

## Comparative Study of the LTM and FLMT Dimensional Equations for Sludge Filtration

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**Abstract:** This study presents, a comparative analysis of the LMT and FLMT dimensional equations for sludge filtration. The main objective of this research, study is to develop a comparative analysis of the LTM and FLMT dimension equations for sludge filtration. In this research study, 2 modified equations for sludge filtration processes were developed for use in routine laboratory investigation. This study was completed using laboratory experimentation and mathematical analysis. The 2 general expressions were obtained for the LMT and the FLMT fundamental dimension. Multiple linear regression was used to obtain the required modified sludge filtration equation for LMT and FLMT. The modified filtration equation obtained for LMT show that the volume of filtrate is directly proportional to area of the funnel, applied vacuum, pressure and time of filtration while inversely proportional to specific resistance of sludge and compressibility coefficient. It was discovered that the LMT equation has a high degree of correlation coefficient between measured volume and calculated volumes, this ranges between 0.957-1. The applicability of the 2 modified equations was compared with the already existing equations predicted by Carman and Anazodo's. The comparative was achieved by evaluating specific resistances of all the existing equations formulated, including the modified LMT and FLMT equation at different pressure points. As a result of this, the specific resistance on pressure, volume, solid contents were verified.

**Key words:** Filtration, sludge, LMT, FLMT, dimensionless, exponent and waste

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### INTRODUCTION

The problem of wastes has increased drastically due to rapid rate of industrialization and increase in population all over the world. The improper management and treatment of these wastes contribute to air, water and soil pollution. They also create breeding places for disease-carrying insects and rodents. These situations result in public nuisance and adversely affect land values and generally interfere with community lives.

In view of tracking these problems, the science research council in 1980 sponsored Dr. Wakenman, R.J of the University of Exeter with the grant of £34.361 for research programme (Afangidehi, 2003). In the process of waste water treatment through proper waste management, there is a huge percentage of sludge generate. The sludges generated are difficult to handle and dispose because of the high percentage of water content (about 97-98%) (Ademiluyi, 1984). Hence, the sludge must be dewatered before disposal.

Sludge is a by-product of the water cleanup process (Andersen, 2001) categorized 3 main classes of sludge:

- Sludge originating from the treatment of urban waste water, consisting of domestic waste water in the mixture of domestic waste with industrial waste water and/or run-off rain water
- Sludge originating from treatment of industrial waste water that is water used in industrial processes
- Sludge from drinking water treatment: Water has to be treated before its consumption. The amount of sludge generated from drinking water treatment is significantly lower than that generated from waste water treatment

The sludge generated by waste treatment must undergo dewatering before disposal to the environment. This can be achieved by filtration. Filtration is the process of separating a heterogeneous mixture of a fluid and particles of a solid by means of a filter medium the particles during the process.

**Concept of specific resistance and sludge filtration:**  
Carman (1934) presents a drive unit rate of flow of unit

viscosity through unit cube of the cake. The study, also added that a filtration equation that assumed specific resistance is constant throughout the sludge cake thickness and that the cake is rigid. This condition is applied if the flow obeys Dawcay Law:

$$\frac{dV}{dt} = \frac{PA}{r\mu L} \quad (1)$$

Where:

$r = 1/K_1$

$r$  = Specific resistance

$K_1$  = Permeability

Ruth (1935) challenged Carman's equation that the specific resistance parameter should be designated as average value. Shirato and Aragaki (1972) made a useful suggestion on the variability of specific resistance along the cake thickness. Bierck *et al.* (1988) found out that the specific resistance of suspensions contains irregular shaped particles which increased with the initial concentration of the suspension while the specific resistance of suspension of spherical particles was constant and independent of concentration. Heertjes (1978) suggested that the variation of specific resistance with solid concentration is due to filter binding. Turoyskil (1974) commented, vividly in the chemical nature of sludge. The study suggested that iron, chromium, copper, acids and alkaline improve filtration while fats, oil and petroleum product resist filtration. Coackley (1958) experimented that freezing alone can result to a small decrease in specific resistance of the order of 1000 lesser than original sludge. Christensen and Donald researched on effect of aging in sludge filtration. O'Shaughnessy presents the effect of phosphate precipitates present in some sledges dewatering. Randall and Kosh researched on the dewatering time of a sludge containing hygroscopic fibrous material could be prolonged considerably by rainfall effects.

