# Theoretical Analysis of Vibrating Rotating Mill with Cross-Longitudinal Motion of Grinding Media

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**Abstract:** The study briefly describes relevance of a vibrating grinding of materials in a construction industry. The problem of increasing the effectiveness of the vibrating grinding of materials as a perspective grinding method is researched and analytically studied. A new design of the vibrating rotating mill which enables a grinding media to move in longitudinal direction in a grinding chamber and the principle of its operation are described. A number of mathematical expressions describing various characteristics of the grinding process and the mill chamber geometry are also presented and discussed. The conclusion that improving the efficiency of the grinding process is possible by an intensification of the grinding media motion in the grinding chamber giving it cross-longitudinal motion maintained by the inclined rotating grinding chamber has been made and substantiated.

**Key words:** Fine grinding, vibrational grinding, vibrating rotating mill, inclined grinding chamber, construction

## INTRODUCTION

Fine and ultrafine grinding of materials in a construction industry is one of the most energy-intensive processes, therefore, increasing efficiency and reducing energy consumption for grinding machines is an urgent issue. For example, up to 60% of electricity consumed is for raw materials grinding for a cement production. Energy-saving technologies have a significant economic effect in combination with large production volumes in Russia is about 40 million tons per year. Ball mills are the main machines for a raw materials grinding and clinker (in Russia and the CIS and the World) but low energy efficiency is their major shortcoming only 3-6% of the energy is consumed by a grinding material.

However, vibrating mills are of interest for fine and ultrafine grinding of various materials including a regrinding of cement. Many technological processes are based on the vibrating grinding such as production of binders without the use of portland cement, dry mixtures, lime binders, ceramics and glass components, pigments, plastics as well as various aggregates.

The vibrating grinding is used as a way of quartz sand concentration which is a common and frequently used aggregate for up to 80% of dry mixtures. Vibrating machines are used for grinding of a long list of raw materials: dry mixture components, silica sand, marble, mica, lime, gypsum; then, cement, lime binders, low water

consumption binders; moreover, ceramics components, glass batch and finally, talc, organic and inorganic pigments, etc. Vibrating mills have a range of advantages in comparison with classical ball mills such as a possibility of grinding a materials with different physical and mechanical properties to a micron and submicron particles sizes; higher grinding speeds and efficiency; possibility of modulating a grinding ratio; smaller machines and less metal consuming; higher levels of product grinding quality as homogeneity of particle size distribution, angular particles content, degree of mechanical activation and others (Mobley, 1999).

The disadvantages of these machines are relatively low productivity up to 1 t/h, high specific energy consumption 80-90 kWh $t^{-1}$, uneven distribution of particle sizes of a grinding product. It is mainly represented by the effect of simultaneous overgrinding and aggregation of material particles being grinded due to overrun of the energy supplied, formation of dead zones and time increase of a material presence in the grinding chamber. The purpose of the study is to analytically examine and improve an effectiveness of the vibrating grinding and a mill design. Improving efficiency is possible by an intensification of the grinding media motion in the grinding chamber giving them cross-longitudinal motion due to an inclined rotating grinding chamber (Bogdanov et al., 2012).

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## MATERIALS AND METHODS

There are two basic modes for fine grinding of materials in vibrating mills-periodic and continuous modes. During the periodic mode, a grinding material is loaded into a mill at once, then crushed and finally unloaded and afterwards loaded again. The periodic mode of vibrating grinding primarily applicable for an ultrafine grinding in relatively small quantities as well as for a scientific research purposes. The continuous mode of the grinding should be used for a heavy-duty industrial production which should be more productive by lacking off equipment downtime.

A way to improve an efficiency of the materials grinding process in vibrating mills is to match sizes of a grinding media to decreasing particle sizes of a product as it passes along the grinding chamber and the usage of different types of the grinding media motion. State-of-the-art way to increase performance of mills and reduce their energy consumption is to create a combined effect of exposure of all the basic mechanisms of particle destruction to the material being grinded involving a largest possible number of a grinding media in a grinding process (Mattan, 1971).

