Journal of Engineering and Applied Sciences 9 (2): 29-38, 2014
ISSN: 1816-949X

Simplification of Failure-Based Composite Design Using General Isotropic Linear FE Softwares for Classical Lamination Shell Theory

Taufiq Rochman, Agoes Soehardjono and Achfas Zacoeb
Civil Engineering Doctoral Program of Brawijaya University, Malang, Indonesia

Abstract: The design of plate and shell structures using special finite element analysis have a certain quite long procedures, especially for multilayered composite cases. Their material complexity lead to expensive analysis time due to degree of freedom caused by an arduous calculation related angle plies configuration, stack sequences, local and global propertiesand discontinuities of stress distribution both normal and interlaminar shear stresses. Composite analysis softwares are very expensive and difficult to operate due their complicated input they ask. Engineers need something practical but accurate enough for design purpose, so the failure limits should be predicted and known well. For design purpose or preliminary design, a very standard displacement based finite element software actually can be used because the stresses is quite independent from material, a well known principal from ordinary structures book. Unlike the displacement and the strain that they have a sharp relationship with the material. In this study, the use of general homogeneous linear isotropic software to solve complex structures, such as multilayered composite plate and shells is introduced. Here, researchers use the internal forces and or stresses found from an ordinary simple software and modified them to take secondary behavior into account and enrich their output into an accurate enough failure stresses for design purpose assessments. Hence, the well known {d} displacement and {ε} strain results are bypassed. After finds the global internal forces, instead of this neglecting, Tsai-Hill criterion will be judged to ensure the safety of each lamina. This proposed design method will be an accurate-efficient method in multilayered composites design.

Key words: Classical laminated theory, shear deformation, laminated composite material, bypassed, Indonesia

INTRODUCTION

The knowledge and computational technology in structural analysis have been rapidly developed. Recent, this subject become approach standard that called DBA (Design By Analysis) based on finite element method that applied whole engineering mechanics that is bridging between theory and practice and also numerical methods using the available resources in a computer code. This general method can simulate structural behavior at various loading condition precisely with the reality or experimental behavior.

Although, this method is usually designed to fulfill mechanical and machine design but it can be used nicely in civil engineering application. Slattery and Riveros (2003) modeled an innovative structures fabricated using in-the-wet construction methods to be applied in Braddock Dam. The floating shell in a dry dock is divided into a 2D grid of hollow cells separated by reinforced concrete walls that most significant structural loads involve hydrostatic pressures on the walls.

It is proved that this concept grow rapidly in that sector and results very effective, efficient, accurate design and the structural component behavior can be known exactly in a linear, static, dynamic and even nonlinearities both geometric and materially. The most important thing in design are safety margin to the failure mode. This is can be helpfully to determine safety-critical points and critical load actions. This method boast the decision judgments while still keeping the resistance, serviceability and durability aspects well and featured with ensurement complement for uncovered cases by DBF (Design by Formula) like superposition combination loading of wind, snow and earthquake.

Several structural analysis software were very easy so only need a few thoughts can results fantastic analysis figures and colorful stresses distribution, very user-friendly to make important analysis for user. But, they have important limitation especially in composite material applications. All of them using linear homogeneous isotropic material. Composite material properties, such as transversely isotropic, orthotropic and even anisotropic cases caused by fiber angle orientation, etc., must included in their analysis have not been considered yet in their shell element material modeling.

Plate and shell design using special finite element analysis have a certain quite long procedures, especially

Corresponding Author: Taufiq Rochman, Civil Engineering Doctoral Program of Brawijaya University, Malang, Indonesia

for multilayered composite cases. Their material complexity lead to expensive analysis time due to degree of freedom caused by an arduous calculation related angle plies configuration, stack sequences, local and global propertiesand discontinuities of stress distribution both normal and interlaminar shear stresses. Composite analysis software is very expensive and difficult to operate due their complicated input they ask.

CLT (Classical Lamination Theory) is the development of classical plate theory that have been discussed in the plate and shell and early finite element analysis literatures (Timoshenko and Woinowsky-Krieger, 1959; Szilard, 1974 Cook, 1981; Weaver and Johnston, 1987; Reddy, 1984), especially for composite laminate structures in the advanced finite element literature (Reddy, 2004) and composite material books (Hull and Clyne, 1981; Vinson and Sierakowski, 1987; Gibson, 1994; Powell, 1994; Hyer, 1998; Jones, 1998). Although, classical model have several lacks of transverse shear strain accuracy (Levinson, 1980; Bert, 1984; He, 1994; Liu and Lin, 1994; Singh *et al*., 1995) but in the thin structure case the transverse shear stresses are small enough. The inplane shear, compression and tension stresses region should be controlled and predominant over transverse shear before interlaminar shear failure occurs, hence it could be neglected in the design scopes.

Based on these problem, it can be describe the work objective is to use general linear isotropic homogeneous FE software to find the deflection of laminated composite structures and then determine their stresses and also their failure modes in certain plies configuration using CLT model. In order to make simplification, several condition and basic assumption is given:

- The element is based on rectangular shell using CLT model
- The properties of each lamina are homogeneous, elastic linear and fibrous transversely isotropic
- Dynamic and temperature is not considered in this analysis
- Transverse shear stress, τ_{xz} and τ_{yz} and also normal stress in z direction, σ_{zz} are ignored
- Bonding among lamina is assumed strong enough to hold any delamination

Literature review

General shell element procedures: A moderate-thick solid continuum is divided into several smaller parts called shell finite element. This process called discretization that shown in Fig. 1. The load will affect the deformations and strain on that continuum and also internal stresses and reaction forces on the restrained joints. All the displacements, strains and stresses results will be an approach value, not the exact ones. Figure 2 shows shell elements, its numbering sequences and its degree of freedoms. Finite element method described here is based on nodal joint displacement, {d} depend on DOF (Degree of Freedoms) shown in Fig. 3.

