Journal of Engineering and Applied Sciences 8 (4): 116-119, 2013
ISSN: 1816-949X

Thermal Boundary Layer Analysis of Nanofluids in a Circular Tube

Seyyed Shahabeddin Azimi and Mansour Kalbasi
Department of Chemical Engineering, Amirkabir University of Technology,
Hafez Avenue, Tehran, Iran

Abstract: In this research, thermal boundary layer in the forced convective heat transfer of nanofluids in fully developed laminar flow in a circular tube is considered in which thermal boundary layer grows. In thermal boundary layer temperature increases gradually. By analysis of conservation laws, it is concluded that the density of the base fluid has important role in the thermal boundary layer because both nanoparticle volume fraction and axial velocity of the nanofluid depend on it. Also to obtain temperature profile, the energy equation does not require to couple with momentum equations despite velocity has radial and axial distribution.

Key words: Heat transfer, nanofluid, laminar flow, fully developed region, velocity, Iran

INTRODUCTION

Conventional heat transfer fluids, e.g., water are poor heat transfer fluids and have low efficiency in cooling or heating processes. Numerous methods have been taken to improve the thermal behavior of these fluids by suspending micro or larger-sized particle materials in liquids (Kakac and Pramuanjaroenkij, 2009). Nanofluid is a liquid (base fluid) in which nanometer-sized solid particles are suspended; these particles are known as nanoparticles. Suspended nanoparticles in various base fluids can alter the fluid flow and heat transfer characteristics of the base fluids (Wang and Mujumdar, 2007).

The nanofuid has feature which is quite different from conventional solid-liquid mixtures in which millimeter or micrometer-sized particles are added, these larger-sized particles settle rapidly, clog flow channels, erode pipelines and cause severe pressure drops (Xuan and Roetzel, 2000).

Nanofluids are proposed for a variety of applications in several important fields, such as microelectronics, aerospace, transportation and medicine (Zhu *et al*., 2006). Forced convective heat transfer of nanofluids is interesting field in literatures (He *et al*., 2009). In these researches, conservation laws were solved and temperature profile was obtained.

In this research, researchers consider forced convective heat transfer of nanofluids when the nanofluid is flowing steadily with laminar motion inside a smooth circular tube in the region in which velocity profile is fully developed. Temperature of the nanofluid is uniform over the flow cross section, the focus is at point where heat transfer begins due to temperature of the tube surface which is larger than nanofluid temperature, following this point which heat transfer takes place, researchers are concerned with the development of the temperature profile named thermal boundary layer (Kays and Crawford, 1993).

MATERIALS AND METHODS

The nanofluid is assumed as a single phase fluid. The nanoparticles can develop a slip velocity with respect to the base fluid. Assumptions in the fully developed laminar flow and thermal entry length region of the tube are:

- Steady state flow
- Newtonian fluid and negligible viscous dissipation
- Velocity of the nanofluid in any point has one component (fully developed region $u_z \neq 0$)
- Nanoparticles and base fluid are locally in thermal equilibrium
- The tube surface has temperature which is larger than the nanofluid bulk temperature in whole of the tube

Governing equations for nanofluids in the thermal boundary layer include the continuity equation (mass balance), equation of motion (momentum balance) and energy equation (energy balance). They are given in the cylindrical coordinates, respectively in the following. Continuity equation of the nanofluid:

$$\frac{\partial(\rho_{nf} u_z)}{\partial z} = 0 \quad (1)$$

Corresponding Author: Mansour Kalbasi, Department of Chemical Engineering, Amirkabir University of Technology, Hafez Avenue, Tehran, Iran

Where:
ρ_{nf} = The density of the nanofluid
u_z = The nanofluid axial velocity

If:

$$\frac{\partial u_z}{\partial z} = 0 \quad \text{then} \quad \frac{\partial \rho_{nf}}{\partial z} = 0$$

this is incorrect result for ρ_{nf}, since temperature is not constant in the thermal boundary layer. Continuity equation of the nanoparticle:

$$\frac{\partial(\phi\rho_p u_z)}{\partial z} + \left[\frac{1}{r}\frac{\partial(rj_r)}{\partial r} + \frac{\partial(j_z)}{\partial z}\right] = 0 \quad (2)$$

Where:
ϕ = The nanoparticle volume fraction
j_z, j_r = Nanofluid mass rates due to diffusion