In this study, the vibrating grinding process is improved by intensifying the motion of the grinding media in the mill chamber. The principle of a combination of different structural features to complicate the way of the motion of grinding media and involve it in the grinding process better had used in the development of ball tube mills which is applicable to vibrating mills as well. Complex circular motion of a mixture of a grinding media and a material in a grinding chamber is designed to intensify and improve the vibrating grinding process.

## RESULTS AND DISCUSSION

In this study, the vibrating rotating mill with cross-longitudinal motion of the grinding media is studied and its laboratory model is depicted in Fig. 1.

In this unit, vibration of the grinding media which creates optimal conditions for fatigue fracture of particles complemented by cross-longitudinal motion of the grinding media. Furthermore, it helps to eliminate dead zones and to involve all balls in the grinding process in addition to a shock abrasive effect on the material. The mill is designed for grinding of materials with a particle size of from 50 microns to 2 mm along with
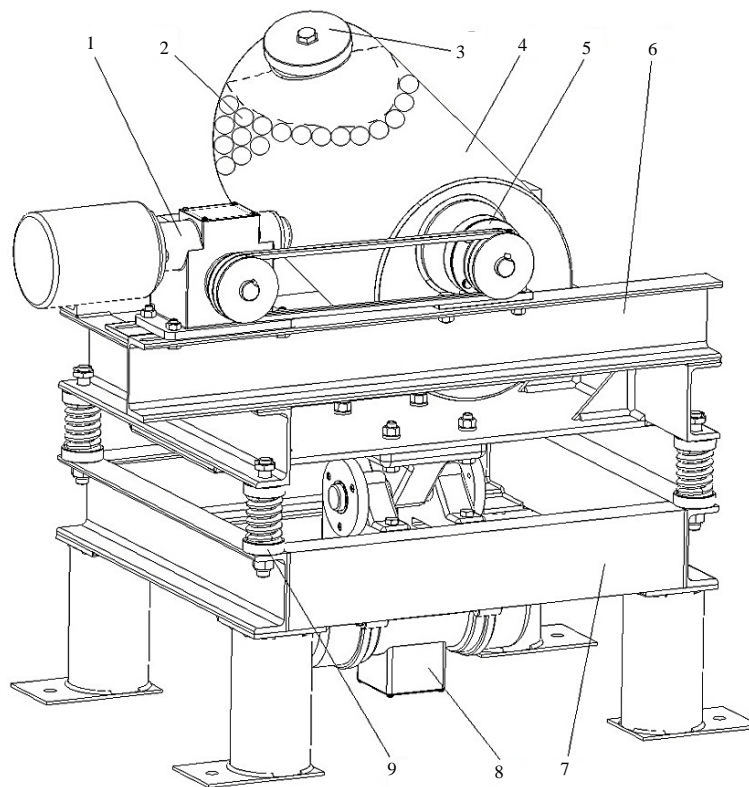


Fig. 1: Structural diagram of the rotating vibrating mill

homogenization, agitation and mechanical activation of several components. The mill has been made a batch-type machine for the research.

The design of the mill consists of a static frame 7, a movable frame 6 which is installed in bearing supports 5 rotatable around their own axis 4 with the grinding chamber 1 filled with grinding bodies 2 and a rotary drive of the grinding chamber 1. The grinding chamber 4 is formed in a shape of an inclined cylinder whose end surfaces are perpendicular to its axis of rotation while its lateral cylindrical surface is angled to a horizontal surface of the movable frame 6. The grinding chamber 4 is fitted into the bearing supports 5 which allows it to be easily rotated by the drive about its axis. In turn, the bearings 5 are part of the movable frame 6 with a vibrating drive 8 mounted to it. The movable frame 6 is rested on the static frame 7 through vibration isolators 9 (the springs). The angle of inclination of the cylinder surface to the horizontal surface of the movable frame 6 must be greater than an angle of repose of the mixture of a material and grinding media but <45° which was found empirically.

