, B), \delta(B, A)), A, B \in clos(C)$$

And:

$$d(x, B) = \inf_{y\in B}|x-y|,\ \delta(A, B) = \sup_{x\in A} d(x, B)$$

where, x∈C is a complex number. Then ρ is a metric on clos(C) and induces a metric topology on the space. All through the remaining part of this research we take η, ξ ∈ D⊗E to be arbitrary except otherwise stated.

Lemma 1: Assume that F: J×E → P(E) is a nonempty, compact-valued, multivalued map such that:

(a) (t, u) → F(t, u) is L ⊗ B measurable
(b) u → F(t, u) is lower semicontinuous for a.e. t ∈ J; for each r > 0, there exists a function $h_r \in L^1(J, R_+)$ such that $\|F(t, u)\|_p := \sup\{|\upsilon| : \upsilon \in F(t, u)\} \le h_r(t)$
for a.e. t∈J and for u∈E with ‖u‖≤r. Then, F is of l.s.c. type.

Theorem 1: Let Y be separable metric space and let N: Y→ P(L^1(J, E)) be a multi-valued operator which has property (BC). Then N has a continuous selection, that is, there exists a continuous function f: Y → L^1(J, E) such that f(x)∈N(x) for every x∈Y.

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PRELIMINARIES

We introduce the following QSDI in integral form:

$$\begin{aligned} X(t) \in X_0 + \int_{t_0}^{t} (E(X(s), s)d\wedge_{\pi}(s) + F(X(s), s)dA_f(s) + \\ G(X(s), s)dA_g^+(s) + H(X(s), s)ds) \\ X(t_0) = X_0,\ t \in [0, T] \end{aligned} \tag{1}$$

Inclusion (Eq. 1) is the usual Hudson and Parthasarathy (1984) formulation of Bo-son quantum stochastic calculus. In the notations and definitions of various spaces of stochastic processes introduced in the research by Ekhaguere (1992), the coefficients E, F, G, H, lie in $L^2_{loc}([0, T]\times\tilde{A})_{mus}$ where $\tilde{A}$ is a locally convex topological space defined by Ekhaguere (1992). The underlying elements of $\tilde{A}$ consists of linear maps from $D\underline{\otimes}E$ into $\Re\otimes\Gamma(L_\gamma^2(\mathbb{R}_+))$ having domains of their adjoints containing $D\underline{\otimes}E$.

The maps f, g, π appearing in Eq. 1 lie in some function spaces defined by Ayoola (2008), Bishop and Ayoola (2015) and Bishop and Anake (2013) while the integrators $\wedge_\pi$, A^+_g, A_f are the gauge, creation and annihilation processes associated with the basic field operators of quantum field theory. D is some pre-Hilbert space whose completion is $\Re$, γ is a fixed Hilbert space and $L_\gamma^2(\mathbb{R}_+)$ is the space of square integrable γ-valued maps on $\mathbb{R}_+$. The inner product of the Hilbert space $\Re\otimes\Gamma(L_\gamma^2(\mathbb{R}_+))$ will be denoted by $\langle ., .\rangle$ and $\|.\|$ the norm induced by $\langle ., .\rangle$. E the linear space generated by the exponential vectors in Fork space $\Gamma(L_\gamma^2(\mathbb{R}_+))$.

Definition 1:
(i) By a multivalued stochastic process indexed by $I = [0, T]\subseteq\mathbb{R}_+$ we mean a multifunction on I with values in $clos(\tilde{A})$.
(ii) If ϕ is a multivalued stochastic process indexed by $I = [0, T]\subseteq R_+$, then a selection of ϕ is a stochastic process $X: I\to\tilde{A}$ with the property that $X(t)\in\phi(t)$ for all $t\in I$. A multivalued stochastic process $\phi(t)$ will be called (iii) Adapted if $\phi(t)\subseteq\tilde{A}_t$ for each $t\in I$.
(iv) Measurable if $t\to d_{\eta\xi}(x, \phi(t))$ is measurable where $x\in\tilde{A}$.
(v) It is L^1-measurable in the sense by Benchohra *et al.* (2006).
(vi) Locally absolutely p-integrable if the map $t\to\|\phi(.)\|_{\eta\xi}$ lies in $L^p_{loc}(\tilde{A})$, $t\in I$, $p\in(0, \infty)$.