$$\{d\} = \begin{Bmatrix} u_1 \\ v_1 \\ \psi_1 \\ \phi_1 \\ \cdots \\ u_4 \\ v_4 \\ w_4 \\ \psi_4 \\ \phi_4 \end{Bmatrix} \quad (1)$$

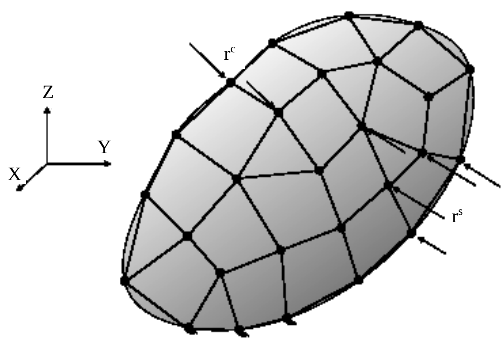

Fig. 1: Discretization on a continuum become shell elements (Stegmann, 2005)

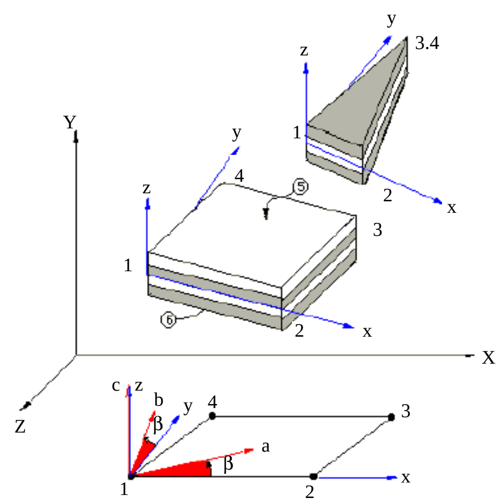

Fig. 2: Joint incidence numbering of layered shell elements: XYZ = Global cartesion coordinate system; xyz = Element coordinate system; abo = Material coordinate system; 5, 8 = Face number for applying loads and boundary conditions pressure is positive when applied (Anonymous, 2004)

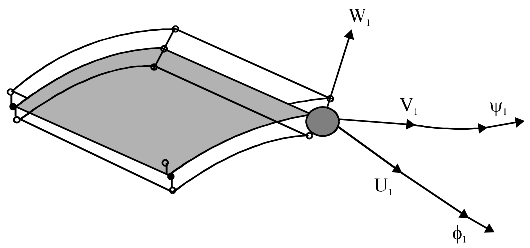

Fig. 3: DOF (Degree of Freedom) of general shell elements

It is generally known that the strains can be determined from multiplication between matrix [B] and displacement matrix {d}:

$$[\varepsilon] = [B]\ \{d\} \tag{2}$$

where [B] is strain-displacement matrix operator;

$$[\sigma] = [D].\{\varepsilon\} \tag{3}$$

Where:

$$D = \frac{E}{(1+v).(1-2v)} \begin{bmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & \frac{1-2v}{2} \end{bmatrix}$$

Element stiffness matrix can be derived using following equation (Weaver and Johnston, 1984):

$$K=\iiint[B]T.[D].[B]\ dr\ ds\ dt \tag{4}$$

This element stiffness matrices then assembled based on joint number into global structure stiffness matrix. Hence, the internal forces and the stresses of linear isotropic homogeneous shell structures can be found and used. The displacement and the strains here neglected due to their fictitious results. In composite material application, then the principal stresses and failure criteria of each layer or lamina can be found either.

Laminated composite material: Major constituents in a fiber-reinforced composite plates and shells are formed by the reinforcing fibers and a matrix which acts as a binder for the fibers. The strength of a composite material depends on the fiber strength and the matrix strength of the chemical bonds which hold them together.

Considering fiber direction, unidirectional composite material is transversely isotropic material that has the same elastic component constants in 2 and 3 direction because 2 perpendicular symmetry planes have the same elastic constants. The relationship between stress and strain for transversely isotropic material hence can be shown in Eq. 5 and 6:

$$\{\sigma\} = \left[Q_{ij}\right].\{\varepsilon\} \tag{5}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \\ \tau_{13} \\ \tau_{23} \end{Bmatrix} = \begin{bmatrix} \frac{E_1}{1-v_{12}.v_{21}} & \frac{E_1.v_{21}}{1-v_{12}.v_{21}} & 0 & 0 & 0 \\ \frac{E_1.v_{21}}{1-v_{12}.v_{21}} & \frac{E_2}{1-v_{12}.v_{21}} & 0 & 0 & 0 \\ 0 & 0 & G_{12} & 0 & 0 \\ 0 & 0 & 0 & G_{12} & 0 \\ 0 & 0 & 0 & 0 & \frac{E_2}{2.(1+v_{23})} \end{bmatrix} . \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} \tag{6}$$

Usually, composite plates and shells are arranged by several stacks of lamina called laminates that have certain orthotropic fiber angles. Fiber orthotropic angle β to the x and y axis shown in Fig. 4.

