Journal of Engineering and Applied Sciences 9 (6): 198-204, 2014
ISSN: 1816-949X

Effect of Slip Condition on Micropolar Fluid Flow in a Stenosed Channel

Gurju Awgichew and G. Radhakrishnamacharya
Department of Mathematics, National Institute of Technology, 506004 Warangal, India

Abstract: Steady incompressible micropolar fluid flow through uniform channel with stenosis is investigated. Assuming the stenosis to be mild and using the slip boundary condition, the equations governing the flow of the proposed model are solved and closed form expressions for the flow characteristics (resistance to flow and wall shear stress) are derived. Both the resistance to flow and the wall shear stress increase with the height of stenosis and slip parameter but decrease with microplolar parameter. The effects of other parameters on the flow characteristics also have been studied.

Key words: Micropolar fluid, stenosis, slip condition, flow resistance, India

INTRODUCTION

Theoretical and experimental studies of the circulatory disorders have been the subject of scientific research, since the investigation by Mann *et al*. (1938). It is realized that the cardiovascular disease is closely associated with the flow conditions in the blood vessels. Stenosis is the abnormal and unnatural growth in the arterial wall that develops at various locations of the cardiovascular system under certain conditions. It may result in serious consequences (cerebral strokes, myocardial infarction leading to heart failure, etc.) by reducing or occluding the blood supply. Also, it has been suggested that the deposits of the cholesterol and proliferation of connective tissue form plaques that enlarge and restrict the blood flow. One may expect that if such an event occurs, the flow characteristics in the vicinity of the resulting protuberances may be significantly altered.

In cardiac-related problems, the affected arteries get hardened, as a result of accumulation of fatty substances inside the lumen or because of formation of plaques as a result of hemorrhage. As the disease progresses, the arteries/arteriole get constricted. Consequently, the flow behavior in the stenosed artery is quite different from that in normal artery. Having knowledge of flow parameters in the stenosed artery, such as the velocity pattern, the flow rate, resistance to flow and the stresses will help bio-medical engineers in developing bio-medical instruments.

Many investigators (Young, 1968; Forrester and Young, 1970; Macdonald, 1979; Chaturani and Ponnalagarsamy, 1983; Ogulu and Abbey, 2005) have studied the characteristics of blood flow through a stenosed artery. However in all these studies, blood has been characterized as a Newtonian fluid and little attention has been given to its suspension nature. Some experimental studies indicate that blood exhibits non-Newtonian behavior at low shear rates in tubes of small diameters (Huckaba and Hahn, 1968). Hence, several researchers have studied blood flow in stenosed region considering blood as a non-Newtonian fluid. For instance Shukla *et al*. (1980), Jung *et al*. (2004), Misra and Shit (2006) and Shah (2012) studied the effects of stenosis on the blood flow through an artery by treating blood as a non-Newtonian fluid.

Micropolar fluid is a non-Newtonian fluid that has received considerable attention from researchers. Micropolar fluids belong to a class of fluids with nonsymmetrical stress tensor and are referred to as polar fluids. The main advantage of using this fluid model compared to other non-Newtonian fluids is that it takes care of the rotation of fluid particles by means of independent kinematic vector called the microrotation vector. Ariman (1971) examined the flow of micropolar fluid in a rigid circular tube and observed that it serves, as a better model in comparison to the Newtonian one for the study of blood flow. Kang and Eringen (1976), Srinivasacharya *et al*. (2003), Mekheimer and El-Kot (2007), Alemayehu and Radhakrishnamacharya (2012) studied micropolar fluids under different conditions.

The existence of slip phenomenon at the boundaries and interfaces has been observed in the flows of rarefied gasses, hypersonic flows of chemically reacting binary mixtures and flows of polymeric liquids (Bhatt and Sacheti, 1979). Several investigators considered the effect of slip (Mehta and Tiwari, 1988; Kwang *et al*., 2000; El Hakeem *et al*., 2007; Srinivas *et al*., 2009; Jadon and Yadav, 2011).

Corresponding Author: Gurju Awgichew, Department of Mathematics, National Institute of Technology, 506004 Warangal, India

Motivated by these studies, the present research attempts to understand the influence of slip boundary condition on steady incompressible micropolar fluid flow in a 2 dimensional uniform channel with stenosis. Assuming that the stenosis is mild and using the slip boundary condition, closed-form solution has been obtained and expressions for resistance to flow and shear stress at the wall have been derived. The effects of various relevant parameters on these flow variables have been studied.

