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Impact of Meta-Heuristic Methods to Solve Multi-Depot Vehicle Routing Problems with Time Windows

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Abstract: Multi-Depot Vehicle Routing Problem with Time Windows (MDVRPTW) is a kind of NP-hard optimization problem which is described, as the problem of creating routes with optimum cost from one depot to a set of customer sites. Each customer has been visited just one time by only one vehicle within a proposed time interval, all routes start and finish at the same depot and the routes cannot violate the capacity constraints on the vehicles. This study addresses, the problem of multi depot vehicle routing in order to minimize the number of vehicles and the total travel cost. The proposed is a mixed integer programming model for the problem and provides a computable MDVRPTW in order to solve the problem; the genetic algorithm is the approach to this model.

Key words: Multi-depot vehicle routing problem, meta-heuristic, mixed integer programing, genetic algorithm, routing, simulated annealing

INTRODUCTION

The most important strategy in the field of supply chain management and logistics industry is to optimize the transportation costs of the product from suppliers to customers. These kinds of problems are called Vehicle Routing Problems (VRP) in which the vehicles leave the depot, serve customers which are assigned and return to the depot. Each customer has own demand (Yoshiike and Takefuji, 2002). In cases with >1 depot, VRPs are known as Multi-Depot VRPs (MDVRP).

Multi-Depot Vehicle Routing Problem with Time Windows (MDVRPTW) is a kind of decision making dilemmas and is used in many optimization problems, such as newspaper problem, school bus routing problem, distribution centers, bank deliveries and security patrol services. The MDVRPTW is the same problem, as the Vehicle Routing Problem (VRP) with additional constraints. In this study, researchers consider a fleet of identical vehicles with known capacities, multi-depots and a set of customers with time windows associated for each customer. Time windows are defined, as an interval wherein the customer with pre-defined demand should be supplied. The interval at the customer sites is called the scheduling horizon. Supplying customers happens with a fleet of identical vehicles by known capacities. The routes cannot violate the capacity constraints on the vehicles and must meet the time windows for each customer which specify the earliest and latest times for the start of service at a customer site. Each vehicle starts from the depot to transport the goods and eventually returns back to the intended depot. The objective of the Vehicle Routing Problem Time Windows (VRPTW) consists of minimizing the number of vehicles and the total travel cost.

The problem is classified as NP-hard optimization problems, so that even finding an optimal solution for small size is very difficult and time consuming. Meta-heuristic solution methods such as genetic algorithms, ant clooney and Simulated Annealing (SA) algorithms have been used to solve this problem. The meta-heuristic methods are to find optimal or near-optimal solutions in reasonable time.

RELATED STUDY

This study addresses literature review for vehicle routing problem. The 1st, researchers review vehicle routing problem. Consequently, this study proceeds to review the literature of vehicle routing problems with multi-depot and then time windows.

Heuristic algorithms such as Simulated Annealing (SA) (Chiang and Russell, 1996; Koulamas *et al*., 1994; Osman, 1993; Tavakkoli-Moghaddam *et al*., 2006),

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Genetic Algorithms (GAs) (Baker and Ayechew, 2003; Osman *et al*., 2005; Thangiah *et al*., 1994; Prins, 2004), Tabu Search (TS) (Gendreau *et al*., 1999; Semet and Taillard, 1993; Renaud *et al*., 1996; Brandao and Mercer, 1997; Osman, 1993) and ant colony optimization (Doerner *et al*., 2002; Reimann *et al*., 2002; Peng *et al*., 2005; Mazzeo and Loiseau, 2004; Bullnheimer *et al*., 1997) are widely used for solving the VRP.

The VRPTW has been the subject of many researches for heuristic and exact methods. Golden and Assad (1986) were earliest researchers in this field (Desrosiers *et al*., 1995; Cordeau *et al*., 2001). They mostly focus on exact techniques. The high complexity level of the VRPTW and its wide applicability to real-life situations leads to use meta-heuristics widely over the last few years to solve the Vehicle Routing Problem with Time Windows (VRPTW). Mehrjerdi (2012) has reviewed several articles and their techniques based on meta-heuristic.

Giosa *et al*. (2002) applied clustering-routing strategy for the Multi-Depot Vehicle Routing Problem with Time Windows (MDVRPTW). Wu *et al*. (2002) applied a Simulated Annealing (SA) approach for solving the multi-depot vehicle routing problem. Crevier *et al*. (2007) proposed an integer programming to model the problem and implied heuristic tabu search algorithm to solve the problem. Ho *et al*. (2008) proposed 2 hybrid genetic algorithms to solve the MDVRP problem. Chen and Xu (2008) applied a combination of a hybrid genetic algorithm with simulated annealing for solving the MDVRP. In this study, a mixed integer programing is proposed to formulate the problem and applied a genetic approach to solve a multi-depot vehicle routing problem with time windows.