The local axis stress for β angle to the global axis is found by transforming structural global stress and strain component, $\{\sigma_s\}$ and $\{\varepsilon_s\}$ to the material local stress and strain, $\{\sigma_m\}$ and $\{\varepsilon_m\}$ as (Chia, 1990):

$$\{\sigma_m\} = [T\sigma].\{\sigma_s\} \quad \{\varepsilon_m\} = [T_\varepsilon].\{\varepsilon_s\} \tag{7}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \\ \tau_{13} \\ \tau_{23} \end{Bmatrix} = [T_\sigma]. \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} \quad \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} = [T_\varepsilon]. \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{8}$$

Where:

$$[T_\sigma] = \begin{bmatrix} c^2 & s^2 & 2cs & 0 & 0 \\ s^2 & c^2 & -2cs & 0 & 0 \\ -cs & cs & c^2-s^2 & 0 & 0 \\ 0 & 0 & 0 & c & -s \\ 0 & 0 & 0 & s & c \end{bmatrix}$$

$$[T_\varepsilon] = \begin{bmatrix} c^2 & s^2 & cs & 0 & 0 \\ s^2 & c^2 & -cs & 0 & 0 \\ -2cs & 2cs & c^2-s^2 & 0 & 0 \\ 0 & 0 & 0 & c & -s \\ 0 & 0 & 0 & s & c \end{bmatrix} \tag{9}$$

Where:

c = cos β

s = sin β

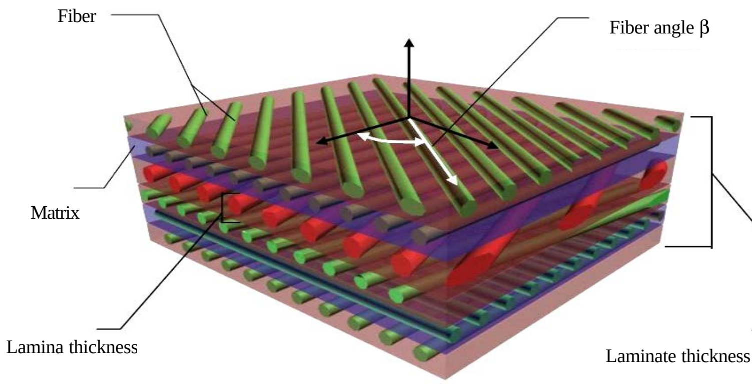

Fig. 4: Fiber angle β in composite flat shell element (Cugnoni, 2004)

So, the structural stress is shown in Eq. 10 (Chia, 1990):

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{Bmatrix} = \left[T_\sigma\right]^{-1}\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\\ \tau_{13}\\ \tau_{23}\end{Bmatrix} = \left[T_\sigma\right]^{-1}.\left[Q\right].$$

$$\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\\ \gamma_{13}\\ \gamma_{23}\end{Bmatrix} = \left[T_\sigma\right]^{-1}.\left[Q\right].\left[T_\varepsilon\right].\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix} \quad (10)$$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{Bmatrix} = \left[\bar{Q}\right].\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix} \quad (11)$$

where $\left[\bar{Q}\right]$ is transformed reduced stiffness matrix;

$$\left[\bar{Q}\right] = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{14} & 0 & 0\\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{24} & 0 & 0\\ \bar{Q}_{41} & \bar{Q}_{42} & \bar{Q}_{44} & 0 & 0\\ 0 & 0 & 0 & \bar{Q}_{55} & \bar{Q}_{56}\\ 0 & 0 & 0 & \bar{Q}_{65} & \bar{Q}_{66}\end{bmatrix} \quad (12)$$

Classical shell theory: This model CST is the well known basic theory in plate/shell analysis that based on the assumption that the normal plane is remain straight and perpendicular before and after deformation. Thus, this model ignores transverse shear deformation in xz and yz plane that occurs between lamina and this is a CST's lack of accuracy. But, this model has enough accuracy for thin plate/shell design. The CST model rotation can be formulated as:

$$\frac{dw}{dx} = -\psi_x; \frac{dw}{dy}\psi_y \quad (13)$$

And the displacement field of CST model in the x, y and z direction is:

$$\begin{aligned} u(x,y,z) &= u_o(x,y,z) + z.\frac{\partial w}{\partial x}(x,y,z)\\ v(x,y,z) &= v_o(x,y,z) + z.\frac{\partial w}{\partial x}(x,y,z)\\ w(x,y,z) &= w_o(x,y,z) \end{aligned} \quad (14)$$

The lamina strain of CST model is described as Eq. 15 where $\kappa_i = \partial^2 w/\partial i^2$:

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z.\begin{Bmatrix}k_x\\ k_y\\ k_{xy}\end{Bmatrix} \quad (15)$$

The stress {σ} of this model found from strain {ε} become:

$$\sigma_x = \frac{E}{1-v^2}\left(\varepsilon_x + v.\varepsilon_y\right)\sigma_Y = \frac{E}{1-v^2}$$
$$\left(V.\varepsilon_x + \varepsilon_y\right)\tau_{xy} = 2.G.\varepsilon_{xy} = G.\gamma_{xy} \quad (16)$$

Where:

$$G = \frac{E}{2.(1+v)}$$

Classical Lamination Theory (CLT): Analysis of CST model for laminated case is called CLT (Classical Lamination Theory). There is the famous ABD matrix that connects the internal forces of CLT model with the strain:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_x^0 \\ k_x \\ k_y \\ k_{xy} \end{Bmatrix} \quad (17)$$

or in partitioned form as;

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} . \begin{Bmatrix} \varepsilon^o \\ k \end{Bmatrix} \quad (18)$$

Hyer (1998) shows the element laminates nomenclature used in ABD matrix where n is the number of lamina and h_k is the related lamina thickness kth and h_{k-1} is the previous lamina thickness (k-1).