The chamber's geometric properties analysis and the definition of coordinates of the grinding media center of gravity have been made in view of existing theories of grinding media motion which consider the grinding media as a pseudo-liquid bulk mixture having viscoelastic properties (Vorob'ev *et al.*, 1990; Beenken *et al.*, 1996; Bogdanov *et al.*, 2008).

The geometric properties of the grinding chamber, representing an inclined cylinder bounded by two vertical planes have been described. The volume and the total surface area of the grinding chamber are defined by the expressions:

$$V_K = \frac{1}{4}\pi D^2 L = \frac{\pi ab H}{\cos\alpha} \qquad (1)$$

$$S_K = \pi D\left(L + \frac{D}{2\cos\alpha}\right) \qquad (2)$$

Where:
a and b = Big and small axes of end surface ellipses
D     = Cylinder diameter
H     = Chamber width
α     = Inclination angle of the chamber

From which it follows that at constant sizes of the chamber D and L its surface area and weight of the chamber increase with the increase of the α angle. The shape and position of a spatial volume occupied by the mixture of the grinding media and the material always change during rotation of the grinding chamber. When loading the chamber by the grinding bodies and the material its center of gravity is shifted from the axis of rotation. When rotating the chamber the mixture has a complex motion in a closed spatial trajectory (Fig. 2).



Fig. 2: Diagram of the grinding chamber

Position of he mixture is determined by the position of the symmetry plane of the chamber. Due to a vibration which causes the mixture to a pseudo-liquid state under the action of gravity forces the mixture to fill the portion of the grinding chamber from the bottom up to a certain level p relative to the axis of rotation. The position of the mixture is determined by the vertical and horizontal position of the symmetry plane of the chamber and the maximum level p of the free surface of the mixture which we assume being horizontal for simplicity and limited by the condition:

$$p_{max} = a\text{-}c + \frac{2cH_2}{H} \tag{3}$$

Where:
c    = Rotation axis shift
$H_2$ = Distance of the point on the rotation axis

Further, the method of determining the magnitude of the vertical and horizontal movements of the grinding mixture in the chamber is considered. To solve this problem the coordinates of the center of gravity of the mixture at the vertical and horizontal symmetry plane of the grinding chamber are defined. As the grinding mixture movement is relatively small in the direction perpendicular to the axis of rotation of the chamber, we take:

$$X_{cm} = 0 \tag{4}$$

The following expressions show the relations between the volume of the mixture of the grinding media and the degree of filling of the grinding chamber φ and its geometrical parameters and the level of the free surface of the mixture p. The volume of the mixture of the grinding media and the material:

$$V_3 = \varphi V_K \tag{5}$$

The filling degree of the grinding chamber, expressed through the geometric parameters:

$$\varphi = \frac{P_a + C_a - 1}{2C_a} + \frac{1}{4C_a}\left(2\text{-}P_a + C_a - \frac{2}{3\pi}\left(2 + \left(P_a - C_a\right)^2\right)\right)\times \\ \sqrt{1-\left(P_a-C_a\right)^2} - \frac{2}{\pi}\left(P_a-C_a\right)\arcsin\left(P_a-C_a\right) \tag{6}$$

Then, the coordinates of a center of mass of the mixture of the grinding media and the material:

$$Y_a = -\frac{C_a}{2\phi}\left(1-\left(\frac{P_a\text{-}1}{C_a}\right)^2\right) - \frac{1}{\pi\phi C_a}\left(\frac{9\pi}{32}-\frac{\pi}{2}P_a + \frac{\pi}{4}P_a(P_a\text{-}C_a)\times \\ \frac{\pi}{8}\left(P_a-C_a\right)^2 + \left(\frac{1}{3}P_a-\frac{13}{48}\left(P_a-C_a\right) + \frac{1}{6}P_a\left(P_a-C_a\right)^2 - \\ \frac{1}{24}\left(P_a-C_a\right)^3\right)\sqrt{1-\left(P_a-C_a\right)^2} - \left(\frac{1}{16}-\frac{1}{2}P_a\left(P_a-C_a\right) + \\ \frac{1}{4}\left(P_a-C_a\right)^2\right)\arcsin\left(P_a-C_a\right)\right) \tag{7}$$