Dimensional analysis is a conceptual tool often applied in physics, chemistry and engineering to understand physical situation involving a mix of different kinds of physical quantities. Dimensional formulas provide a useful catalogue system for physical quantities (Rajput, 2004). The principle of dimensional homogeneity states that in a physical equation consisting of an algebraic sum of 2 or more terms the exponent of the dimension of Length. Mass or time in any term of the equation must be the same as that in any other term. In effect, this principle states that quantities of different kinds cannot be added together (Anazodo, 1974). The system of fundamental units commonly used in Newtonian mechanics is the LMT system. The

mathematical methods used for dimension is LMT and FMLT which are base on the fourier's principle of homogeneity. In this study, Buckingham's  $\pi$  Method was adopted in developing the new equation for FMLT and LMT. It is necessary to assign any arbitrary value to the exponent of the variables of interest and at the same time to express this variable, as a product of the other variables with their exponents undetermined. Thus:

$$V = P^a A^b \mu^c C^d R^e S^f T^g \quad (2)$$

Where, a-g are numerical exponents.

## MATERIALS AND METHODS

The summary of dimensional formulae for developing the new filtration equation for LMT and FLMT is show in Table 1.

**LMT Method (Buckingham's  $\pi$  Method theories):** The following equation after was used to determine the LMT for sludge filtration:

$$V = f(P, A, \mu, C, R, t, S) \quad (3)$$

$$\text{Or } f(P, A, \mu, C, V, R, t, S) = 0 \quad (4)$$

The total of variables  $n = 8$ . Thus, number of fundamental dimensions  $m = 3$ , No. of  $\pi$ -terms =  $n-m = 8-3 = 5$ . Hence, the number of  $\pi$ -terms in the equation can be written as:

$$f(\pi_1 \pi_2 \pi_3 \pi_4 \pi_5) = 0 \quad (5)$$

$$\pi_1 = P^a A^b \mu^c V \quad (6)$$

$$\pi_2 = P^a A^b \mu^c C \quad (7)$$

Table 1: Summary of dimensional formulae for LMT and FLMT

Physical variables	Symbols	Dimensions	
		LMT	FMT $L_x L_y L_z$
Volume of filtrate	V	$L^3$	$L_x L_y L_z$
Filtration area	A	$L^2$	$L_x L_y$
Time for filtration	T	T	T
Mass of cake dry solid per unit volume of filtrate	C	$ML^{-3}$	$ML_x^{-3} ML_y^{-3} ML_z^{-3}$
Net filtration pressure	P	$ML^{-1}T^{-2}$	$FL_x^{-1} L_y^{-1} L_z^{-1} T^{-2}$
Viscosity of filtrate	$\mu$	$ML^{-1}T^{-1}$	$FL_x^{-1} L_y^{-1} L_z^{-1} T^{-1}$
Average specific resistance of filter cake	R	$LM^{-1}$	$L_z M^{-1}$
Compressibility coefficient	S	$M^{-1}LT^{-2}$	$M^{-1}L_z T^{-2}$

$$\pi_3 = P^a A^b \mu^c R \quad (8)$$

$$\pi_4 = P^a A^b \mu^c t \quad (9)$$

$$\pi_5 = P^a A^b \mu^c S \quad (10)$$

Where,  $\pi$ 's are dimensionless terms. Now consider  $\pi_1$ -term, Eq. 6 can be analysed as follows. The dimensions in Eq. 6 are thus:

$$M^a L^b T^c = (ML^{-1}T^{-2})^a (L^2)^b (ML^{-1}T^{-1})^c (L^3) \quad (11)$$

Equating the exponents of M, L and T, respectively researchers get:

$$\text{For, } M: 0 = a + c \quad (12)$$

$$L: 0 = -a + 2b - c + 3 \quad (13)$$

$$T: 0 = -2a - c \quad (14)$$

In Eq. 12:

$$a = -c \quad (15)$$

Substituting Eq. 15 in Eq. 14:

$$0 = 2c - c, \quad c = 0$$

Similarly, substituting Eq. 12 and 14 in Eq. 13:

$$0 = c + 2b + 3 \quad \text{but } c = 0$$

That implies:

$$2b = -3, \quad b = -\frac{3}{2}$$

$$\therefore \pi_1 = P^a A^{-3/2} \mu^c V, \quad \pi_1 = \frac{V}{A^{3/2}}$$

Solving for  $\pi_2$ - $\pi_5$ , researchers get:

$$\pi_2 = \frac{PAC}{\mu^2}, \quad \pi_3 = \frac{\mu^2 R}{P}, \quad \pi_4 = \frac{Pt}{\mu} \quad \text{and } \pi_5 = PS$$

Substituting the value of  $\pi_1$ - $\pi_5$  into Eq. 5 this is:

$$f\left(\frac{V}{A^{3/2}}, \frac{PAC}{\mu^2}, \frac{\mu^2 R}{P}, \frac{Pt}{\mu}, PS\right) \quad (16)$$

Following Buckingham's  $\pi$  Method, any of the dimensionless terms of Eq. 16 can be written as function of the others as in Eq. 21. Also:

$$\frac{V}{A^{3/2}} = K \left(\frac{PAC}{\mu^2}\right)^a \left(\frac{\mu^2 R}{P}\right)^b \left(\frac{Pt}{\mu}\right)^c (PS)^d \quad (17)$$

The exponents in Eq. 17 can be obtained by regression analysis using experimental data:

$$\ln K + a \ln \frac{PAC}{\mu^2} + b \ln \frac{\mu^2 R}{P} + c \ln \frac{Pt}{\mu} + d \ln PS \quad (18)$$

Equation 18 can be written as:

$$Y = \ln K + ax_1 + bx_2 + cx_3 + dx_4 \quad (19)$$

Where:

$$Y = \ln V/A^{3/2}$$

$$x_1 = \ln PAC/\mu^2$$

$$x_2 = \ln \mu^2 R/P$$

$$x_3 = \ln Pt/\mu$$

$$x_4 = \ln PS$$

Using results obtained from filtration experiment (data too large to reproduce) the values of a-d were obtained by regression (Table 2 and 3). From Table 3, researchers have:

$$\ln K = 8.022; a = -0.48; b = 0.689;$$

$$c = 0.481; d = 0.000961$$

Hence;

$$K = e^{8.022} = 3047.266$$

$$\frac{V}{A^{3/2}} = 3047.266 \left(\frac{PAC}{\mu^2}\right)^{-0.48} \left(\frac{\mu^2 R}{P}\right)^{0.68} \left(\frac{Pt}{\mu}\right)^{0.481} (PS)^{-0.000961}$$

$$V = 3047.266 \frac{A^{1.02} P^{0.689} t^{0.48}}{\mu^{0.889} R^{0.689} \mu^{0.481} S^{0.000961}} \quad (20)$$

**Table 2: Model coefficient for LMT**

Un-standardized				
Constant	-----			
(LnK)	$\beta$ (8.022)	SE (0.415)	Standardized $\beta$	Sig. (0.000)
a	-0.480	0.009	-1.136	0.000
b	-0.689	0.026	-0.590	0.000
c	-0.481	0.021	-0.607	0.000
d	-9.610 E-04	0.017	-0.001	0.956

**Table 3: Model summary of LMT**

Model	R	R <sup>2</sup>	Adjusted R	SE of the estimate
1	0.916	0.839	0.838	0.1692

Squaring both side of Eq. 20:

$$V^2 = 9.29 \times 10^6 \left[ \frac{P^{1.38} A^{2.04} t^{0.96}}{R^{1.38} C^{0.96} \mu^{1.8} S^{0.002}} \right] \quad (21)$$

Equation 21 is new filtration in LMT Method.