PROBLEM FORMULATION AND DEFINITIONS

Vehicle routing problem with time windows, consisting of a fleet of vehicles with multi-depot supplies a certain number of customers with different demands and time constraints. It contains network connecting all the depots to all of the customers. In order to solve this problem, following assumptions are made:

- Supplying customers after the upper bound of its time window makes the solution infeasible
- If a vehicle arrives before the lower bound of a customer's time window makes additional waiting time on the route
- Each route must start and end within the time window associated with the depot

The routing approach of each vehicle is as follows: The vehicle starts moving from deports, meets the customers and eventually returns to the intended depot. Every customer has demanded d_j and can only be met once and by only 1 vehicle.

Vehicle k has capacity Q_k, must be greater than or equal to the total demand of all customers that will be met by the vehicle. Overload is not allowed. Time window means that every customer has a predefined time interval in order to meet customer that includes earliest arrival time $e_i$and latest arrival time l_i for vehicles. Vehicles must enter the customer site before latest arrival time l_i and if arriving before the earliest arrival time e_i they must then wait there. Then researchers defined waiting time W_i. Each customer has service time f_i, includes loading-unloading products. The distance between every customer and every vehicle is calculated by Euclidean distance on the straight-line. Vehicle speed is one unit of distance per unit of time. This is supposed to make problems easier. Mathematical formulation of the problem is stated as follows: In this issue, researchers have a distribution company with several depots. The number and depot locations are defined. Each depot is large enough that can contain all products ordered by customers. Each customer is met by a vehicle exactly one time. In this problem, three decisions will be decided:

- Grouping: Allocation of customers to the depot
- Routing: Assigning customers in each depot to routes
- Scheduling: Sequencing each route in every depot

The 1st need is to decide on clustering customers in order to serve them. In general, the purpose of MDVRP is minimizing distance or time. It can be also reduced the number of vehicles.

Parameters:

Nu = Number of customers
C_{ij} = Distance between point i and j
V_i = Capacity of depot i
d_j = Demand of customer j
Q_k = Capacity of vehicle k
t_i = Arrival time to customer i
t_{ij} = Travel time between customer i and j
f_i = Travel time between customer i and j
w_i = Waiting time at the customer i location
l_i = Latest arrival time to customer i
e_i = Earliest arrival time to customer i

Sets:

Names	Sets	Objects
I	Set of depots	I = 1, 2,..., i
J	Set of customers	J = 1, 2,..., j
K	Set of vehicles	K = 1, 2,..., k

Decision variable:

$X_{ijk}=\{^1_0\}$ If the point i and j on route k is connected the value = 1, otherwise 0

$\mu_j=\{^1_0\}$ If point j is assigned to the depot i the value of this variable = 1, otherwise 0

U_{lk} Auxiliary variables in constraints to prevent generating sub-tour which takes only positive values (sub-tour elimination)

MATHEMATICAL FORMULATION

$$\text{Min} \sum_i \sum_j \sum_k C_{ijk} X_{ijk} \tag{1}$$

The objective function minimizes the total delivery distance:

$$\sum_k \sum_i X_{ijk} = 1, \quad j \in J \tag{2}$$

Constraint (Eq. 2) ensures that each customer is assigned to a single route:

$$\sum_j \sum_i d_j X_{ijk} \leq Q_k, \quad k \in K \tag{3}$$

Constraint (Eq. 3) is the vehicle capacity constraints:

$$U_{lk} - U_{jk} + NuX_{ijk} \leq Nu - 1, \quad i, j \in J, k \in K \tag{4}$$

Constraint (Eq. 4) ensures new sub-tour elimination:

$$\sum_j d_i \mu_{ij} \leq V_i, \quad i \in I \tag{5}$$

Constraint (Eq. 5) required capacity constraints for the depots:

$$\sum_{u \in I \cup J} \left(X_{iuk} + X_{ujk}\right) - \mu_{ij} \leq 1, \quad i \in I, \ j \in J, \ k \in K \tag{6}$$

This constraint ensures that the customer is assigned to a depot only if that depot is on the route passes the customer:

$$\sum_k \sum_j X_{ijk} \left(t_i + t_{ij} + f_i + W_i\right) = t_j, \quad j \in J, k \in K \tag{7}$$

$$\sum_i \sum_j X_{ijk} \left(t_{ij} + f_i + W_i\right) \leq l_O, \quad j \in J, k \in K \tag{8}$$

$$e_i \leq t_i + W_i \leq l_i, \quad i \in I \tag{9}$$

Constraints (Eq. 7-9) refer to time windows constraints:

$$X_{ijk} \in \{0,1\}, \quad i \in I, j \in J, k \in K \tag{10}$$

$$Z_{ij} \in \{0,1\}, \quad i \in I, j \in J \tag{11}$$

$$U_{lk} \geq 0, \quad l \in j, \ k \in K \tag{12}$$

Finally, constraints (Eq. 10-12) specifying binary and non-negative constraint variables.