$$Z_H = \frac{1}{84}\left(\frac{P_a + C_a - 1}{C_a}\right)^2 - \frac{1}{4\pi\phi C_a^2}\left(\frac{7\pi}{16}-\pi\left(P_a + C_a\right) - \\ \frac{\pi}{4}\left(P_a-C_a\right)^2 + \frac{\pi}{2}\left(P_a^2 + C_a^2\right) - \frac{P_a + 7C_a}{12}\left(1-\left(P_a-C_a\right)^2\right)^{\frac{3}{2}} + \\ \frac{5P_a + 11C_a}{8}\sqrt{1-\left(P_a-C_a\right)^2} + \\ \left(\frac{1}{8}+P_a^2-C_a^2-\frac{1}{2}\left(P_a-C_a\right)^2\right)\arcsin\left(P_a-C_a\right)\right) \tag{8}$$

where, $P_a$ p/a, $C_a$ = c/a, $Y_a$ = $y_{cm}$/a, $Z_H$ = $z_{cm}$/H the dimensionless geometrical parameters. Expressions have been calculated and presented in the dimensionless form as well as the $P_a$ range of a level variation of the free surface of the mixture p.

Expressions for the horizontal plane of symmetry are defined. Vertical coordinate of the center of mass of the mixture determined by the degree of filling of the mill chamber φ:

$$Y_R = -\frac{2}{3\pi\varphi}\left(1\text{-}P_R^2\right)^{\frac{3}{2}} \tag{9}$$

where, $Y_R = y_{cm}/R$, $P_R = p/R$ the dimensionless geometrical parameters. Movement of the center of mass of the mixture during rotation:

$$\Delta x = 0 \tag{10}$$

$$\Delta y = \left(\frac{Y_a}{\cos\alpha}\text{-}Y_R\right) \tag{11}$$

$$\Delta z = \frac{H}{2}\text{-}Z_H H \tag{12}$$

Expressions above are used to determine the efficient volume of the grinding media while designing the vibrating rotating mill which ensures the effectiveness of the grinding process (Fig. 3). The analysis performed shows that for all angles of rotation of the grinding chamber with increasing degree of filling of grinding media, the position of the center of mass of the mixture of
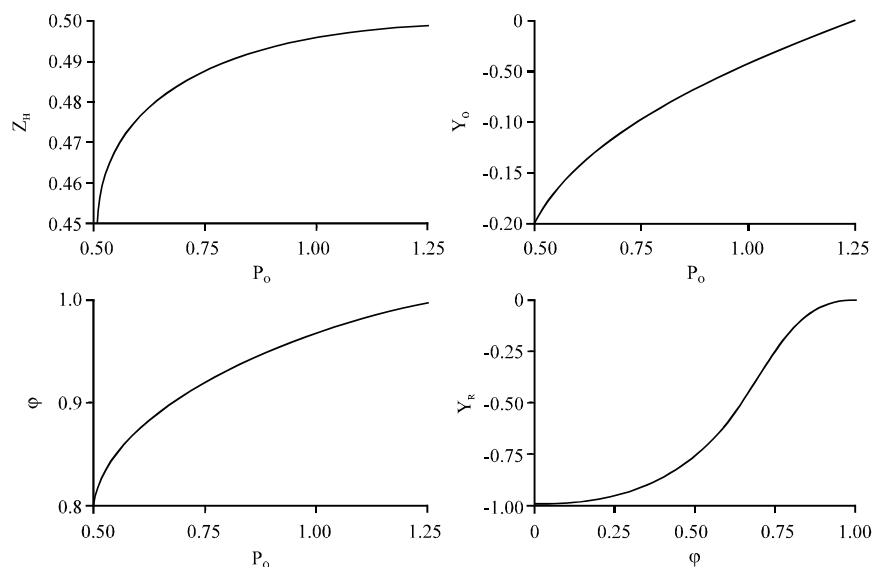
Fig. 3: Graphic plots of the filling factor $\varphi$ and the coordinates of the center of grinding media mass and $Y_a$ and $Z_n$ and the level of the free surface of the mixture $P_a$; the vertical coordinate of the center of mass $Y_R$ and the filling factor $\varphi$

material and grinding media closer to the geometric center of the grinding chamber the grinding mixture movement decreases. Consequently, lower coefficients of filling degree of the chamber are applicable.