**FLMT-Method:**

$$f(V, P, A, \mu, C, R, t, S) = 0 \quad (22)$$

Using Buckingham's  $\pi$  Method number of  $\pi$ -terms =  $n-m = 8-5 = 3$ . Hence, the No. of  $\pi$ -terms in the equation can be written as:

$$f_1(\pi_1, \pi_2, \pi_3) = 0 \quad (23)$$

$$\pi_1 = P^a \mu^b C^c A^d S^e V \quad (24)$$

$$\pi_2 = P^a \mu^b C^c A^d S^e R \quad (25)$$

$$\pi_3 = P^a \mu^b C^c A^d S^e t \quad (26)$$

The dimensions in Eq. 22 can be analyzed as follows:

$$F^a M^b T^c L_x^d L_y^e L_z^f = (FL_x^{-1} L_y^{-1})^a (FL_z^2 T)^b (ML_x^{-1} L_y^{-1} L_z^{-1})^c (L_x L_y)^d (M^1 L_z T^2)^e (L_x L_y L_z)^f$$

Equating the exponents of F, M, L<sub>x</sub>, L<sub>y</sub>, L<sub>z</sub> and T, respectively researchers get:

$$\text{For, } F^0: 0 = a + b = a = -b \quad (27)$$

$$L_x: 0 = -a - c + d + 1 \quad (28)$$

$$L_y: 0 = -a - c + d + 1 \quad (29)$$

$$L_z: 0 = 2b - c + e + 1 \quad (30)$$

$$M^0: 0 = c - e = c = e \quad (31)$$

$$T^0: 0 = b + 2e \quad (32)$$

Solving Eq. 27-29 simultaneously, researchers get:

$$a = -\frac{1}{2}; b = \frac{1}{2}; c = -\frac{1}{4}; d = -\frac{7}{4}; e = -\frac{1}{4}$$

Substituting the value of a-e into Eq. 37 gives:

$$\pi_1 = P^{-1/2} \mu^{1/2} C^{-1/4} A^{-7/4} S^{-1/4} V$$

$$\pi_1 = \frac{\mu^{1/2} V}{P^{-1/2} C^{-1/4} A^{-7/4} S^{-1/4}}$$

Solving  $\pi_2$  and  $\pi_3$ , researchers get:

$$\pi_2 = CAR$$

And:

$$\pi_3 = \frac{t}{(CAS)^{1/2}}$$

Finally:

$$(\pi_1, \pi_2, \pi_3) = 0$$

$$\left[ \frac{\mu^{1/2} V}{P^{-1/2} C^{-1/4} A^{-7/4} S^{-1/4}}, CAR, \frac{t}{(CAS)^{1/2}} \right] = 0 \quad (33)$$

$$\frac{V \mu^{1/2}}{(CS)^{1/4} P^{1/2} A^{7/4}} = K (CAR)^a \left( \frac{t}{(CAS)^{1/2}} \right)^b \quad (34)$$

$$\ln \frac{V \mu^{1/2}}{(CS)^{1/4} P^{1/2} A^{7/4}} = \ln K + a \ln CAR + b \ln \frac{t}{(CAS)^{1/2}} \quad (35)$$

But  $y = ax + bx$ , also  $x = \ln RCA$ :

$$Z = \ln \frac{t}{(CAS)^{1/2}}$$

Using results obtained from filtration experiment (data too large to reproduce) the values of a and b were obtained by regression (Table 4 and 5).