The A_{ij} is coefficient of the laminate normal extensional in-plane stiffnessand B_{ij} is coefficient of the laminate normal-curvature and moment-strain coupling stiffness while D_{ij} is coefficient of the laminate bending stiffness. The ABD matrix components are found from integration through the plates or shell thicknesses and results:

$$A_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \bar{Q}_{ij}^{(k)} dz = \sum_{k=1}^{n} \bar{Q}_{ij}^{(k)} . (h_k - h_{k-1})$$

where i, j = 1, 2, 4, 5, 6.

$$B_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \bar{Q}_{ij}^{(k)} z.dz = \frac{1}{2} . \sum_{k=1}^{n} \bar{Q}_{ij}^{(k)} . (h_k^2 - h_{k-1}^2)$$

where i, j = 1, 2, 4, 5, 6.

$$D_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \bar{Q}_{ij}^{(k)} z^2\, dz = \frac{1}{3} . \sum_{k=1}^{n} \bar{Q}_{ij}^{(k)} . (h_k^3 - h_{k-1}^3) \quad (19)$$

where i, j =1, 2, 4.

Failure stress criterion

Isotropic case: In determining failure stress criterion for isotropic case, there are 3 main concept, first maximum principal stress theory from Rankineand famous, as maximum stress criterion. Secondly is the Von Mises stress criterionand the last is tresca stress criterion based on the maximum shear stress concept. Figure 5 shows the diagram of 3 failure stress criterion. The maximum stress criterion has the fully plastic concept:

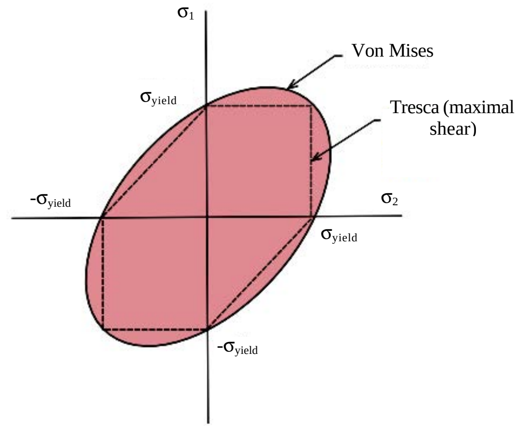

Fig. 5: The diagram of different failure criterion (http://en.wikipedia.org)

$$\sigma_o(\text{tension}) < \sigma_1 < \sigma_o(\text{compression}) \quad (20)$$

While Von Mises stress criterion or famous, as equivalent or effective stress derived from critical distortional energy theory and in Eq. 21:

$$\sigma_{vme} = \sqrt{\frac{1}{2}\left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_y)^2 + 6(\tau_{xy}^2 + \tau_{vz}^2 + \tau_{zx}^2) \right]}$$
$$= \frac{1}{\sqrt{2}} \sqrt{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{22} - \sigma_{33})^2} \quad (21)$$

Where σ_{11} and σ_{22} are the principal stresses for isotropic material. For plane stress case, σ_3 = 0, Eq. 19 reduced to Eq. 22 that why for this case, the equation represents an ellipse:

$$\sigma_{vme} = \sqrt{\sigma_{11}^2 - \sigma_{11}\sigma_{22} + \sigma_{22}^2} \quad (22)$$

Finally, the maximum shear stress criterion tresca have involve plastic region in Eq. 23:

$$\tau_{vme} = \frac{\sigma_{11} - \sigma_{33}}{2} = \frac{\sigma_{max} - \sigma_{min}}{2} \quad (23)$$

Tresca is more conservative than Von Mises and also valid for isotropic and ductile materials.

Laminated composite case: For laminated composite case, the structure should be ensured below stress limit in each layer. There are several theories that suitable for laminated composites material: The maximum stress theory, the Tsai-Hill theory, Tsai-Wu criterion, maximum strain, hoffman, hashin, puck, cuntze, chang, azzi-tsai, etc. But in this study, only describe the first three theory. The symbol used X, X_C, X_T, Y, Y_C, Y_T and S are allowable lamina direct normal strength in the fiber 1-direction, for compression and tension, allowable direct strength normal to the fiber 2-direction (in-plane), for compression and tensionand also in-plane shear strength, respectively in the material coordinate. Sometimes other books mention σ_{LU} and σ_{TU} and τ_{LT} instead of X, Y and S where L, T and U are longitudinal, transverse and ultimate, respectively. (Parhi *et al*., 2001).

In the maximum stress theory, the notion is that layer failure is judged to occur if any one of the stresses σ_{11}, σ_{22} or τ_{12} in the material coordinates exceeds the corresponding allowable stress value. In order to avoid failure, the following inequalities must be satisfied:

$$\frac{\sigma_{11}}{X} \leq 1, \frac{\sigma_{22}}{Y} \leq 1, \frac{\tau_{12}}{S} \leq 1 \qquad (24)$$

The allowable direct stress X and Y should be replaced by X_C and Y_C in compression or X_T and Y_T in tension according to the signs of σ_{11} and σ_{22}, respectively. In one way transversely isotropic or each longitudinal layer, these equations represent fiber failure, transverse matrix cracking and shear matrix cracking, respectively.