To predict the properties of a product, it is required to know the total power expended for transporting the grinding mixture. The power spent on moving the grinding mixture in the vibrating rotating mill is a sum of the powers of two drives of the grinding chamber, taking into account the overall efficiency of the unit. To assess the total useful power the expressions for each of the power components are defined separately.

**The rotational component of the grinding process:** Additionally, the rotation of the grinding chamber activates the material grinding mechanism that exists in the ball drum mill with inclined septum. Movements of the center of mixture mass are given above. The maximum rotation speed and rotation frequency of the grinding chamber:

$$n_{max} = 0.2\frac{42.3}{\sqrt{D}} \tag{13}$$

$$\omega_v = 2\pi\frac{n}{60} \tag{14}$$

where, n rotation speed of the chamber.

**The vibrational component of the grinding process:** Vertical oscillation driving force is described by:

$$F_V = m_d r_d \omega_d^2 \cos\omega t \tag{15}$$

Where:

$m_d r_d$ = Static moment of vibrating drive

$\omega$ = Angular frequency of the grinding chamber rotation

The force between the grinding media and the grinding chamber:

$$F_{12} = -c_1\left(y_1 - y_2 - y_{cm1}\right) - b_1\left(\dot{y}_1 - \dot{y}_2\right) \tag{16}$$

The elastic force and the damping force of the springs:

$$F_2 = -c_1 y_2 - b_1 \dot{y}_2 \tag{17}$$

The interaction system of the grinding media and the movable frame of the mill:

$$\left(1 - K_{at}\right)m_1\ddot{y}_1 + b_1\left(\dot{y}_1 - \dot{y}_2\right) + c_1\left(y_1 - y_2 - y_{cm1}\right) = 0$$
$$\left(m_2 + K_{at}m_1\right)\ddot{y}_2 + b_2\dot{y}_2 + c_2 y_2 + b_1\left(\dot{y}_2 - \dot{y}_1\right) + \tag{18}$$
$$c_1\left(y_2 - y_1 + y_{cm1}\right) = M_d\omega^2\cos\omega t$$

The oscillation damping coefficients of the grinding media and the movable frame of the mill:

$$2\delta_1 = \frac{b_1}{\left(1 - K_{at}\right)m_1} \tag{19}$$

$$2\delta_2 = \frac{b_2}{m_2 + K_{at}m_1} \qquad (20)$$

Where:

$c_1\ c_2$ = Constant coefficient of stiffness of the grinding media and the movable frame

$b_1\ b_2$ = Constant coefficient of viscous resistance of the grinding media and the movable frame

$y_1, y_2,$ = Coordinates and velocities of the centers of

$\dot{y}_1, \dot{y}_2$ the grinding media and the movable frame

$m_1$ = Mass of the grinding media

$m_2$ = Mass of the movable frame

$K_{at}$ = Coefficient of the mass attachment to the mass of movable frame

$\delta_1, \delta_2$ = Oscillation damping coefficient of the grinding media and the movable frame

The proper angular oscillation frequency of the grinding media and the movable frame of the mill:

$$\omega_{01}^2 = \frac{c_1}{(1\text{-}K_{at})m_1} \qquad (21)$$

$$\omega_{02}^2 = \frac{c_2}{m_2 + K_{at}m_1} \qquad (22)$$

The oscillation amplitude of the grinding media with a phase lag of angle $\alpha_1$:

$$A_1 = A\sqrt{\frac{\omega^4\left(\omega_{01}^4 + 4\delta_1^2\omega^2\right)}{\Delta_1^2 + \Delta_2^2}} \qquad (23)$$