We denote this set by $L^p_{loc}(\tilde{A})_{mus}$ and for $p\in(0, \infty)$, $I\subseteq\mathbb{R}_+$, the set $L^2_{loc}(I\times\tilde{A})_{mus}$ denotes the set of maps ϕ: $I\times\tilde{A}\to clos(\tilde{A})$ such that the map $t\to\phi(t, X(t))$ lies in $L^p_{loc}(\tilde{A})_{mus}$ for every $X\in L^p_{loc}(\tilde{A})$. It has be shown by Ekhaguere (1992) that the following first order initial value non-classical ordinary differential inclusion in integral form:

$$\begin{aligned} &\frac{d}{dt}\langle\eta, X(t)\xi\rangle \in P(X(s), s)(\eta, \xi) \\ &\langle\eta, X(t)\xi\rangle = \langle\eta, X_0\xi\rangle \end{aligned} \tag{2}$$

is equivalent to integral inclusion (Eq. 1). As sexplained by Ekhaguere (1992), the map P appearing in Eq. 2 has the form:

$$\begin{aligned} P(x, t)(\eta, \xi) = (\mu E)(x, t)(\eta, \xi) + (\gamma F)(x, t)(\eta, \xi) + \\ (\sigma G)(x, t)(\eta, \xi) + H(x, t)(\eta, \xi) \end{aligned}$$

$\eta, \xi\in D\underline{\otimes}E$ $(x, t)\in\tilde{A}\times I$. For the definition of a solution of (Eq. 1) (Bishop and Agboola, 2014; Ekhaguere, 1992) and the references therein. Next we introduce the impulsive QSDI. Let the intervals $I_0, I_1, ..., I_m$, I' and I_k; k = 1, 2, ..., m be as defined by Ogundiran (2013). where, $t_0 = 0$, $t_{m+1} = b$, $I = [0, b]$, $0<t_1<...<t_{m+1} = b$. For $x: I\to sesq(D\underline{\otimes}E)$, the space $PC(I, sesq(D\underline{\otimes}E)$ equip with the norm:

$$\|x\|_{PC} = \sup\{|x(t)(\eta, \xi)|: t\in I\}$$

is a Banach space. We consider the following impulsive QSDI given by:

$$\begin{aligned} dx(t) \in (E(x(t), t)d\wedge_{\pi}(t) + F(x(t), t)dA_f(t) + \\ G(x(t), t)dA_g^+(t) + H(x(t), t)dt), \\ \text{almost all } t\in I,\ t\neq t_k,\ k = 1, ..., m \\ \Delta x\big|_{t=t_k} = J_k(x(t_k^-)),\ k = 1, ..., m \\ x(0) = x_0,\ t\in[0, T] \end{aligned} \tag{3}$$

where, $P: I\times\tilde{A}\to clos(\tilde{A})$ is a multivalued map with non-empty compact values and $J_k\in C(\tilde{A}, \tilde{A})$, (k = 1, 2, ..., m) and $\Delta x(t_k) = x(t_k^+)-x(t_k^-)$ represents the jump in the state x at t_k.

Definition 2: A stochastic process $x\in PC(I, \tilde{A})\cap((t_k, t_{k+1}), \tilde{A})_{wac}$, $0\leq k\leq m$ is called a solution of (Eq. 3) if x satisfies the differential inclusion (Eq. 2) for almost all $I-\{t_1, ..., t_m\}$ and the conditions $\Delta x\big|_{t=t_k} = J_k(x(t_k^-))$ and $x(0) = x_0$. Next, we establish the following useful result.

Theorem 2: Suppose that the following is satisfied: (H_1) there exists a continuous non-decreasing function ψ: $[0, \infty)\to(0, \infty)$ and $p^\phi\in L^1(I, R_+)$ such that:

$$\left|\Phi(t, x(\eta\xi))\right| \le p^{\phi}_{\eta\xi}(t)\psi\left(\left\|x\right\|_{\eta\xi}\right) \quad (4)$$

for a.e. $t \in I$, $\eta, \xi \in D \underline{\otimes} E$ and $x \in \tilde{A}$ with:

$$\int_{t_{k-1}}^{t_k} p^{\phi}_{\eta\xi}(s)ds < \int_{N_{k-1}}^{\infty} \frac{du}{\psi(u)} \quad (5)$$

where, $\left\|x_0\right\|_{\eta\xi} \le N_0$, $N_{\eta\xi, k-1} \le \sup_{x\in[-M_{\eta\xi, k-2}, M_{\eta\xi, k-2}]}\left|J_{k-1}(x)\right| + M_{\eta\xi, k-2}$:

$$M_{\eta\xi, k-2} = \gamma^{-1}\left(\int_{t_{k-2}}^{t_{k-1}} p^{\phi}_{\eta\xi}(s)ds\right) \quad (6)$$

for k = 1, ..., m+1 and:

$$\gamma_l(z) = \int_{N_{l-1}}^{z} \frac{du}{\psi(u)},\ z \ge N_{l-1}, l \in \{1, ..., m+1\} \quad (7)$$