MATHEMATICAL FORMULATION

Researchers consider steady, incompressible micropolar fluid flow through a 2 dimensional uniform channel with local stenosis. Cartesian coordinate system is chosen so that the x-axis coincides with the center line of the channel and the y-axis normal to it. The stenosis is supposed to be mild and develops in a symmetric manner. The boundary of the channel is taken as (Shukla *et al*., 1980):

$$\eta(x)=\begin{cases} d_0-\dfrac{\delta}{2}\left(1+\cos\dfrac{2\pi}{L_0}\left(x-d_1-\dfrac{L_0}{2}\right)\right) \\ d_1\le x\le d_1+L_0 \\ d_0 \qquad ; \text{otherwise} \end{cases} \tag{1}$$

Where:
d_0 = The mean half width of the non-stenotic region of the channel
L = The length of the channel
L_0 = The length of stenosis
δ = The maximum height of stenosis (Fig. 1)

The equations governing the flow of incompressible micropolar fluid (neglecting body forces and body couples) are given by Muthu *et al*. (2003):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\left(\frac{2\mu+k}{2}\right)\nabla^2 u+k\frac{\partial g}{\partial y} \tag{3}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\left(\frac{2\mu+k}{2}\right)\nabla^2 v-k\frac{\partial g}{\partial x} \tag{4}$$

$$\rho J\left(\frac{\partial g}{\partial t}+u\frac{\partial g}{\partial x}+v\frac{\partial g}{\partial y}\right)=-2kg+\gamma\nabla^2 g+k\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) \tag{5}$$

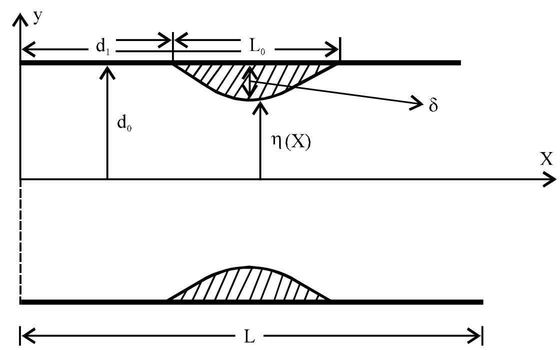

Fig. 1: Geometry of the channel with local stenosis

Where:
u and v = The velocity components along the x and y directions
p = The pressure
g = The microrotation
ρ = The density
μ = The coefficient viscosity of classical fluid dynamics
J = The micro inertia constant
γ = The viscosity coefficients for the micropolar fluid
t = The time

$$\nabla^2=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)$$

Assuming the stenosis to be mild (Young, 1968), the Eq. 2-5 get reduced as:

$$-\frac{\partial p}{\partial x}+\left(\frac{\partial\mu+k}{2}\right)\frac{\partial^2 u}{\partial y^2}+k\frac{\partial g}{\partial y}=0 \tag{6}$$

$$-2kg+\gamma\frac{\partial^2 g}{\partial y^2}-k\frac{\partial u}{\partial y}=0 \tag{7}$$

The slip boundary conditions for the velocity and microrotation are respectively given by Bhatt and Sacheti (1979):

$$u=\frac{-d_0\sqrt{Da}}{\alpha_1}\frac{\partial u}{\partial y}\text{ and } g=\frac{-d_0\sqrt{Da}}{\alpha_1}\frac{\partial g}{\partial y}\text{ at } y=\pm\eta \tag{8}$$

Where:
Da = The permeability parameter (or Darcy number)
α_1 = The slip parameter

Solving Eq. 6-7 under the boundary conditions (Eq. 8), the velocity is given as:

$$u(y) = -\frac{1}{\mu}\frac{\partial p}{\partial x}\left[\begin{array}{l}\frac{\eta^2}{2}-\frac{y^2}{2}+\frac{\eta d_0\sqrt{Da}}{\alpha_1}+\left(\frac{d_0\sqrt{Da}}{\alpha_1}+\eta\right)\times \\ \left(\frac{k}{2\mu+k}\right)\left(\frac{-s_1^*+\cosh(my)}{ms_2^*}\right)\end{array}\right] \quad (9)$$