The Tsai-Hill theory was a Von Mises-like criterion for composites developed from hill yield criterion for anisotropic material. The Tsai-Hill theory provides a single fungction to predict strength. In the case of biaxial plane stress for a single homogeneous orthotropic layer the criterion becomes Eq. 25, failure is initiated when the inequality below is violated.

$$\left(\frac{\sigma_{11}}{X}\right)^2 - \left(\frac{\sigma_{11}\sigma_{22}}{X^2}\right) + \left(\frac{\sigma_{22}}{Y}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2 \leq 1 \qquad (25)$$

Tsai-Hill theory will reduces to the Von Mises theory back for isotropic material in the Eq. 22 by setting the substitutions for $\tau_{12} = 0$. More complex failure criteria for composites is Tsai-Wu stress criterion.

$$\begin{aligned} &F_1\sigma_{11} + F_2\sigma_{22} + F_3\sigma_{33} + 2F_{12}\sigma_{11}\sigma_{22} + \\ &2F_{13}\sigma_{11}\sigma_{22} + 2F_{23}\sigma_{22}\sigma_{33} + F_{11}\sigma_{11}^2 + \\ &F_{22}\sigma_{22}^2 + F_{33}\sigma_{33}^2 + F_{44}\tau_{13}^2 + F_{55}\tau_{23}^2 + F_{66}\tau_{12}^2 \geq 1 \end{aligned} \qquad (26)$$

Where, σ_{11}, σ_{22} and σ_{66} are limit stress situation related to the symmetry directions. For the case of orthotropic lamina under plane stress condition, Eq. 26 can be reduced into 27:

$$\begin{aligned} \phi = &F_1\sigma_{11} + F_2\sigma_{22} + 2F_{12}\sigma_{11}\sigma_{22} + \\ &F_{11}\sigma_{11}^2 + F_{22}\sigma_{22}^2 + F_{66}\sigma_{66}^2 \quad \geq 1 \end{aligned} \qquad (27)$$

Where, F_i and F_{ij} are strength tensor of the 2nd and 4th rank and can be expressed in the unidirectional critical stresses which can be derived from uniaxial experiments:

$$\begin{aligned} &F_1 = \frac{1}{X_T} - \frac{1}{X_C},\ F_2 = \frac{1}{Y_T} - \frac{1}{Y_C},\ F_{11} = \frac{1}{X_T X_C} \\ &F_{22} = \frac{1}{Y_T Y_C},\ F_{66} = \frac{1}{S^2}, \\ &F_{12} = -\frac{1}{2}\sqrt{F_{11}F_{22}} = -\frac{1}{2}\sqrt{\frac{1}{X_T X_C Y_T Y_C}} \end{aligned}$$

Interaction coefficient F_{12} is assumed such, represent the interaction of extensional stresses while quadratic strength coefficient represent an ellipsoid in the stress space and linear strength coefficient represent different strength in tension and compression. Hence, this ability to represent all earliar interaction provide more generality than one achieved by Tsai-Hill theory but Tsai-Wu criterion requires more experimental data test, especially biaxial test to determine F_{12} term, simple uniaxial and pure shear testing to determine another coefficients.

MATERIALS AND METHODS

Using general FEA software: In order to find a usual internal forces moment M, shear forces Q and normal forces N without any arduous computational procedures involve a long matrix multiplication, many linear isotropic homogeneous structural DBA softwares have been developed. Indeed, almost all the results, such as the displacements and the strains are inaccurate for composite materials but actually global stress is quite independent from the material properties. A well known principal from ordinary structures book that internal forces and stresses were not affected by the material properties but it were affected the loads. On the contrary, the material properties will strongly affect the displacements and it strains. So, the stresses result, hence adopted to find the principal stress for each lamina and then backward finding to determine the strain. The displacement results can be overruled due to their inaccurate results, furthermore, unlike steel material; composite material has a brittle behavior, so the failure is first reached before displacement limit passed over.

Researchers use the simplest famous one that is STAADPro v8i 2007 from Bentley System Inc. because of its simplicity and it has the biggest user in the civil engineering area.

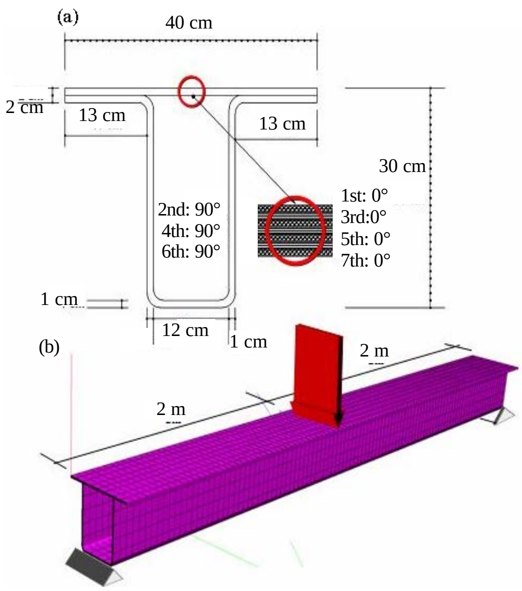

Fig. 6: a) Combo-girder cross section (top); b) STAADPro v8i 2007 3D geometric coordinates (bottom)

Composite box girder structures data: The proposed composite box girder COMBO-girder under 3 point bending load is used as benchmark validation. The geometric structure data and its configuration are given in Fig. 6. The length of girder was 4 m, the depth of girder was 30 cm and it has 40 cm width. The girder laminates consist of 7 layers and the thickness was 1 cm thick. The sequence of lay-up configuration is (0°/90°/0°/90°/0°/90°/0°).

Each laminates is the combination of unidirectional (0°) L900E fiber mat and bidirectional (90°/0°) LT600 M225E, both of them are E-glass type products from Triaxis composites.

Fabric specifications and laminate properties: The L900E is a unidirectional product that suitable for hand lay-up, vacuum bagging of relatively flat component, vacuum injection and other RTM-process (Fig. 7a; Table 1 and 2). Ideal for components with unidirectional loads such as beam, flanges, etc. The product is easily cut in tapes to requested widths in order to fit the application exactly. Fiber content above 55% is easily obtained with ordinary hand lay-up. While LT600M225E is a biaxial products that has the same properties mentioned earlier and fiber content above 45% is easily also obtained with ordinary hand lay-up (Fig. 7b). Table 3 shows the fabric specifications.