Then, for each k = 1, ..., m+1 there exists a constant $M_{\eta\xi, k-1}$ such that:

$$\sup\left\{\left\|x(t)\right\|_{\eta\xi}: t \in [t_k, t_{k-1}]\right\} \le M_{\eta\xi, k-1} \quad (8)$$

for each solution x of the problem (Eq. 3).

Proof: Assume x is a solution of (Eq. 3), then $x_{[0, t_1]}$ is a solution of (Eq. 2) for a.e. $t \in [0, t_1]$. Let $t^* \in [0, t_1]$ such that:

$$\sup\left\{\left\|x(t)\right\|_{\eta\xi}: t \in [0, t_1]\right\} = \left\|x(t^*)\right\|_{\eta\xi} \quad (9)$$

then from (Eq. 4), we get:

$$\frac{\frac{d}{dt}\langle \eta, X(t)\xi\rangle}{\psi\left(\langle \eta, X(t)\xi\rangle\right)} \le p^{\phi}(t), \text{ for a.e. } t \in [0, t_1] \quad (10)$$

$$\int_{t_1}^{t^*} \frac{\frac{d}{dt}\langle \eta, X(s)\xi\rangle}{\psi\left(\langle \eta, X(s)\xi\rangle\right)} ds \le \int_{t_1}^{t^*} p^{\phi}_{\eta\xi}(s)ds \quad (11)$$

continuing as above, we have:

$$\gamma_2\left(\left|x(t^*)(\eta, \xi)\right|\right) = \int_{N\eta\xi, 1}^{\left|x(t^*)(\eta, \xi)\right|} \frac{du}{\phi(u)} \le \int_{t_1}^{t^*} p^{\phi}_{\eta\xi}(s)ds \le \int_{t_1}^{t_2} p^{\phi}_{\eta\xi}(s)ds \quad (12)$$

And:

$$\left|x(t^*)(\eta, \xi)\right| = \sup\left\{\left|x(t)(\eta, \xi)\right|: t \in [t_1, t_2]\right\} \le \gamma_2^{-1}\left(\int_{t_1}^{t_2} p^{\phi}_{\eta\xi}(s)ds\right) := M_{\eta\xi, 1} \quad (13)$$

we continue in this order and allow $x\big|_{[t_m, T]}$ be a solution of Eq. 3, where we replace J, t [0, T] with J_m, t_m [t_m, T], respectively. We then obtain a constant $M_{\eta\xi, m}$ such that:

$$\sup\left\{\left|x(t)(\eta, \xi)\right|: t \in [t_m, T]\right\} \le \gamma^{-1}_{m+1}\left(\int_{t_m}^{T} p^{\phi}_{\eta\xi}(s)ds\right) := M_{\eta\xi, m} \quad (14)$$

This yields:

$$\left\|x\right\|_{PC} \le \max\left\{\left|y(0)(\eta, \xi)\right|\right\}, M_{k-1}: k = 1, ..., m+1 := \hat{b} \quad (15)$$

MAIN RESULTS

Theorem 3: Assume that the following conditions and the hypothesis of theorem 2 are satisfied:

H_1) P: $I \times \tilde{A} \to clos(\tilde{A})$ is a non-empty compact valued multivalued map (Bishop and Anake, 2013) such that:
H_2) $(t, x) \to P(t, x)(\eta\xi)$ is L^1-measurable.
H3) for a.e. $t \in I$ and $\eta, \xi \in D \underline{\otimes} E$, the maps $x \to \psi(t, x)(\eta\xi)$, $\psi \in \{\mu E, \upsilon F, \sigma F, H\}$ are non-empty upper semicontinuous (resp. lower semi-continuous) multivalued stochastic process in the sense by Ogundiran (2013).
H4) for each r>0 there exist a function $h_{\eta\xi, r} \in (I, R_+)$ such that:

$$\left|P(t, x)(\eta, \xi)\right| := \sup\left\{\left|\upsilon(\eta, \xi)\right|: \upsilon \in P(t, x)\right\} \le h_{\eta\xi, r}(t)$$