Where:

$$m = 2\sqrt{\frac{\mu k}{\gamma(2\mu+k)}}$$

$$s_1^* = \cosh(m\eta)+\frac{d_0\sqrt{Da}}{\alpha_1} m\sinh(m\eta)$$

$$s_2^* = \sinh(m\eta)+\frac{d_0\sqrt{Da}}{\alpha_1} m\cosh(m\eta)$$

ANALYSIS

The flux Q of the fluid is given by:

$$Q = 2\int_0^\eta u\,dy = -\frac{2}{\mu}\frac{\partial p}{\partial x}\left[\begin{array}{l}\frac{\eta^3}{3}+\frac{\eta^2 d_0\sqrt{Da}}{\alpha_1}+\left(\frac{d_0\sqrt{Da}}{\alpha_1}+\eta\right) \\ \left(\frac{k}{2\mu+k}\right)\left(\frac{-s_1^*\eta}{ms_2^*}+\frac{\sinh(m\eta)}{m^2 s_2^*}\right)\end{array}\right] \quad (10)$$

Now introducing the following dimensionless quantities:

$$\delta' = \frac{\delta}{d_0}, x' = \frac{x}{L}, d_1' = \frac{d_1}{L}, L_0' = \frac{L_0}{L},$$
$$H = \frac{\eta}{d_0}, p' = \frac{p}{\left(\frac{\mu UL}{d_0^2}\right)}, Q' = \frac{Q}{Ud_0} \quad (11)$$

In Eq. 1 and 10, researchers get (after dropping primes):

$$Q = -2\frac{\partial p}{\partial x}\left[\begin{array}{l}\frac{H^3}{3}+\frac{H^2\sqrt{Da}}{\alpha_1}+\left(\frac{\sqrt{Da}}{\alpha_1}+H\right)\times \\ \left(\frac{N}{Ms_2}\right)\left(-s_1H+\frac{\sinh(MNH)}{MN}\right)\end{array}\right] \quad (12)$$

Where:

$$M = 2d_0\sqrt{\frac{\mu}{\gamma}}, \mu_1 = \frac{k}{\mu}, N = \sqrt{\frac{\mu_1}{2+\mu_1}}$$

$$s_1 = \cosh(MNH)+\frac{\sqrt{Da}}{\alpha_1}MN\sinh(MNH)$$

$$s_2 = \sinh(MNH)+\frac{\sqrt{Da}}{\alpha_1}MN\cosh(MNH)$$

M = The micropolar parameter
μ_1 = The cross viscosity coefficient

From Eq. 12, researchers obtain:

$$\frac{\partial p}{\partial x} = -\frac{Q}{2R_1} \quad (13)$$

Where:

$$R_1 = \frac{H^3}{3}+\frac{H^2\sqrt{Da}}{\alpha_1}+\left(\frac{\sqrt{Da}}{\alpha_1}+H\right)\times \left(\frac{N}{Ms_2}\right)\left(-s_1H+\frac{\sinh(MNH)}{MN}\right)$$

Integrating Eq. 13 with respect to x, researchers get pressure difference Δp along the total length of a channel as:

$$\Delta p = \frac{Q}{2}\int_0^1\frac{dx}{R_1} \quad (14)$$

The resistance to flow, denoted by λ, is defined by:

$$\lambda = \frac{\Delta p}{Q} \quad (15)$$

Using Eq. 14 in 15, researchers get:

$$\lambda = \frac{1}{2}\int_0^1\frac{dx}{R_1} \quad (16)$$

The pressure drop in the case of no stenosis (H = 1), denoted by Δp_n is obtained from Eq. 14 as:

$$\Delta p_n = \frac{Q}{2}\int_0^1\frac{dx}{R_2} \quad (17)$$

Where:

$$R_2 = \frac{1}{3}+\frac{\sqrt{Da}}{\alpha_1}+\left(\frac{\sqrt{Da}}{\alpha_1}+1\right)\left(\frac{N}{Ms_{22}}\right)\times \left(-s_{11}+\frac{\sinh(MN)}{MN}\right)$$

$$s_{11} = \cosh(MN)+\frac{\sqrt{Da}}{\alpha_1}MN\sinh(MN)$$

$$s_{22} = \sinh(MN)+\frac{\sqrt{Da}}{\alpha_1}MN\cosh(MN)$$

The resistance to flow in the absence of stenosis, λ_n is defined by:

$$\lambda_n = \frac{\Delta p_n}{Q} \quad (18)$$

Using Eq. 17 in 18, researchers obtain:

$$\lambda_n = \frac{1}{2}\int_0^1 \frac{dx}{R_2} \quad (19)$$

The normalized resistance to the flow, $\bar{\lambda}$ is given by:

$$\bar{\lambda} = \frac{\lambda}{\lambda_n} \quad (20)$$

The shear stress acting on the wall of the channel is given by:

$$\tau_w = -\mu \frac{\partial u}{\partial y}\Big|_{y=\eta} \quad (21)$$

Introducing the dimensionless quantity:

$$\tau'_w = \frac{\tau_w}{\mu U/d_0} \quad (22)$$

In Eq. 21 and using Eq. 9, researchers get (after dropping primes):

$$\tau_w = -\frac{Q}{2}\left[\frac{\left(\frac{\sqrt{Da}}{\alpha_1}+H\right)\left(\frac{N^2 \sinh(MNH)}{s_2}\right)-H}{R_1}\right] \quad (23)$$

The shear stress at the wall in the absence of stenosis (H = 1), denoted by $(\tau_w)_n$ can be obtained from Eq. 23 as:

$$(\tau_w)_n = -\frac{Q}{2}\left[\frac{\left(\frac{\sqrt{Da}}{\alpha_1}+1\right)\left(\frac{N^2 \sinh(MNH)}{s_{22}}\right)-1}{R_2}\right] \quad (24)$$

The normalized shear stress at the wall, $\bar{\tau}_w$ is given by:

$$\bar{\tau}_w = \frac{\tau_w}{(\tau_w)_n} \quad (25)$$

RESULTS AND DISCUSSION

The resistance to the flow and the wall shear stress are the 2 important characteristics in the study of fluid flow through a stenosed artery. The expressions for resistance to the flow and wall shear stress, given by Eq. 20 and 25, respectively have been numerically evaluated using mathematica software for different values of relevant parameters and presented graphically. The important parameters involved in the expressions are:

- The height of stenosis, δ
- The slip parameter, α_1
- The Darcy number, Da
- The cross viscosity μ_1 which denotes the ratio of viscosity coefficient for micropolar fluid k to classical viscosity coefficient of the fluid μ
- The microploar parameter M which can be thought of as a fluid property depending on the size of the microstricture; this is due to the factor $(\gamma/\mu)^{1/2}$ which has the dimension of length

Figure 2-9 show the effects of various parameters on the resistance to flow in a uniform channel with mild

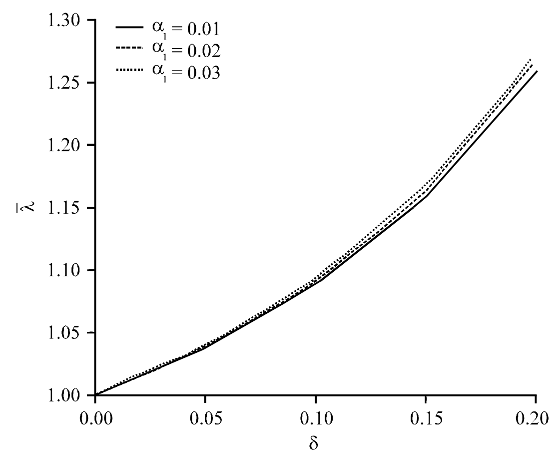

Fig. 2: Effect of α_1 on $\bar{\lambda}$ (d_1 = 0.4, Da = 0.002, L_0 = 0.2, μ_1 = 4, M = 2)

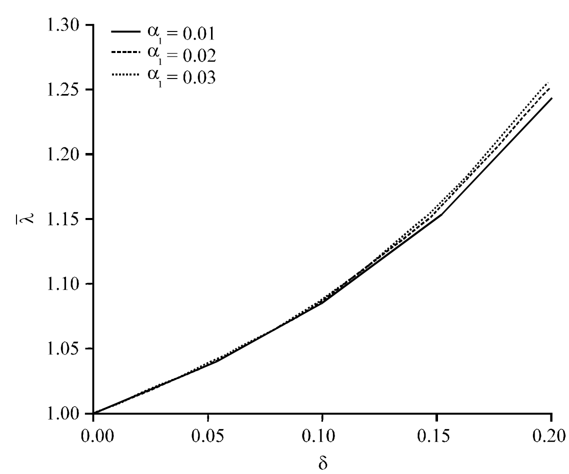

Fig. 3: Effect of α_1 on $\bar{\lambda}$ (d_1 = 0.4, Da = 0.002, L_0 = 0.2, μ_1 = 4, M = 4)

