

## Performance Analysis of an Irreversible Dual Cycle Based on an Ecological Coefficient of Performance Criterion

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**Abstract:** The effects of compression, cut-off and pressure ratios on the performance of a dual cycle during the finite-time thermodynamic are investigated. In the model, the linear relation between the specific heat ratio of the working fluid and its temperature, the friction loss computed according to the mean velocity of the piston and the heat transfer loss are considered. The relations between thermal efficiency and compression ratio and between an Ecological Coefficient of Performance (ECOP) and compression ratio are derived. Moreover, the effects of cut-off and pressure ratios on the cycle performance are analyzed. The results show that the cut-off and pressure ratios are more effective on performance of a dual engine. Also, the points of maximum ECOP and thermal efficiency of dual cycle will decrease with an increase of cut-off and pressure ratios. The results are of importance to provide good guidance for the performance evaluation, improvement and design of practical dual engines.

**Key words:** Ecological coefficient, dual cycle, internal irreversibility, finite-time processes, design, Iran

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### INTRODUCTION

A series of achievements have been made since finite-time thermodynamics was used to simulate, analyze and optimize the performance of ideal thermodynamic processes (Abbassi *et al.*, 2012; Parlak, 2005), devices and cycles (Jesudason, 2009; Chen *et al.*, 2008). An interesting ecological optimization function was proposed by Angulo-Brown (1991) to assess the environmental impact of heat engines. Chen *et al.* (2004a) recently reported a study on the optimal ecological performance for generalized irreversible Carnot engines. In their study, the ecological-optimization criterion was considered as an objective function and the effects of finite-rate heat transfer, heat leakage and internal irreversibility on the optimal performance of cycle were investigated. Optimization studies for air-standard reciprocating cycles, i.e., otto, diesel and dual cycles with rate-dependent loss mechanisms have appeared as early as in the 1980s physics literature. In the fundamental analysis of modern diesel engines, the dual cycle is commonly employed as it includes the heat addition processes both at volume and constant pressure. Hoffman (1985) and Mozurkewich (1982) incorporated the major loss terms, such as; friction loss, heat leak and incomplete combustion in a simple model based on an air-standard cycle. Then using optimal control theory, the piston trajectory which yields maximum power output was computed. Aizenbud *et al.*

(1982) determined the optimal motion of a piston fitted to a cylinder which contained a gas pumped with a given heating rate and coupled to a heat bath for finite periods.

Approaches developed by Angulo-Brown *et al.* (1996) and Klein (1991) have been applied to air-standard diesel, otto and dual cycles by numerous researchers to determine the optimal conditions at maximum work/maximum power output. Two recent publications refer to applications for the dual cycle. Lin *et al.* (1999) considered the effect of heat transfer through a cylinder wall on the work output of a dual cycle and Chen *et al.* (2004b) carried out a thermodynamic performance analysis of an air-standard dual cycle with heat transfer and frictional losses.

The optimization of the ecological function is claimed to represent a compromise between the power output ( $\dot{w}$ ) and the loss power ( $T_0\dot{s}_g$ ) which is produced by entropy generation in the system and its surroundings and also between power output and the thermal efficiency. A common characteristic of the ecological function curves is that the function can take negative values. At this condition, loss power term is greater than the actual power produced. Therefore, it needs the interpretation to comprehend this situation thermodynamically. In a study, Ust (2004) proposed a new ecological objective function that always has positive values to identify the effect of loss rate of availability on the power output. The objective function is called the

Ecological Coefficient of Performance (ECOP) and defined as the power output per unit loss rate of availability, i.e.,  $ECOP = \dot{W}/T_0 \dot{S}_g$ .

As can be seen in the relevant literature, the investigation of the effects of compression, pressure and cut-off ratio on the performance of the dual cycle based on an ecological coefficient of performance criterion does not appear to have been published. Therefore, the objective of this study is to examine the effect of compression, pressure and cut-off ratio on the power output and the thermal efficiency of the air-standard dual cycle based on an ecological coefficient of performance.

### CYCLE ANALYSIS AND PERFORMANCE ANALYSIS

The Pressure-Volume (P-V) diagram of an irreversible dual cycle is shown in Fig. 1 where  $T_1$ - $T_5$  are the temperatures of the working fluid in state points 1-5. Process 1→2 is an isentropic compression. The heat addition occurs in the constant volume process 2→3 and the constant pressure process 3→4. The process 4→5 is an isentropic expansion process. Heat rejection occurs in the constant volume process 5→1.