for a.e. $t \in I$ and for $x \in \tilde{A}$ with $\|x\| \le r$.
H5) there exists a continuous non-decreasing function defined in preliminaries and $p^{\phi} \in L^1(I, R_+)$ such that (Eq. 4) holds for a.e. $t \in I$, $\eta, \xi \in D \underline{\otimes} E$ and $x \in \tilde{A}$ with:

$$\int_0^b p^{\phi}_{\eta\xi}(s)ds < \int_{\infty}^{c} \frac{du}{\psi(u)},\ c = \left\|x_0\right\|_{\eta\xi} + \sum_{k=1}^{m} c_k \quad (16)$$

where, $\left\|x_0\right\|_{\eta\xi} \le N_0$, $N_{\eta\xi, k-1} \le \sup_{[-M_{k-2}, M_{k-2}]}\left|J_{k-1}(x)\right| + M_{\eta\xi, k-2}$:

$$M_{\eta\xi, k-2} = \gamma^{-1}\left(\int_{t_{k-2}}^{t_{k-1}} p^{\phi}_{\eta\xi}(s)ds\right) \quad (17)$$

for k = 1, ..., m+1 and then the impulsive QSDI (Eq. 3) has at least one solution.

Proof: By (H_1) and (H_3) imply Lemma 1 (Benchohra *et al*., 2006) and by Theorem 1 (Benchohra *et al*., 2006) we have that there exists a continuous map f: $PC(I, \tilde{A}) \to L^1(I, \tilde{A})$ such that $f(x_0(\eta, \xi)) \in P(x_0(\eta, \xi))$ for all $x \in PC(I, \tilde{A})$. Consider the following problem:

$$\begin{aligned} dx(t) &= (E(x(t), t)d\wedge_{\pi}(t) + F(x(t), t)dA_f(t) + \\ &\quad G(x(t), t)dA_g^+(t) + H(x(t), t)dt), \\ &\quad \text{almost all } t\in I,\ t\neq t_k,\ k = 1, ..., m \\ &\quad \Delta x\big|_{t=t_k} = J_k(x(t_k^-)),\ k = 1, ..., m \\ &\quad x(0) = x_0,\ t\in[0, T] \end{aligned} \tag{18}$$

Remark: If x∈PC(I, Ã) is a solution of problem (Eq. 3), then x is a solution of problem (Eq. 3). Since, existence of solution for problem (Eq. 3) have been established by Ogundiran (2013), we follow a similar procedure.

Next we transform the problem to a fixed point problem and use the schauder-tychonov fixed point theorem to establish our result. Consider the operator $N: PC(I, sesq(D\underline{\otimes}E)) \to PC(I, sesq(D\underline{\otimes}E))$ be defined by:

$$N(x)(t)(\eta, \xi) = \langle \eta, x_0\xi\rangle + \int_0^t \phi(s, x(s))(\eta, \xi)ds + \sum_{0<t_k<t} J_k(x(t_k)) \tag{19}$$

we show that N is compact.

Step 1: N is continuous:

$$\begin{aligned} &\big|N(x)_n(t)(\eta, \xi)-N(x)(t)(\eta, \xi)\big| \leq \\ &\int_0^t \big|P(s, x_n(s))(\eta, \xi)-P(s, x(s))(\eta, \xi)\big|ds + \\ &\sum_{0<t_k<t} \big|J_k(x_n(t_k^-)(\eta, \xi)-J_k(y(t_k^-)(\eta, \xi)\big| \leq \\ &\int_0^b \big|P(s, x_n(s))(\eta, \xi)-P(s, x(s))(\eta, \xi)\big|ds + \\ &\sum_{0<t_k<t} \big|J_k(x_n(t_k^-))(\eta, \xi)-J_k(y(t_k^-)(\eta, \xi)\big| \end{aligned}$$

Since, the map P and J_k, k = 1, ..., m are continuous then:

$$\big\|N(x)_n(t)-N(x)(t)\big\|_{PC} \leq \big|P(s, x_n(s))(\eta, \xi)-P(s, x(s))(\eta, \xi)\big|_{L^1} + \sum_{k=1}^{m}\big|J_k(x_n(t_k^-)(\eta, \xi)-J_k(y(t_k^-)(\eta, \xi)\big| \to 0$$

as n→0.