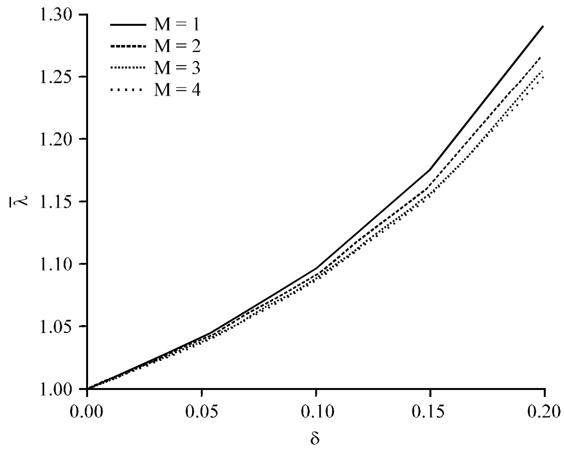

Fig. 4: Effect of M on $\bar{\lambda}$ (d_1 = 0.4, L_0 = 0.2, Da = 0.002, α_1 = 0.02, μ_1 = 4)

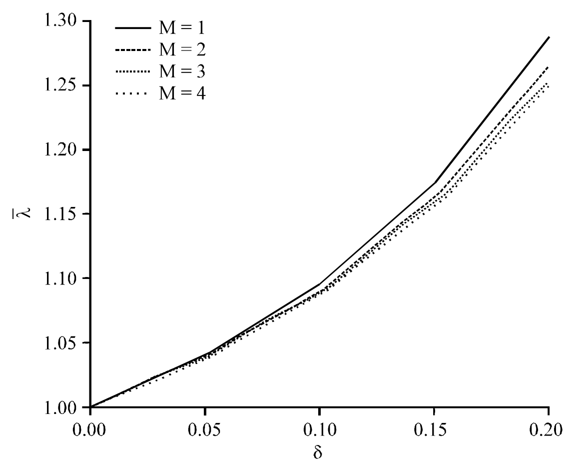

Fig. 5: Effect of M on $\bar{\lambda}$ (d_1 = 0.4, L_0 = 0.2, Da = 0.002, α_1 = 0.02, μ_1 = 8)

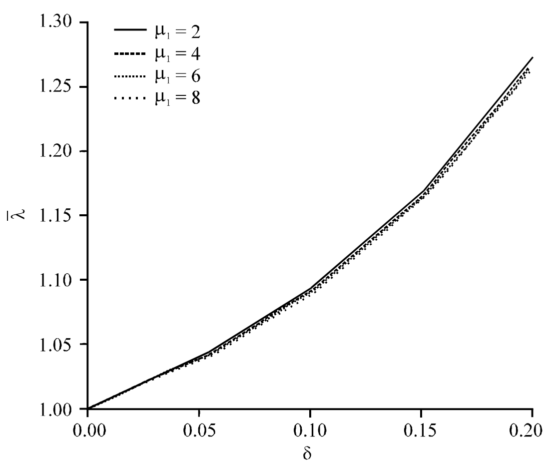

Fig. 6: Effect of μ_1 on $\bar{\lambda}$ (d_1 = 0.4, L_0 = 0.2, Da = 0.002, α_1 = 0.02, M = 2)

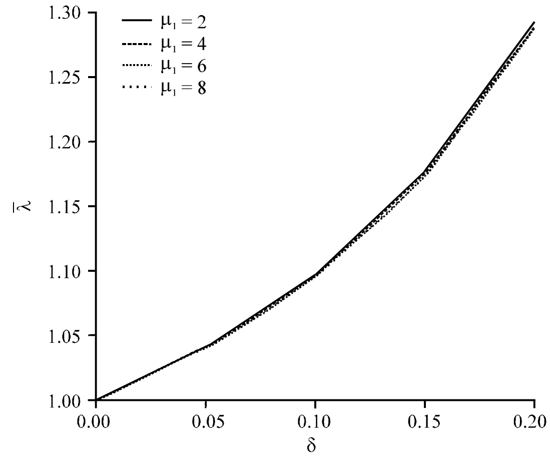

Fig. 7: Effect of μ_1 on $\bar{\lambda}$ (d_1 = 0.4, L_0 = 0.2, Da = 0.002, α_1 = 0.02, M = 1)