NUMERICAL CALCULATION

To demonstrate the effectiveness of the mathematical model in MDVRPs a numerical example is provided and reviewed. In the proposed GA Algorithm, researchers consider the maximum number of generations to 1000, the population size of 100 and a 5% probability of mutation. The selection operator used is tournament selector. Tournament selection is an essential selection operator for GAs. It is to code and implement on nonparallel or parallel structures, strong against noisy data and the selection rat is tunable. The selection pressure of tournament selection directly changes with the tournament size. More adversaries lead to higher resulting selection pressure.

In this example, the numbers of customers are 52, the numbers of stores are 4 and the numbers of vehicles are 8. The number of vehicles is 28 and maximum capacity per vehicle is considered 175. In order to computations simplicity, researchers used an optimizer software Heuristic Lab version 3.3.9.

The data are sampled pr06-tw which summarized in a 252×252 matrix and described the distance between every customer's locations. The number of iterations are supposed 1000 and in every iteration the number of population size is 100. In each iteration the best distance for all populations are saved as best distance and in the next step if the best improved will be replaced by new one otherwise the previous one will be kept. Figure 1 and 2 shows the best distances between 252 customers.

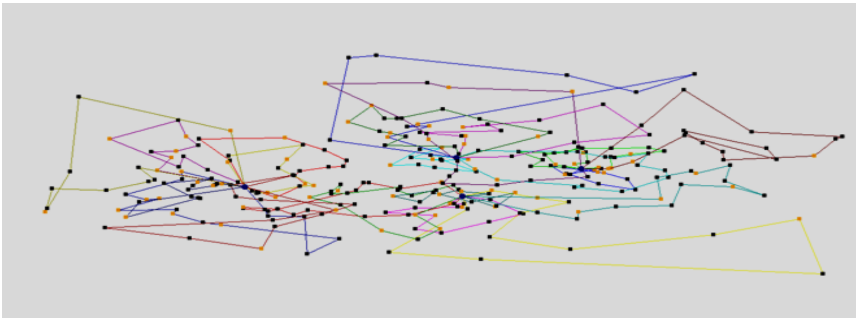

Fig. 1: The distances between 252 customers

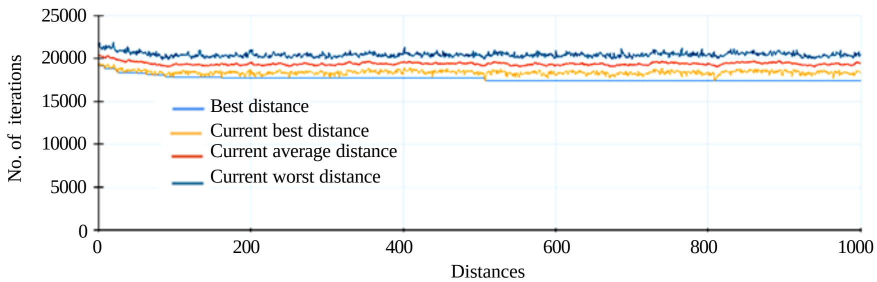

Fig. 2: Best output for distances

CONCLUSION

The implementation of the SA is easier than GA, as there are some disadvantages in comparison to GA. The simple structure of SA makes it difficult to search the big solution space. GA method characteristic as mutation and crossover makes it simple to search the huge solution space. Mutation allows it to jump to another part of the solution space, regardless of the recent position. Mutation occurs during evolution according to a pre-defined mutation probability. Researchers should set this probability to low. Otherwise the search will turn into a primitive random search. The main purpose of mutation in GAs is achieving vast diversity in the problem space. Mutation should allow the algorithm to avoid local minima. It prevents similarity in the population of chromosomes. And it also leads to preventing GA systems from taking just the fittest chromosomes in the population for generating the next random selection with weighting toward those that are in best fit. The crossover operator tends to widespread exploration in the solution space. As the high fitness solutions develop, the crossover operator provides exploration in the neighborhood of them. It allows us to create new results from previously calculated fit test results. While GA operates with these 2 effective operators and huge solution spaces, SA only replaces some of its elements of the defined neighborhood function and works with smaller problem spaces; therefore the result of the GA is likely to be much superior to SA.

Computational results illustrate that the model is efficient in solving MDVRP-TW. This approach can be extended for further research on the varied number of vehicles and depots and it will be studied how it can affect the optimum distances.

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