Step 2: N maps bounded sets into bounded sets in PC(I, sesq(D⊗E)). Let B be a bounded subset of PC(I, sesq(D⊗E)) for any x∈B let l be a constant such that:

$$\begin{aligned} N(x)(t)(\eta, \xi) &= \langle \eta, x_0\xi\rangle + \int_0^t \phi(s, x(s))(\eta, \xi)ds + \\ &\quad \sum_{0<t_k<t} J_k(x(t_k))(\eta, \xi) \leq \\ &\quad \|x_0\| + \big|h_{\eta\xi}\big|L^1 + \sum_{k=1}^{m}\big|J_k(x(t_k)\big| := 1 \end{aligned}$$

Step 3: N maps bounded sets into equicontinuous sets in PC(I, sesq(D⊗E)). Let $\tau_1, \tau_2\in I'$, $\tau_1<\tau_2$ and set $B_q(0) = \{x_{\eta\xi}\in PC(I, sesq(D\underline{\otimes}E)): \|x\|_{PC} \leq q\}$. Case 1; $t\neq t_i$, i = 1, ..., m:

$$\big|N(x)(\tau_2)(\eta, \xi)-N(x)(\tau_1)(\eta, \xi)\big| \leq \int_{\tau_2}^{\tau_1} h_{\eta\xi, q}ds + \sum_{0<t_k<\tau_2-\tau_1}\big|J_k(x(t_k))\big|$$

The right hand side of the above inequality tends to zero as $\tau_2\to\tau_1$. This proves equicontinuity for the first case. Case 2; $t = t_i$. The proof is similar to that given in Theorem 3 by Ogundiran (2013). Then by hypothesis (iv) by Ogundiran (2013) together with the Arzela-Ascoli theorem N(Bq) is equicontinuous. Lastly, we show that the set:

$$\begin{aligned} R(N) = \{&x\in PC(I, sesq(D\underline{\otimes}E)): \\ &\lambda x = \lambda Nx,\ 0<\lambda<1\} \subseteq \overline{B_q(0)} \end{aligned}$$

is bounded, q>0. Let x∈R(N), be defined as above. Then for each t∈I, we get:

$$\begin{aligned} x(t)(\eta, \xi) &= \lambda^{-1}[\langle \eta, x_0\xi\rangle + \\ &\quad \int_0^t P(s, x(s))(\eta, \xi)ds + \sum_{0<t_k<t} J_k(x(t_k^-))] \end{aligned}$$

$$\begin{aligned} \big|x(t)(\eta, \xi)\big| &\leq \big|x_0(\eta, \xi)\big| + \\ &\quad \int_0^t p(s)\psi\left|x(s)(\eta, \xi)\right|ds + \sum_{k=1}^{m} J_k(x(t_k^-)) \end{aligned}$$

Let υ(t) represent the left hand side of the above inequality so that:

$$\big|\upsilon(t)(\eta, \xi)\big| \leq \big|x_0(\eta, \xi)\big| + \int_0^t p(s)\psi\big|x(s)(\eta, \xi)\big|ds + \sum_{k=1}^{m} c_k$$

and at t = 0 we get:

$$\begin{aligned} \upsilon(0) &= \big|x_0\big| + \sum_{k=1}^{m} c_k \\ \upsilon'(t)(\eta, \xi) &= p(t)\psi\big|x(t)(\eta, \xi)\big| \end{aligned}$$

for a.e. t∈I and by the increasing property of ψ, we have:

$$\big|\upsilon'(\eta, \xi)\big| \leq \big|p(t)\psi(x(t)(\eta, \xi))\big|$$

For each t∈I, we get:

$$\int_0^t \frac{\upsilon'(s)(\eta, \xi)}{\psi(\upsilon(s)(\eta, \xi))}ds \leq \int_0^t p_{\eta\xi}(s)ds$$

We then have:

$$\int_{\upsilon(0)}^{\upsilon(t)} \frac{du}{\psi(u)} \leq \int_0^t p(s)ds \leq \int_0^b p(s)ds \leq \int_{\upsilon(0)}^{\infty} \frac{du}{\psi(u)}$$

and $|\upsilon(t)(\eta, \xi)| \leq d$ and hence $\|x\|_{PC} := \sup\{|x(t)(\eta, \xi)|: 0\} \leq d$ for a constant d depending only on p and ϕ and therefore, R(N) is bounded.

CONCLUSION

Having satisfied all assumptions of the Schauder-Tychonov's fixed point theorem, we conclude that N has a fixed point x which is a solution of the problem (Eq. 8) and by the above remark, x is a solution of the problem (Eq. 3).

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