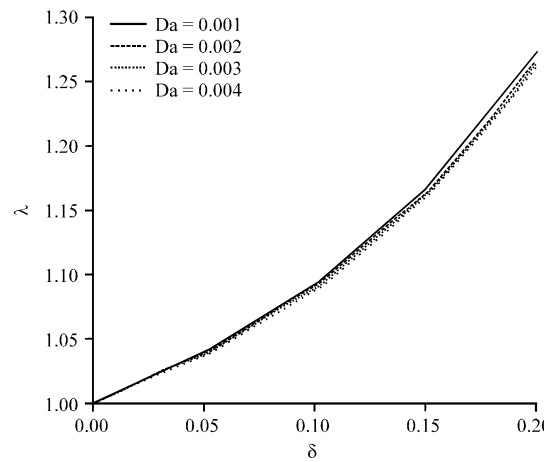

Fig. 8: Effect of Da on $\bar{\lambda}$ (d_1 = 0.4, L_0 = 0.2, μ_1 = 4, α_1 = 0.02, M = 2)

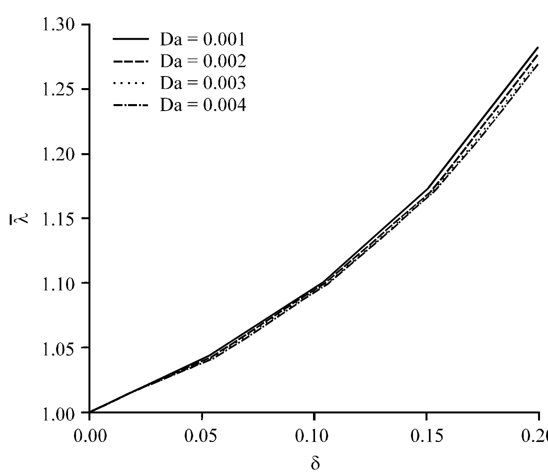

Fig. 9: Effect of Da on $\bar{\lambda}$ (d_1 = 0.4, L_0 = 0.2, μ_1 = 4, α_1 = 0.03, M = 2)

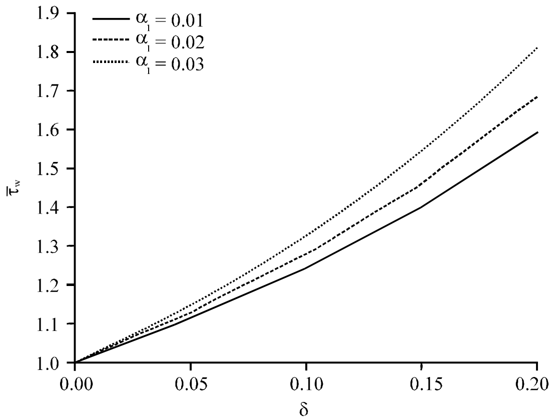

Fig. 10: Effect of α_1 on $\bar{\tau}_w$ (d_1 = 0.4, L_0 = 0.2, μ_1 = 4, Da = 0.002, x = 0.5, M = 2)

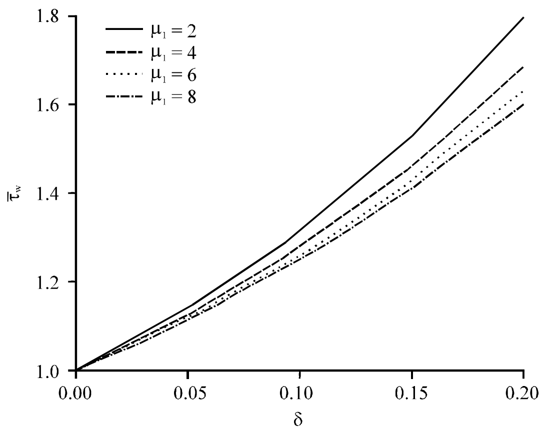

Fig. 11: Effect of μ_1 on $\bar{\tau}_w$ (d_1 = 0.4, L_0 = 0.2, α_1 = 0.03, Da = 0.002, x = 0.5, M = 2)

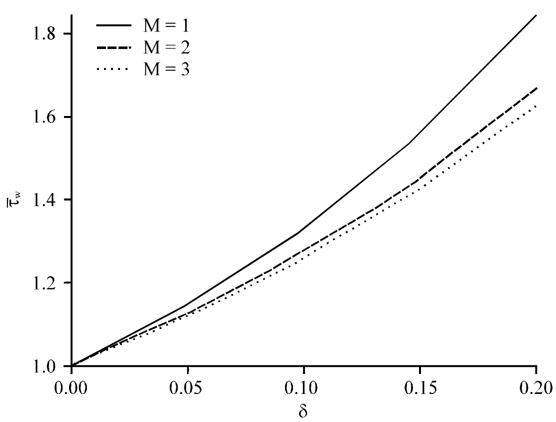

Fig. 12: Effect of M on $\bar{\tau}_w$ (d_1 = 0.4, L_0 = 0.2, α_1 = 0.02, Da = 0.002, x = 0.5, μ_1 = 4)