Table 1: Laminate properties

Matrix type	Resin type	Fiber weight fraction (%)	Fiber volume fraction (%)	Layer thickness (mm)
L 900-E	Isophthalic polyester	58.00	40.00	0.9
LT 600 M 225-E	Isophthalic polyester	44.73	27.58	1.165

Table 2: Mechanical properties of lamina fabric mat

		Layer orientation degree					
		L900E			LT600 M225E		
Properties	Standard test	0°	45°	90°	0°	45°	90°
Tensile modulus (GPa)	ASTM D3039-76	27.9	9.1	8.5	11.1	5.5	11.2
Compressive modulus (GPa)	ASTM D695	30.5	9.1	12.4	9.5	5.0	9.5
Tensile strength (MPa)	ASTM D3039-76	708.0	94.0	46.8	311.7	153.7	312.8
Compressive strength (MPa)	ASTM D695	417.0	141.0	119.0	265.2	139.0	266.1
Elongation at break (%)	ASTM D3039-76	2.8		2.3			
Shear strength, inplane (MPa)	ASTM 4255-83	62.0	110.0	62.0			
Shear strength, interlaminar (MPa)	ASTM 2344-84	40.8					

Table 3: Fabric speci fications

Specification	Unit	L900E		LT600 M225E	
Layer orientation degree	(°)	0	90	0	90
Layer weight	(gm^{-2})	827	45	295	297
Material (E-glass)					
Weight, knitting yarn	(gm^{-2})	12		16	
Weight, total	(gm^{-2})	884±3%		833±5%	
Standard roll width	(cm)	126		137	
Standard roll length (approx.)	(m)	45		44	
Standard roll weight (approx.)	(kg)	50		50	

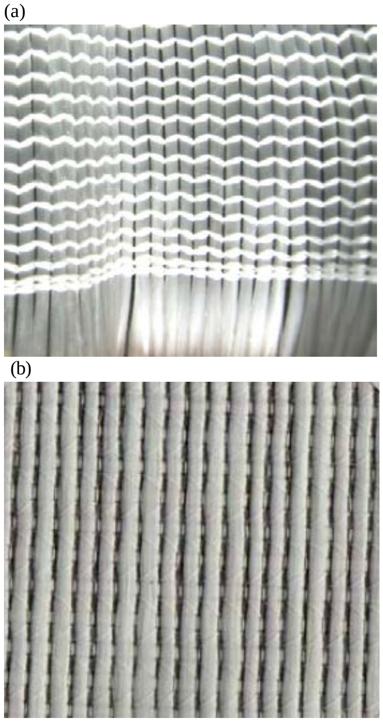

Fig. 7: a) L900E fiber mat; b) LT600 M225E fiber mat

RESULTS AND DISCUSSION

Global stress result: From the isotropic linear homogenous software, STAADPro v8i 2007, using an arbitrary material properties such as steel or concrete, it is found the displacement, strains, stresses and internal forces and or reactions. The displacements and strains result will be disused due to their fictitious results but the internal forces, especially stresses will be used due to their quite independent results from material properties, as mentioned (Table 4).

Hence, normal stresses per unit width F_x, F_y and inplane shear stress F_{xy} and also moment M_x, M_y and M_{xy} shown is Fig. 8 and they were used to find principle stress along fiber direction and or it strains.

Based on Fig. 8, the compression normal stress is roughly predicted as 22.2 MPa, below the compressive strength of the composite that is 74.32 MPa for the composites. For tension area, the stress occurred is 29.5 MPa also below 148.64 MPa as the tension strength of the composites.

Based on Fig. 9, the compression normal stress is roughly predicted, especially for support area as 16.5 MPa is indeed, below the compressive strength of the composite that is 74.32 MPa.

Based on Fig. 10, the in-plane shear stress is roughly predicted, especially for support area, as 12.8 Mpa is

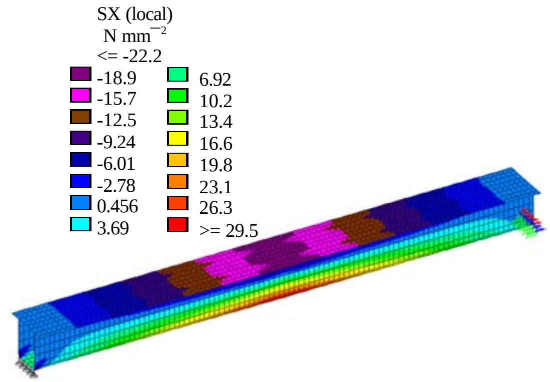

Fig. 8: The maximum longitudinal normal stress at the direction of x local axis, σ_x is predicted as 29.5 MPa for tension and -22.2 MPa for compression

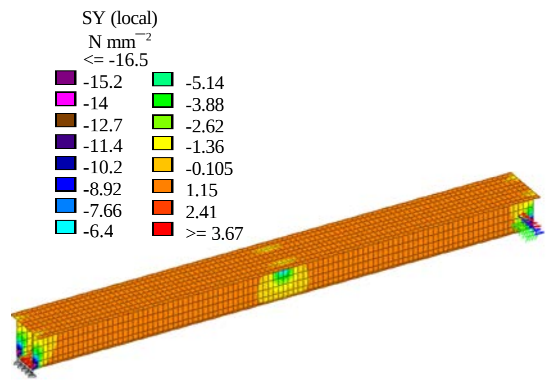

Fig. 9: The maximum normal stress at the direction of y local axis, σ_y is predicted as 3.67 MPa for tension and -16.5 MPa for compression

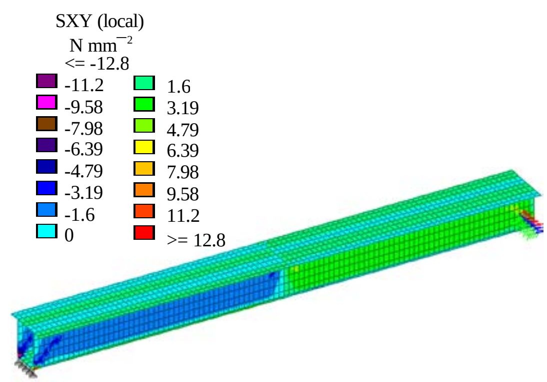

Fig. 10: The maximum in-plane shear stress τ_{xy} is predicted as 12.8 MPa at the support area

indeed, below the in-plane shear strength of the composite that is 103.46 Mpa for composites.