$$\alpha_1 = \text{arctg}\frac{\omega_{01}^2\Delta_2\text{-}2\delta_1\omega\Delta_1}{\omega_{02}^2\Delta_1\text{-}2\delta_1\omega\Delta_2} \qquad (24)$$

where, $\Delta_1$, $\Delta_2$ intermediate parameters with specific dimensions. The oscillation amplitude of the movable frame with a phase lag of angle $\alpha_2$:

$$A_2 = A\sqrt{\frac{\omega^4\left(\omega_{01}^2 + \omega^2\right)^2 + 4\delta_1^2\omega^2}{\Delta_1^2 + \Delta_2^2}} \qquad (25)$$

$$\alpha_1 = \text{arctg}\frac{\left(\omega_{01}^2\text{-}\omega^2\right)\Delta_2\text{-}2\delta_1\omega\Delta_1}{\left(\omega_{01}^2\text{-}\omega^2\right)\Delta_1 + 2\delta_1\omega\Delta_2} \qquad (26)$$

The expressions above are used to calculate the required stiffness and damping coefficients while designing the vibrating rotating mill out of the oscillation

amplitude measured at different degrees of filling of the grinding chamber. The power expended on the movement of grinding media as a result of rotation of the grinding chamber:

$$N_1^r = \frac{2m_3g\Delta y + 4m_3gf\Delta z}{T} \qquad (27)$$

The power expended on the movement of grinding media as a result of vibration of the grinding chamber:

$$N_1^v = \frac{1}{2}(1\text{-}K_{at})m_1\omega^3 A_1 A_2 \left|\sin\left(\alpha_1\text{-}\alpha_2\right)\right| \qquad (28)$$

Herein, the mechanical efficiency coefficient which depends on various parameters of the mill which are represented graphically below:

$$\eta_m = \frac{N_1^v}{N_{dr}} \qquad (29)$$

As a result, the energy from the vibrating drive is distributed between the moving frame of the mill and the grinding media. While the part of the energy imparted to the mixture of grinding media and material increases when oscillations of the movable frame mass decrease. This must be considered when designing the vibrating rotating mill (Fig. 4).

A grindability of materials is defined by their physical and mechanical properties, namely by their strength, hardness (abrasiveness) and fragility. Kinetics modelling of the vibrating grinding in this study based on the condition according to which the destruction of a grain of the grinding material occurs only when you create a deformation stress limit $\sigma_{max}$:

$$\sigma_{max} = \sigma_{max0}\sqrt{\frac{S}{S_0}}\left(\frac{1\text{-}\dfrac{S_0}{S_{max}}}{1\text{-}\dfrac{S}{S_{max}}}\right)^{\frac{n}{2}} \qquad (30)$$

Where:

$\sigma_{max0}$ = Initial stress limit

$S_0$ = Initial specific surface value

$S_{max}$ = Maximum specific surface value

The kinetic dependence of the specific surface area of the energy in the grinding process has been offered which allows performing a rational design of vibrating rotating mills and estimating of a dispersion of the grinding product:
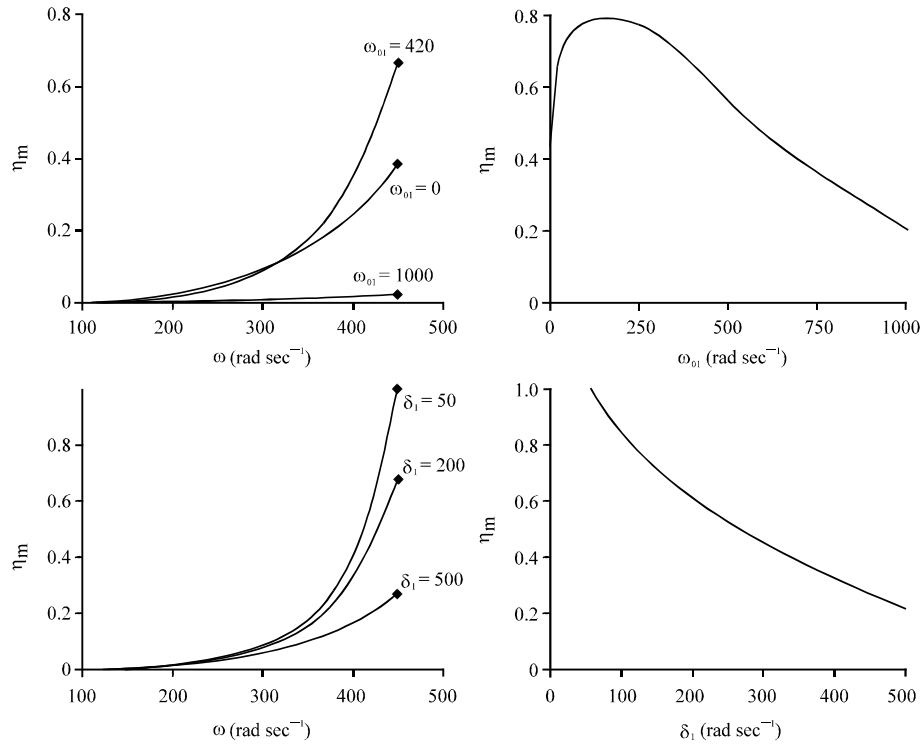
Fig. 4: Graphic plots of the mechanical efficiency coefficient and vibration frequency; proper oscillation frequency of the grinding media; vibration frequency with different damping coefficients; damping coefficient of the grinding media
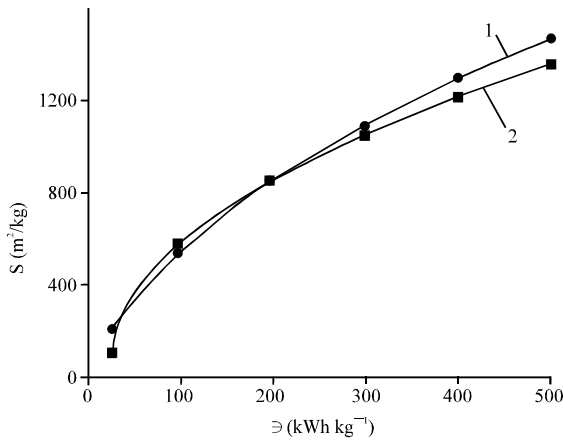


Fig. 5: Graphic plot of the specific surface and the energy consumption while grinding of quartz sand (1: experimental 2: calculated kinetic)

$$S = S_{max} - \frac{S_{max} - S_0}{\left(1 + \frac{K(n-1)S_0}{S_{max} - S_0} \, \Theta\right)^{\frac{1}{n-1}}} \qquad (31)$$

where, $\Theta$ energy required for a grain destruction. Finally, the dead zones are eliminated because of the cross-longitudinal movement of the grinding media occurring due to the inclined grinding chamber rotation. The energy supplied to the grinding media is more evenly distributed between it that leads to the intensification of not only a rotating grinding but also a vibrating impact grinding (Fig. 5).

**CONCLUSION**

Contemporary designs of vibrating mills have been examined in this study which served as prototypes for the development of a new design of the vibrating rotating mill; the relevance of the study of vibrating grinding process has been showed. Major directions of improving a vibrating grinding process are considered and one of them has been taken in this study.

The new design of the vibrating rotating mill is proposed and patented (patent for utility model RU105199). The innovative design of the grinding chamber enables the grinding media to move reciprocatingly in the longitudinal direction. During the grinding process the material is reduced efficiently by a complex mode of cross-longitudinal movement of the grinding media, comparing to a classic grinding chamber which only performs oscillating movements.

The mathematical expressions describing the geometric properties of the grinding chamber, the system of vibration mechanics for the grinding media and the grinding chamber, dependencies for determination of the amplitudes of the oscillations of a mixture of grinding media and the movable frame of the mill, the power spent on the movement of the grinding media in the grinding chamber, the mechanical efficiency and kinetic dependence which allows designing the machine rationally have been obtained in the study.

The new design of the vibrating rotating mill enhances the grinding process and provides a basis for further improvement of the technology of materials grinding.

## ACKNOWLEDGEMENT

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