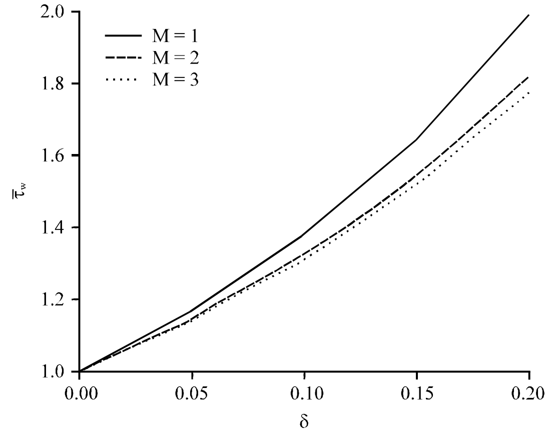

Fig. 13: Effect of M on $\bar{\tau}_w$ (d_1 = 0.4, L_0 = 0.2, α_1 = 0.03, Da = 0.002, x = 0.5, μ_1 = 4)

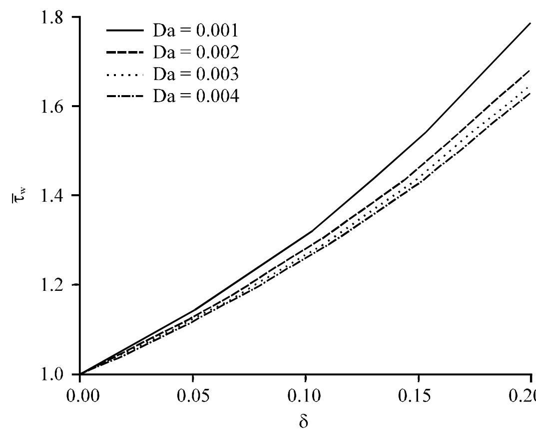

Fig. 14: Effect of Da on $\bar{\tau}_w$ (d_1 = 0.4, L_0 = 0.2, α_1 = 0.02, x = 0.5, μ_1 = 4, M = 2)

stenosis. It can be observed that the resistance to flow increases with the height of stenosis (Fig. 2-9). This result agrees with the previous results obtained by Young (1968), Shukla *et al.* (1980) and Chaturani and Ponnalagarsamy (1983). Further, it can be noticed that the resistance to flow increases with slip parameter α_1 (Fig. 2-3) but decreases with micropolar parameter M (Fig. 4 and 5), cross viscosity coefficient μ (Fig. 6 and 7) and Darcy number Da (Fig. 8 and 9). However, the decrease with cross viscosity coefficient μ_1 is not very significant (Fig. 6 and 7).

Figure 10-15 show the effects of various parameters on the wall shear stress. The wall shear stress increases with the height of stenosis (Fig. 10-15).

This result agrees with previous results obtained by Young (1968), Shukla *et al.* (1980) and Chaturani and Ponnalagarsamy (1983). Moreover, the wall shear stress increases with slip parameter (Fig. 10) but decreases with viscosity coefficient μ_1 (Fig. 11), micropolar parameter M (Fig. 12 and 13) and Darcy number Da (Fig. 14 and 15).

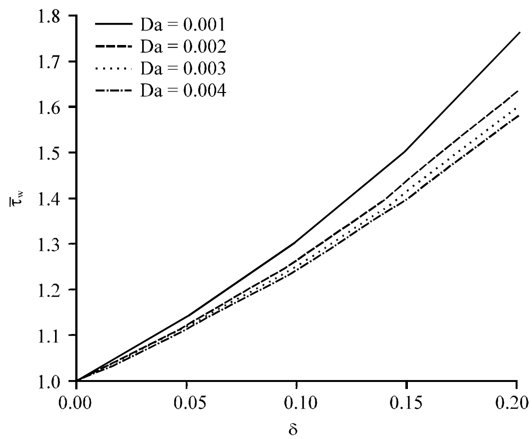

Fig. 15: Effect of Da on $\bar{\tau}_w$ (d_1 = 0.4, L_0 = 0.2, α_1 = 0.02, x = 0.5, μ_1 = 4, M = 4)

CONCLUSION

A mathematical model for the steady flow of an incompressible micropolar fluid in a 2 dimensional channel with stenosis has been studied by assuming the stenosis to mild and using slip boundary condition. It is observed that both the resistance to flow and the wall shear stress increase with the height of stenosis δ and the slip parameter α_1 but decrease with the micropolar parameter M, the cross viscosity μ_1 and Darcy number Da.