Table 4: Model coefficient for FLMT

Parameters	Unstandardized coefficients		Standardized coefficients $\beta$	t	Sig.	95% confidence interval for B		Correlations	
	$\beta$	SE				Lower bound	Upper bound	Zero-order	Partial
Constant	12.987	0.380	11.500	34.1	0.00	12.240	13.733	-	-
a	-0.467	0.013	-0.641	-36.2	0.00	-0.492	-0.442	-0.917	-0.819
b	0.472	0.022	0.377	21.3	0.00	0.428	0.515	0.847	0.644

Table 5: Model summary of FLMT

Model	R	R <sup>2</sup>	Adjusted R <sup>2</sup>	SE of the estimate	Change statistics				
					R <sup>2</sup> change	F change	df1	df2	Sig. F change
1	0.953	0.907	0.907	0.18245 99473 4451	0.907	3142.0 65.0	2	642	0.000

From Table 4, K = 11.513, a = -0.641 and b = 0.377. Substituting a and b in Eq. 35 researchers get:

$$\ln \frac{V\mu^{1/2}}{(CS)^{1/4} P^{1/2} A^{7/4}} = K + \ln(CAR)^{-0.641} + \left( \frac{t}{(CAS)^{1/2}} \right)^{0.377}$$

$$\ln \frac{V\mu^{1/2}}{(CS)^{1/4} P^{1/2} A^{7/4}} = \ln e^K + \ln(CAR)^{-0.641} + \left( \frac{t}{(CAS)^{1/2}} \right)^{0.377}$$

But K = 11.513, therefore exp (K) = 10<sup>5</sup>:

$$V = 10^5 \times \left( \frac{(CS)^{1/4} P^{1/2} A^{7/4}}{\mu^{1/2}} \right) (CAR)^{-0.641} + \left( \frac{t}{(CAS)^{1/2}} \right)^{0.377}$$

$$V = \frac{P^{1/2} A^{0.9205} S^{0.0615} t^{0.377}}{R^{0.641} C^{0.58} \mu^{1/2}} \times 10^5$$

Squaring both side of the equation researchers get:

$$V^2 = \frac{10^{10} P A^{184.0} S^{0.123} t^{0.754}}{R^{1.28} C^{1.16} \mu} \quad (36)$$

Equation 36 is the new filtration equation in FLMT Method. Equation 36 can be expressed as:

$$\frac{t}{v} = QV$$

Where:

$$Q = \frac{10^{10} P A^{184.0} S^{0.123} t^{0.754}}{R^{1.28} C^{1.16} \mu} = 14.57 \text{ being constant}$$

## RESULTS AND DISCUSSION

**Evaluation of the specific resistance R and compressibility coefficient S:** There are 8 parameters involved in the formulation of these new equations. The parameters include, V, P, A, T,  $\mu$ , R, S. The 1st 6 parameters were experimentally measured while the last 2 parameters were evaluated. The evaluation of specific

Table 6: Expected treatment of raw sludge t/V versus V

Time (t, min)	Time (t, sec)	Volume (mL)	Corrected volume (v)	t/V (sec mL <sup>-1</sup> )
-2	-	0.0	-	-
0	0	56.0	0	0.0
1	60	62.0	6	10.0
2	120	67.0	11	10.9
3	180	71.0	15	12.0
4	200	75.0	19	12.5
5	300	79.0	23	13.0
6	360	82.0	26	13.8
7	420	85.0	29	14.5
8	480	88.0	32	15.0
9	540	91.0	35	15.4
10	600	93.0	37	16.2
11	660	96.0	40	16.5
12	720	99.0	43	16.7
13	780	101.0	45	17.3

resistance was done by first experimenting the sludge sample obtained from University of Nigeria treatment plant in the laboratory. The result of the laboratory experiment and the raw data obtained prior to the evaluation of specific resistance. Table 6 shows the expected treatment of raw sludge t/V versus V.