Momentum equation of the nanofluid in the radial direction (Bird *et al*., 2007):

$$-\frac{\partial p}{\partial r} - \frac{1}{r}\frac{\partial(r\tau_{rr})}{\partial r} - \frac{\partial \tau_{zr}}{\partial z} + \frac{\tau_{\theta\theta}}{r} + \rho_{nf} g_r = 0$$
$$\tau_{rr} = \left(\frac{2}{3}\mu_{nf} - \kappa_{nf}\right)\left(\frac{\partial u_z}{\partial z}\right),\ \tau_{zr} = -\mu_{nf}\left[\frac{\partial u_z}{\partial r}\right] \quad (3)$$
$$\tau_{\theta\theta} = \left(\frac{2}{3}\mu_{nf} - \kappa_{nf}\right)\left(\frac{\partial u_z}{\partial z}\right)$$

Where:
p = The nanofluid pressure
g = The gravitational acceleration
τ = The shear stress
μ_{nf} = The viscosity of the nanofluid
κ_{nf} = The bulk viscosity of the nanofluid

Momentum equation of the nanofluid in the axial direction:

$$\rho_{nf}\left(u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \frac{\partial \tau_{zz}}{\partial z} - \frac{1}{r}\frac{\partial(r\tau_{rz})}{\partial r}$$
$$\tau_{rz} = -\mu_{nf}\left[\frac{\partial u_z}{\partial r}\right],\ \tau_{zz} = -\mu_{nf}\left[2\frac{\partial u_z}{\partial z}\right] + \quad (5)$$
$$\left(\frac{2}{3}\mu_{nf} - \kappa_{nf}\right)\left(\frac{\partial u_z}{\partial z}\right)$$

Energy equation of the nanofluid:

$$\frac{\partial}{\partial z}\left(u_z(\rho c_p)_{nf} T\right) = -\begin{bmatrix}\frac{1}{r}\frac{\partial}{\partial r}\left(-rk_{nf}\frac{\partial T}{\partial r}\right) + \\ \frac{\partial}{\partial z}\left(-k_{nf}\frac{\partial T}{\partial z}\right)\end{bmatrix} \quad (5)$$

Where:
T = The nanofluid temperature
$(c_p)_{nf}$ = The specific heat of the nanofluid
k_{nf} = The thermal conductivity of the nanofluid

To calculate $(c_p)_{nf}$, researchers can use Eq. 6:

$$c_{p,nf} = \frac{\varphi(\rho_p c_{p,p}) + (1-\varphi)(\rho_f c_{p,f})}{\varphi\rho_p + (1-\varphi)\rho_f} \quad (6)$$

Where:
$c_{p,p}$ = The specific heat of the nanoparticle (it can be assumed as constant)
$c_{p,f}$ = The specific heat of the base fluid (function of temperature)
ρ_p = The density of the nanoparticle (it can be assumed as constant)
ρ_f = The density of the base fluid (function of temperature)

The density of the nanofluid can be calculated by the mixing rule as:

$$\rho_{nf} = \varphi\rho_p + (1-\varphi)\rho_f$$

In Eq. 5, thermal effects due to the nanoparticle has been incorporated in k_{nf}, as $k_{nf} = k_{static} + k_{dynamic}$. k_{static} is the static thermal conductivity when nanoparticles are stationary to the base fluid (classical models), as an example, it can be calculated by well-known model of Maxwell (1904):

$$k_{Maxwell} = \frac{k_p + 2k_f + 2(2k_p - k_f)\varphi}{k_p + 2k_f - (k_p - k_f)\varphi} k_f$$

Where:
k_f = Function of temperature is the thermal conductivity of base fluid
k_p = It can be assumed as constant is the thermal conductivity of nanoparticles

$k_{dynamic}$ is the enhanced thermal conductivity generated by movement of nanoparticles (Kleinstreuer and Feng, 2011). Jang and Choi (2004) derived following equation for $k_{dynamic}$ as:

$$k_{dynamic} = 3C_1\frac{d_{bf}}{d_p} Re_{d_p}^2 \Pr\varphi \quad (9)$$

Where:
C_1 = A proportional constant
d_{bf} = The diameter of the base fluid molecule
d_p = The diameter of the nanoparticle
Re_{dp} = The Reynolds number defined for random motion velocity of nanoparticles and Pr is Prandtl number

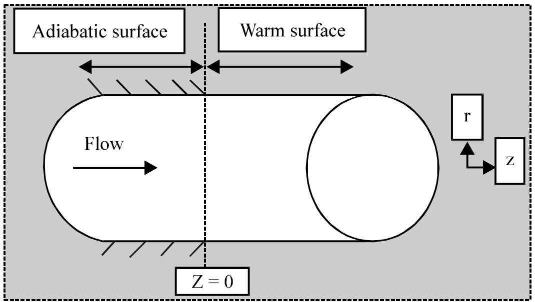

Fig. 1: The circular tube at z = 0 heat transfer begins

Unknowns in the partial differential equations (Eq. 1-5) are:

- Nanoparticle volume fraction (ϕ)
- Nanofluid temperature (T)
- Nanofluid pressure (ρ)
- Nanofluid axial velocity (u_z)

Boundary conditions of the problem are:

$$\left.\begin{array}{l} u_z = f(r) \\ T = T_i \\ p = h(r) \\ \phi = g(r) \end{array}\right\} \text{at } z = 0, \quad \left.\begin{array}{l} \dfrac{\partial T}{\partial r} = 0 \\ \dfrac{\partial u_z}{\partial r} = 0 \\ \dfrac{\partial p}{\partial r} = 0 \\ \dfrac{\partial \phi}{\partial r} = 0 \end{array}\right\} \text{at } r = 0 \tag{10}$$

$$\left.\begin{array}{l} \text{Thermal condition} \\ \text{in tube wall} \\ u_z = 0 \end{array}\right\} \text{at } r = R(\text{tube radius})$$

Thermal condition of tube surface is considered in the heat transfer case (e.g., constant wall temperature). Profiles of f(r), g(r) and h(r) are profiles of velocity, nanoparticle volume fraction and pressure at z = 0 (Fig. 1) which are functions of radius and T_i is the initial temperature of the nanofluid at z = 0.

RESULTS AND DISCUSSION

By considering Eq. 1, it can be obtained as:

$$\frac{\partial(\rho_{nf} u_z)}{\partial z} = 0 \rightarrow \rho_{nf} u_z = (\rho_{nf} u_z)_{z=0} = (\rho_{nf} u_z)_i \rightarrow u_z = \frac{(\rho_{nf} u_z)_i}{\rho_{nf}} \tag{11}$$

Subscript i denotes boundary conditions at z = 0, as shown in Fig. 1 which are given. Applying scaling analysis to Eq. 2, diffusion terms (j_z and j_r) can be neglected comparing with convective term as (Buongiorno, 2006):

$$\frac{\partial(\phi \rho_p u_z)}{\partial z} + \left[\frac{1}{r}\frac{\partial(r j_r)}{\partial r} + \frac{\partial(j_z)}{\partial z}\right] = 0 \rightarrow \frac{\partial(\phi \rho_p u_z)}{\partial z} = 0$$

$$\xrightarrow{\rho_p = \text{Constant}} \phi u_z = (\phi u_z)_{z=0} = (\phi u_z)_i$$

$$\rightarrow \phi = \frac{(\phi u_z)_i}{u_z} = \frac{(\phi u_z)_i}{\dfrac{(\rho_{nf} u_z)_i}{\rho_{nf}}}$$

$$\rho_{nf} \text{ from Eq. 7} \rightarrow \phi = \frac{(\phi)_i \rho_f}{(\rho_{nf})_i - (\phi)_i (\rho_p - \rho_f)}$$

$$\rightarrow \phi = \frac{(\phi)_i \rho_f}{(\rho_f)_i + (\phi)_i (\rho_f - (\rho_f)_i)} = \frac{g(r)}{\dfrac{(\rho_f)_{z=0}}{\rho_f}(1 - g(r)) + g(r)} \tag{12}$$

In Eq. 12, term $(\rho_f)_{z=0}$ is known since ρ_f is function of temperature and the temperature is given in z = 0, therefore ϕ in the thermal boundary layer depends on ρ_f. Since, $(\rho_f)_{z=0}/\rho_f>1\phi$ decreases in all points of the thermal boundary layer comparing with their corresponding point at z = 0 and with the same radius. With derivation of Eq. 12, researchers can obtain as:

$$\frac{d\varphi}{d\rho_f} = \frac{g(r)(1 - g(r))(\rho_f)_{z=0}}{(\rho_f)^2 \left(\dfrac{(\rho_f)_{z=0}}{\rho_f}(1 - g(r)) + g(r)\right)^2} > 0 \tag{13}$$

In Eq. 13, term (1 - g(r)) is positive since g(r) as nanoparticle volume fraction is always between 0 and 1 and therefore $d\phi/d\rho_f>0$.

$$\frac{d\varphi}{dT} = \overbrace{\frac{d\varphi}{d\rho_f}}^{\geq 0} \times \overbrace{\frac{d\rho_f}{dT}}^{\leq 0} \rightarrow \frac{d\varphi}{dT} < 0 \tag{14}$$

$$\frac{d\varphi}{dr} = \underbrace{\frac{d\varphi}{dT}}_{<0} \times \underbrace{\frac{dT}{dr}}_{>0} \rightarrow \frac{d\varphi}{dr} < 0, \quad \frac{d\varphi}{dz} = \underbrace{\frac{d\varphi}{dT}}_{<0} \times \underbrace{\frac{dT}{dz}}_{<0} \rightarrow \frac{d\varphi}{dz} > 0 \tag{15}$$