More accurate stress failure prediction like Tsai-Hill failure criterion still needed to ensure the safety, although

Table 4: Experimental material properties for saturated polyester resin composite

Properties	Units	Values
Longitudinal young modulus	E_{11}	45 GPa
Transverse young modulus	E_{22}	2.72 GPa
In-plane shear modulus	G_{12}	1.72 GPa
Poisson's ratio	ν_{12}	0.25
Longitudinal tensile strength	X_T	148.64 MPa
Longitudinal compressive strength	X_C	74.32 MPa
Transverse tensile strength	Y_T	40.21 MPa
Transverse compressive strength	Y_C	37.16 MPa
In-plane shear strength	S	103.46 Mpa

its stress far below 148.64 MPa as the tension strength of the composites and show adequate value both in compression and tension, respectively.

$$\left(\frac{-22.2}{-74.32}\right)^2 - \frac{-22.2(-16.5)}{74.32^2} + \left(\frac{-16.5}{-37.16}\right)^2 + \left(\frac{12.8}{103.46}\right)^2 = 0.344 < 1;$$

$$\left(\frac{29.5}{148.64}\right)^2 - \frac{29.5(3.67)}{148.64^2} + \left(\frac{3.67}{40.21}\right)^2 + \left(\frac{12.8}{103.46}\right)^2 = 0.167 < 1$$

While Tsai-Wu criterion for compression and tension strength, respectively gives:

$$\left(\frac{1}{148.64} - \frac{1}{-74.32}\right).(-22.2) + \left(\frac{1}{40.21} - \frac{1}{-37.16}\right).(-16.5) - 2.\left(\frac{1}{2}\sqrt{\frac{1}{148.64.(-74.32).40.21.(-37.16)}}\right).(-22.2).(-16.5) + \left(\frac{1}{148.64.(-74.32)}\right)(-22.2)^2 + \left(\frac{1}{40.21.(-37.16)}\right)(-16.5)^2 + \frac{1}{103.46^2}.12.8^2 = -1.514 < 1$$

Indeed, Tsai-Wu criterion seems to be overestimate in the failure judgment over the Tsai-Hill criterion, so according to, Tsai-Wu criterion, this compressive stress are too large in overall multidirectional stresses or structure compression strength are not adequate to overcome compressive stress occurred:

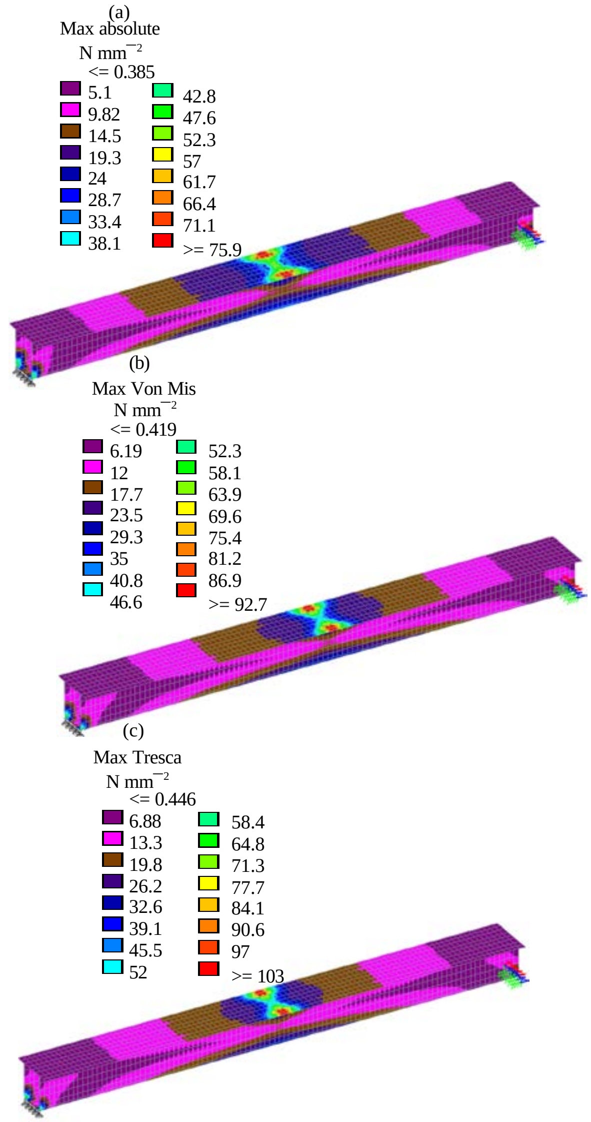

Fig. 11: Several failure criterion for isotropic case: a) Maximum stress; b) Von Mises; c) Tresca criterions

$$\left(\frac{1}{148.64} - \frac{1}{-74.32}\right).(-29.5) + \left(\frac{1}{40.21} - \frac{1}{-37.16}\right)(3.67) - 2.\left(\frac{1}{2}\sqrt{\frac{1}{148.64.(-74.32).40.21.(-37.16)}}\right)(-29.5).3.67 + \left(\frac{1}{148.64.(-74.32)}\right)(29.5)^2 + \left(\frac{1}{40.21.(-37.16)}\right)(3.67)^2 + \frac{1}{103.46^2}.12.8^2 = 0.713 < 1$$

But, Tsai-Wu criterion seems to meet Tsai-Hill criterion in the tension failure judgment, hence according to Tsai-Wu criterion, this tension stress are little enough in overall multidirectional stresses or structure tensile strength are adequate enough to overcome tension stress occurred.