As already mentioned in this study, it can be supposed that the specific heat ratio of the working fluid is a function of temperature alone and has the linear forms:

$$\gamma = \gamma_0 - k_1 T \quad (1)$$

Where:

$\gamma$  = The specific heat ratio ( $\gamma = c_p/c_v$ )

$T$  = The absolute temperature

$\gamma_0$  = Constant

$k_1$  = Constant

Assuming that the heat engine is operated at the rate of  $N$  revolutions per second, the heat added per second in the isochoric 2→3 heat addition process may be written as:

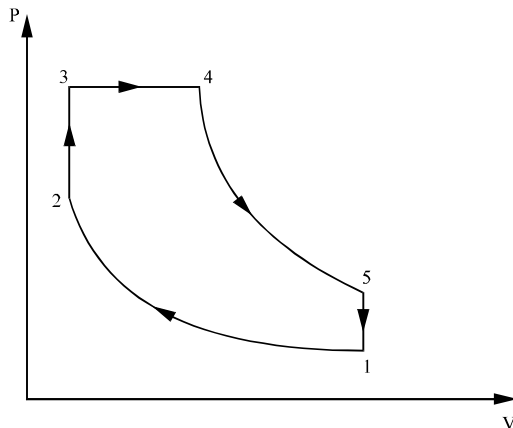


Fig. 1: P-V diagram for the air-standard dual cycle

$$\begin{aligned} \dot{Q}_{in} &= NM \left( \int_{T_2}^{T_3} c_v dT + \int_{T_3}^{T_4} c_p dT \right) \\ &= NM \left[ \int_{T_2}^{T_3} \left( \frac{R}{\gamma_0 - k_1 T - 1} \right) dT + \int_{T_3}^{T_4} \left( \frac{(\gamma_0 - k_1 T)R}{\gamma_0 - k_1 T - 1} \right) dT \right] \\ &= NMR \left[ T_4 - T_3 + \frac{1}{k_1} \ln \left( \frac{\gamma_0 - k_1 T_2 - 1}{\gamma_0 - k_1 T_4 - 1} \right) \right] \end{aligned} \quad (2)$$

Where:

$M$  = The molar number of the working fluid

$R$  = Molar gas constant at constant volume for the working fluid

$c_v$  = Molar specific heat at constant volume for the working fluid

The heat rejected per sec by the working fluid during processes 4→5 and 5→1 is:

$$\begin{aligned} \dot{Q}_{out} &= NM \int_{T_1}^{T_5} c_v dT = NM \int_{T_1}^{T_5} \left( \frac{R}{\gamma_0 - k_1 T - 1} \right) dT \\ &= \frac{NMR}{k_1} \ln \left( \frac{\gamma_0 - k_1 T_1 - 1}{\gamma_0 - k_1 T_5 - 1} \right) \end{aligned} \quad (3)$$

Where  $c_p$  is the molar specific heat at constant pressure for the working fluid.

According to Ge *et al.* (2005) and Ebrahimi (2010), the equation for a reversible adiabatic process with variable specific heat ratio can be written as:

$$TV^{\gamma-1} = (T + dT)(V + dV)^{\gamma-1} \quad (4)$$

Re-arranging Eq. 1 and 4, researchers get the following Eq. 5:

$$T_1(\gamma_0 - k_1 T_1 - 1) = T_2(\gamma_0 - k_1 T_2 - 1)(V_2/V_1)^{\gamma_0-1} \quad (5)$$

The compression ratio ( $r_c$ ), the pressure ratio ( $r_p$ ) and the cut-off ratio ( $\beta$ ) are defined as:

$$r_c = \frac{V_1}{V_2} \quad (6)$$

$$r_p = \frac{P_3}{P_2} = \frac{T_3}{T_2} \quad (7)$$

and;

$$\beta = \frac{V_4}{V_3} = \frac{T_4}{T_3} \quad (8)$$

Therefore, the equations for processes (1→2) and (4→5) are shown, respectively by Eq. 9:

$$T_1(\gamma_0 - k_1 T_2 - 1)(r_c)^{\gamma_0 - 1} = T_2(\gamma_0 - k_1 T_1 - 1) \quad (9)$$

and;

$$T_4(\gamma_0 - k_1 T_5 - 1) = T_5(\gamma_0 - k_1 T_4 - 1)\left(\frac{r_c}{\beta}\right)^{\gamma_0 - 1} \quad (10)$$

For an ideal Dual Cycle Model, there are no losses. However, for a real internal combustion engine cycle, the heat transfer irreversibility between the working fluid, the cylinder wall and friction like term loss are not negligible. The heat loss through the cylinder wall is assumed to be proportional to the average temperature of both the working fluid and the cylinder wall and the wall temperature is constant. The energy transferred to the working fluid during combustion is shown by the following linear relation (Chen *et al.*, 2006; Lin *et al.*, 1999):

$$\dot{Q}_{in} = NM[A - B(T_2 + T_4)] \quad (11)$$

Where A and B are two constants related to combustion and heat transfer which are functions of engine speed.