REFERENCES

Alemayehu, H. and G. Radhakrishnamacharya, 2012. The dispersion on peristaltic flow of micropolar fluid in a porous medium. J. Porous. Media., 15: 1067-1077.

Ariman, T., 1971. On the analysis of blood flow. J. Biomech., 4: 185-192.

Bhatt, B.S. and N.C. Sacheti, 1979. On the analogy in slip flows. Indian J. Pure Appl. Math., 10: 303-306.

Chaturani, P. and R. Ponnalagarsamy, 1983. Blood flow through stenosed arteries. Proceedings of the 1st International Conference on Physiological Fluid Dynamics, September 5-7, 1983, Madras, India, pp: 63-67.

El Hakeem, A., A. El Naby and I.I.E. El. Shamy, 2007. Slip effect on peristaltic transport of power-law fluid through an inclined tube. Appl. Math. Sci., 1: 2967-2980.

Forrester, J.H. and D.F. Young, 1970. Flow through a converging-diverging tube and its implications in occlusive vascular disease. J. Biomech., 3: 297-305.

Huckaba, C.E. and A.W. Hahn, 1968. A generalized approach to the modeling of arterial blood. Bull. Math. Biophys., 30: 645-662.

Jadon, V.K. and S.S. Yadav, 2011. Effects of chemical reaction on mhd free convective flow of viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime. J. Eng. Appl. Sci., 6: 349-353.

Jung, H., J.W. Choil and C.G. Park, 2004. Asymmetric flows of non-Newtonian fluids in symmetric stenosed artey. Korea Aust. Rheol. J., 16: 101-108.

Kang, C.K. and A.C. Eringen, 1976. The effect of microstructure on the rheological properties of blood. Bull. Math. Biol., 38: 135-159.

Kwang, H., W. Chua and J. Fang, 2000. Peristaltic transport in a slip flow. Eur. Phys. J. B., 16: 543-547.

Macdonald, D.A., 1979. On steady flow through modeled vascular stenosis. J. Biomech., 12: 13-20.

Mann, F. G., J. F. Herrick, H. Essex and E. J. Blades, 1938. Effects of blood flow on decreasing the lumen of a blood vessel. Surgery, 4: 249-252.

Mehta, K.N. and M.C. Tiwari, 1988. Dispersion in presence of slip and chemical reactions in porous wall tube flow. Def. Sci. J., 38: 1-11.

Mekheimer, K.S. and M.A. El-Kot, 2007. The micropolar fluid model for blood flow through stenotic arteries. Int. J. Pure Appl. Math., 36: 393-405.

Misra, J.C. and G.C. Shit, 2006. Blood flow through arteries in a pathological state: A theoretical Study. Int. J. Engng. Sci., 44: 662-671.

Muthu, P., B.V. Rathishkumar and Beeyush, 2003. On the influence of wall properties in the peristaltic motion of micropolar fluid. ANZIAM. J., 45: 245-260.

Ogulu, A. and T.M. Abbey, 2005. Simulation of heat transfer on an oscillatory blood flow in an indented porous artery. Int. Commun. Heat Mass Transf., 32: 983-989.

Shah, S.R., 2012. A biomechanical approach for the study of two-phase blood flow through stenosed artery. J. Eng. Appl. Sci., 7: 159-164.

Shukla, J.B., R.S. Parihar and B.R.P. Rao, 1980. Effects of stenosis on non-Newtonian flow of blood in an artery. Bull. Math. Biol., 42: 283-294.

Srinivas, S., R. Gayathri and M. Kothandapani, 2009. The influence of slip conditions, wall properties and heat transfer on MHD peristaltic transport. Comput. Phys. Comm., 180: 2115-2122.

Srinivasacharya, D., M. Mishra and A.R. Rao, 2003. Peristaltic pumping of a micropolar fluid in a tube. Acta Mech., 161: 165-178.

Young, D.F., 1968. Effects of a time-dependent stenosis on flow through a tube. J. Eng. Ind. Trans. AMSE., 90: 248-254.