Pressure of filtration = 5251.239 cm<sup>-2</sup>

Mean temperature = 26.3°C

Viscosity  $\mu$  = 0.00870 poises

Slope  $\times$  = Only applicable to LMT Method

Solid content (C) = n 0.08533/g/cm<sup>3</sup>

Filter Area (A) = 38.48 cm<sup>2</sup>

**LMT Method:** For the purpose of this parameter, raw data were 1st analyzed. Table 5 and 6 show the example of the mode of analyses given to all data obtained from the filtration experiments. The slope of t/V against V could not be obtained by ordinary regression analysis as t/V against V are independent. Graphical method was found useful because of this nature where slopes are required. The graph of t/V against V is plotted in Fig. 1. Table 6 was used to plot a graph of t/V versus V.

**FMLT Method:** For the purpose of obtaining the specific resistance equation for FLMT, researchers need to plot a graph of 2ln V versus ln t. The reason for this will be in clear in the mathematical operations show after. Recall the new Eq. 36 developed by FMLT Method is thus (Table 7):

Table 7: Table of 2ln V versus ln t

Time (t, min)	Time (t, sec)	V (mL)	Corrected V	2*ln V	ln t
-2	-	0	-	-	-
0	0	56	0	-	-
1	60	62	6	3.583	3.0871
2	120	67	11	4.796	3.6098
3	180	71	15	5.416	3.9155
4	240	75	19	5.889	4.1324
5	300	79	23	6.271	4.3007
6	360	82	26	6.516	4.4381
7	420	85	29	6.735	4.5544
8	480	88	32	6.931	4.6550
9	540	91	35	7.111	4.7438
10	600	93	37	7.222	4.8233
11	660	96	40	7.378	4.8952
12	720	99	43	7.522	4.9608
13	780	101	45	7.613	5.0211

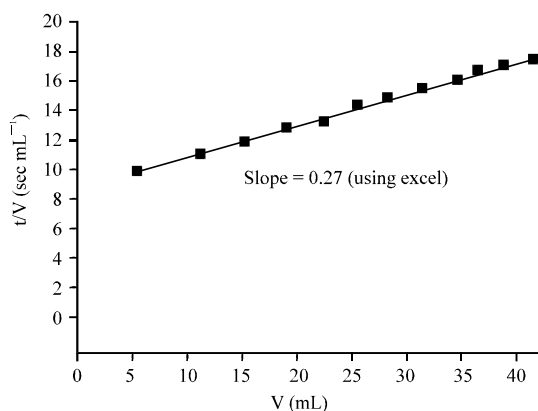


Fig. 1: Graph of t/V versus V

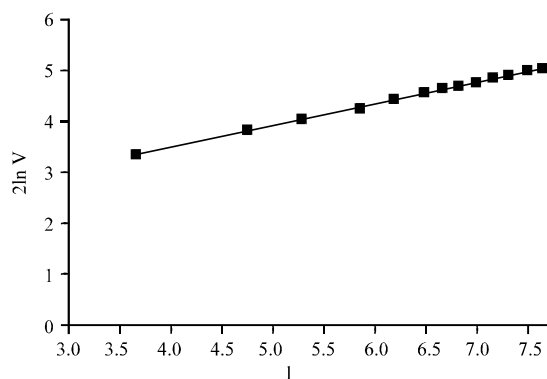


Fig. 2: Graph of 2ln V versus ln t

$$V^2 = \frac{10^{10} \text{PA}^{184.0} \text{S}^{0.123} t^{0.754}}{R^{1.28} C^{1.16} \mu}, V^2 = t^{0.754} \left[ \frac{10^{10} \text{PA}^{184.0} \text{S}^{0.123}}{R^{1.28} C^{1.16} \mu} \right]$$

$$V^2 = \phi t^{0.754} \quad (37)$$

Where:

$$\phi = \frac{10^{10} \text{PA}^{184.0} \text{S}^{0.123}}{R^{1.28} C^{1.16} \mu}$$

$$\ln V^2 = \ln \phi + \ln t^{0.754}, 2\ln V = \ln \phi + 0.754 \ln t \quad (38)$$