Equation 15 indicates that from center of the tube (r = 0) to surface of the tube (r = R) nanoparticle volume fraction (ϕ) decreases at each cross section of the tube. Using Eq. 12 for ϕ, u_z can be obtained as:

$$\left(u_z\right)_i = f(r),\ \left(\rho_{nf}\right)_i = \left(\varphi_i\rho_p + (1-\varphi_i)\left(\rho_f\right)_{T=T_i}\right),\ \ \varphi_i = g(r)$$

$$\rightarrow u_z = \frac{\left(\rho_{nf}u_z\right)_i}{\rho_{nf}} = f(r)\frac{\left(g(r)\rho_p + (1-g(r))\left(\rho_f\right)_{z=0}\right)}{\dfrac{g(r)}{\dfrac{\left(\rho_f\right)_{z=0}}{\rho_f}\left(1-g(r)\right)+g(r)}\left(\rho_p-\rho_f\right)+\rho_f} \quad (16)$$

Equation 16 shows that u_z has explicit dependency to the temperature since ρ_f is function of the temperature. Now to obtain temperature profile in the thermal boundary layer, only Eq. 5 must be solved since all parameters of it are functions of temperature as:

$$\frac{\partial}{\partial z}\left(\underbrace{u_z}_{\text{function of temperature}}\overbrace{\left(\rho c_p\right)_{nf}}^{\text{function of temperature}}T\right) = -\left[\frac{1}{r}\frac{\partial}{\partial r}\left(-r\underbrace{k_{nf}}_{\text{function of temperature}}\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(-k_{nf}\frac{\partial T}{\partial z}\right)\right] \quad (17)$$

Therefore, without coupling five differential equations all unknowns can be obtained, firstly temperature can be calculated by solving Eq. 17 then axial velocity (u_z) can be obtained by Eq. 9 since ρ_f is function of temperature. Using Eq. 4 and 5, the pressure distribution can be obtained.

CONCLUSION

One qualitative analysis of conservation laws of forced convective heat transfer of nanofluids in the fully developed laminar flow in a horizontal circular tube with warm surface in which thermal boundary layer grows has been presented. Mathematical analysis of conservation laws indicates that both profiles of nanoparticle volume fraction and axial velocity of the nanofluid depend on density of base fluid in whole of the tube. In other words when base fluid has constant thermophysical properties, distribution of nanoparticle volume fraction and axial velocity of nanofluid are unhanded comparing with corresponding boundary conditions.

REFERENCES

Bird, R.B., W.E. Stewart and E.N. Lightfoot, 2007. Transport Phenomena. 2nd Edn., John Wiley and Sons, New York.

Buongiorno, J., 2006. Convective transport in nanofluids. ASME J. Heat Transfer, 128: 240-250.

He, Y., Y. Mena, Y. Zhao, H. Lu and Y. Ding, 2009. Numerical investigation into the convective heat transfer of TiO2 nanofluids flowing through a straight tube under the laminar flow conditions. Applied Therm. Eng., 29: 1965-1972.

Jang, S.P. and S.U.S. Choi, 2004. Role of Brownian motion in the enhanced thermal conductivity of nanofluids. Applied Phys. Lett., Vol. 84 10.1063/1.1756684.

Kakac, S. and A. Pramuanjaroenkij, 2009. Review of convective heat transfer enhancement with nanofluids. Int. J. Heat Mass Transfer, 52: 3187-3196.

Kays, M.W. and M.E. Crawford, 1993. Convective Heat and Mass Transfer. 3nd Edn., McGraw-Hill Book Co., New York.

Kleinstreuer, C. and Y. Feng, 2011. Experimental and theoretical studies of nanofluid thermal conductivity enhancement: A review. Nanoscale Res. Lett., 10.1186/1556-276X-6-229.

Maxwell, J.C., 1904. A Treatise on Electricity and Magnetism Clarendon Press, Oxford, pp: 108-111.

Wang, X.Q. and A.S. Mujumdar, 2007. Heat transfer characteristics of nano?uids: A review. Int. J. Therm. Sci., 46: 1-19.

Xuan, Y. and W. Roetzel, 2000. Conceptions for heat transfer correlations of nanofluids. Int. J. Heat Mass Transfer, 43: 3701-3707.

Zhu, H., C. Zhang, S. Liu, Y. Tang and Y. Yin, 2006. Effects of nanoparticle clustering and alignment on thermal conductivities of Fe_3O_4 aqueous nanofluids. Applied Phys. Lett., 10.1063/1.2221905.