Figure 11 shows different failure stress criterion for isotropic homogeneous material, first, absolute maximum stress criterion then Von Mises stress criterion and finally Tresca stress criterion. Tresca failure stress criterion seems to be the most conservative criterion, over-estimate the stress occur but it is the safest criterion for design. But, for laminated composite material indeed Tsai-Hill and Tsai-Wu criterion present more accurate results than Von-Mises and Tresca because last mentioned above are both used for homogeneous isotropic material.

CONCLUSION

The use of general homogeneous linear isotropic software to solve complex structures, such as multilayered composite plate and shells is proposed and it can be concluded regarding this analysis that: The standard and conventional displacement based structural analysis software is sufficient enough to preliminary analyze the global behavior of layered composite structures for design purpose because the stresses is quite independent from material. The stresses found then modified to take secondary behavior into account and enrich their output into an accurate enough failure stresses for design purpose assessments.

Further, criterion like Tsai-Hill and Tsai-Wu criterion for composite material should be judged to ensure the safety of each lamina of multilayered composite structures. This proposed design method will be an accurate-efficient method in multilayered composites design, especially for preliminary design.

REFERENCES

Anonymous, 2004. ANSYS Structural Analysis Guide: ANSYS Release 9.0. ANSYS, Incorporated, New York, USA.

Bert, C.W., 1984. A critical evaluation of new plate theories applied to laminated composites. Composite Struct., 2: 329-347.

Chia, C.Y., 1990. Nonlinear Analysis of Plates. Mc Graw Hill Co., New York, ISBN-13: 9780070107465, Pages: 422.

Cook, R.D., 1981. Concept and Applications of Finite Element Analysis. 2nd Edn., John Wily and Sons, Inc., New York, ISBN: 0-471-03050-3, pp: 137-139.

Cugnoni, J., 2004. Identification and frequency modal registration of constitutive properties of composite shells. Ph.D. Thesis, Ecole Polytechnique Federale de Lausanne, Switzerland.

Gibson, R., 1994. Principles of Composite Material Mechanics. McGraw Hill, New York.

He, J.F., 1994. A refined shear deformation theory of bending of isotropic plates. J. Mech. Struct. Mach., 22: 397-427.

Hull, D. and T.W. Clyne, 1987. An Introduction to Composite Materials (Cambridge Solid State Sciences Series). 2 Edn., Cambridge University Press, Cambridge, UK.

Hyer, M.W., 1998. Stress Analysis of Fiber-Reinforced Composite Materials. WCB Mc-Graw Hill, Singapore.

Jones, R.M., 1998. Mechanics of Composite Materials. Taylor and Francis Inc., USA.

Levinson, M., 1980. An accurate, simple theory of the statics and dynamics of elastic plates. Mech. Res. Commun., 7: 343-350.

Liu, I.W. and C.C. Lin, 1994. A new conforming plate bending element for the analysis of composite laminates. Proceedings of the National Science Council, Republic of China: Physical Science and Engineering, Vol. 18, November 1994, Atlanta, Georgia, pp: 561-569.

Parhi, P.K., S.K Bhattacharyya and P.K. Sinha, 2001. Failure analysis of multiple delaminated composite plates due to bending and impact. Bull. Mater. Sci., 24: 143-149.

Powell, P.C., 1994. Engineering with Fibre-Polymer Laminates. Springer, New York, ISBN-13: 9780412496103, Pages: 441.

Reddy, J.N., 1984. Energy and Variational Methods in Applied Mechanics. John Wiley and Sons Ltd., New York, USA.

Reddy, J.N., 2004. Mechanics of Laminated Composite Plates and Shells: Theory and Analysis. 2nd Edn., CRC Press, Boca Raton, FL., USA.

Singh, G., G.V. Rao and N.G.R. Iyengar, 1995. A discussion on simple third order theories and elasticity approaches for flexure of laminated plates. J. Struct. Eng. Mech., 3: 121-133.

Slattery, K.T. and G.A. Riveros, 2003. Design by analysis of innovative navigation structures. Innovations for Navigation Projects Research Program. ERDC/ITL TR-03-5. US Army Corps of Engineers, August 2003.

Stegmann, J., 2005. Analysis and optimization of laminated composite shell structures. Ph.D. Thesis, Aalborg University, Denmark.

Szilard, R., 1974. Theories and Applications of Plate Analysis: Classical Numerical and Engineering Methods. Wiley, New York, ISBN-13: 9780471429890, Pages: 1056.

Timoshenko, S. and S. Woinowsky-Krieger, 1959. Theory of Plates and Shells. 2nd Edn., McGraw-Hill Book Co., New York, pp: 580.

Vinson, J.R. and R.L. Sierakowski, 1987. The Behavior of Structures Composed of Composite Materials. Martinus Nijhoff Publishers, Dordrecht.

Weaver, W. and P.R. Johnston, 1984. Finite Elements for Structural Analysis. Prentice-Hall Englewood Cliffs, New Jersey, ISBN-13: 9780133170993, Pages: 403.