The graph of 2ln V versus ln t is displaced in Fig. 2. Where  $\ln \phi_1$  is the intercept of 2ln V versus ln t:

$$\ln \phi_1 = -2.679, \phi_1 = \text{Exp}(-2.679), \phi_1 = 0.069$$

$$0.069 = \frac{10^{10} \text{PA}^{184.0} \text{S}^{0.123}}{R^{1.28} C^{1.16} \mu} \quad (39)$$

$$R = \left[ \frac{14.49 \times 10^{10} \text{PA}^{184.0} \text{S}^{0.123}}{C^{1.16} \mu} \right]^{0.78} \quad (40)$$

**Comparing LMT and FLMT with others:** Carman's tradition equation states thus:

$$V^2 = \frac{\text{PA}^2 t}{\mu r C} \quad (41)$$

Where, all parameters have been defined in the symbolic notions. But:

$$r = \frac{\text{PA}^2 t}{V^2 \mu r C} \quad (42)$$

Anazodo's specific resistance and dimensional equation was:

$$V^2 = \frac{\text{PA}^{5/4} \phi}{\mu C^{1/2} r^{1/2}} \quad (43)$$

Where,  $\phi$  = time (sec). Other parameters were already defined in the literature but:

$$r^{1/2} = \frac{\text{PA}^{5/4} \phi}{\mu C^{1/2} V^2} \quad (44)$$

$$r = \left[ \frac{\text{PA}^{5/4} \phi}{\mu C^{1/2} V^2} \right]^2 \quad (45)$$

## CONCLUSION

The modified filtration equation obtained for LMT shows that the volume of filtrate is directly proportional to area of funnel, applied vacuum pressure and time of filtration while being inversely proportional to sludge solid content, specific resistance of the filtrate, viscosity of filtrate and compressibility coefficient of the sludge.

This is in agreement with scientific reasoning and experimental observation. It is worthy to note that as volume of filtrate decreases the compressibility attributes increases. This is because the void ratio or compression pores will be larger, the resistance to the flow of filtrate will be less and thus more filtrate will be received. As the filtration continues, more cakes are being built up until the cake cracks. This means that as the sludge compression increase, the void ratio decreases and the specific resistance of the filtrate increases and the resulted to decrease in volume of the filtrate.

The LMT equation formulated satisfied this experimental observations and it is therefore recommended, as a more valid equation when compared with FLMT equations which shows that filtrate volume is directly proportional to compressible coefficient area of funnel, applied vacuum pressure and time of filtration of sludge while being inversely proportional to solid content of sludge, specific resistance and viscosity of filtrate. It is obvious that FLMT relationship is not in consonance with the filtration theory and therefore invalid by making compressibility coefficient directly proportional to cumulative filtrate volume. It is worth commendable that these 2 equations account for the compressibility coefficient  $S$ , as an attribute of the factors influencing sludge dewatering process. This theory renders FLMT equations invalid when the 2 equations are compared with other existing filtration equation by Anazodo, Ademiluyi and Carman. It was observed that the LMT equation conforms favourably to filtration theory. Another alternative means of validating the 2 new equation derived by LMT and FLMT is to compare already computed values of specific resistance parameter by Carman's and Anazodo with the actual value determined from the new equations. For the purpose of comparison, the Carman traditional equation has been used as a basis for comparison. In view of the mention earlier reasons, the 2 new equations by LMT and FLMT tends to conform amicably in all the 2 graphs plotted to compare effect of specific resistance versus  $V$ ,  $C$  and  $P$  with other equations by Carman and Anazodo. The 2 new equations have been compared analytically and graphically with Anazodo and Carman's traditional equations and all the factors used in measuring the effect of specific resistance show a high level of correlation.

## RECOMMENDATIONS

For further research on this topic should help develop a yield equation based on the new theory to enable the comparison between theoretical yield and the yield obtained from industrial vacuum filter plant. There is need to derive another equation to described sludge dewatering in drying beds.

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