Taking into account the friction loss of the piston and assuming a dissipation term represented by a friction force that is a linear function of the piston velocity gives (Ebrahimi, 2011; Ceviz and Kaymaz, 2005):

$$f_\mu = \mu \bar{S}_p = \mu \frac{dx}{dt} \quad (12)$$

Where:

$\mu$  = Coefficient of friction that takes into account the global losses

$\bar{S}_p$  = The mean piston speed

$x$  = The piston displacement

Then, the lost power is:

$$P_\mu = \frac{dW_\mu}{dt} = \mu \frac{dx}{dt} \frac{dx}{dt} = \mu \bar{S}_p^2 \quad (13)$$

The net actual power output of the dual cycle engine can be written as:

$$\begin{aligned} P_{output} &= \dot{Q}_{in} - \dot{Q}_{out} - P_\mu \\ &= \frac{NMR}{k_1} \left[ \ln \left( \frac{(\gamma_0 - k_1 T_2 - 1)(\gamma_0 - k_1 T_5 - 1)}{(\gamma_0 - k_1 T_4 - 1)(\gamma_0 - k_1 T_1 - 1)} \right) + \right] - \mu \bar{S}_p^2 \\ &\quad \left[ \frac{1}{k_1 (T_4 - T_3)} \right] \end{aligned} \quad (14)$$

The thermal efficiency of the dual cycle engine is expressed by:

$$\eta_{th} = \frac{P_{output}}{\dot{Q}_{in}} = \frac{\frac{NMR}{k_1} \left[ \ln \left( \frac{(\gamma_0 - k_1 T_2 - 1)(\gamma_0 - k_1 T_5 - 1)}{(\gamma_0 - k_1 T_4 - 1)(\gamma_0 - k_1 T_1 - 1)} \right) + \right] - \mu \bar{S}_p^2}{\frac{MR}{k_1} \left[ \ln \left( \frac{(\gamma_0 - k_1 T_2 - 1)}{(\gamma_0 - k_1 T_4 - 1)} \right) + \right] \frac{1}{k_1 (T_4 - T_3)}} \quad (15)$$

The entropy generation rate of the dual cycle is:

$$\dot{S}_g = \frac{\dot{Q}_{LT}}{T_L} - \frac{\dot{Q}_{HT}}{T_H} \quad (16)$$

The ECOP (Ecological Coefficient of Performance) is defined as the ratio of power output to the loss rate of availability, i.e.:

$$ECOP = \frac{P_{output}}{T_0 \dot{S}_g} \quad (17)$$

Where  $T_0$  is the environment temperature and it is assumed to be equal to the cold reservoir temperature,  $T_L$  and researchers assume  $T_L$  is equal to the minimum temperature of the cycle,  $T_1$  ( $T_L = T_1$ ) and  $T_H$  is the temperature of the hot reservoir and it is assumed to be equal to the maximum temperature of the cycle,  $T_4$  ( $T_H = T_4$ ).

Also, researchers assume that  $\dot{Q}_{LT} = \dot{Q}_{out}$ ,  $\dot{Q}_{HT} = \dot{Q}_{in}$ . Thus, the entropy generation rate and ecological coefficient of performance of the dual cycle can be written as:

$$\dot{S}_g = \frac{NMR}{k_1} \left[ \frac{1}{T_1} \ln \left( \frac{\gamma_0 - k_1 T_1 - 1}{\gamma_0 - k_1 T_5 - 1} \right) - \frac{k_1}{T_4} (T_4 - T_3) \right] - \frac{1}{T_4} \ln \left( \frac{\gamma_0 - k_1 T_2 - 1}{\gamma_0 - k_1 T_4 - 1} \right) \quad (18)$$

$$ECOP = \frac{\frac{NMR}{k_1} \left[ \ln \left( \frac{(\gamma_0 - k_1 T_2 - 1)(\gamma_0 - k_1 T_5 - 1)}{(\gamma_0 - k_1 T_4 - 1)(\gamma_0 - k_1 T_1 - 1)} \right) + \right] - \mu \bar{S}_p^2}{\frac{NMR}{k_1} \left[ \ln \left( \frac{\gamma_0 - k_1 T_1 - 1}{\gamma_0 - k_1 T_5 - 1} \right) - \frac{T_1 k_1}{T_4} (T_4 - T_3) - \frac{T_1}{T_4} \ln \left( \frac{\gamma_0 - k_1 T_2 - 1}{\gamma_0 - k_1 T_4 - 1} \right) \right]} \quad (19)$$

When  $r_c$ ,  $\beta$ ,  $r_p$  and  $T_1$  are given,  $T_2$  can be obtained from Eq. 9 and  $T_3$  can be found from Eq. 7 then substituting Eq. 2 into Eq. 11 yields  $T_4$ ; at last,  $T_5$  can be found from Eq. 10. Substituting  $T_1$ - $T_5$  into Eq. 14, 15 and

19 yields the power output, thermal efficiency and the ECOP. Therefore, the relations between the power output, the thermal efficiency and the ECOP can be derived.

### NUMERICAL EXAMPLE AND DISCUSSION

The following constants and parameters have been used in this exercise:  $r_p = 1.05-1.2$ ,  $T_1 = 300$  K,  $B = 25$  Jmol<sup>-1</sup> k<sup>-1</sup>,  $\bar{s}_p = 10$  msec<sup>-1</sup>,  $A = 60000$  Jmol<sup>-1</sup>,  $\mu = 12.9$  NS m<sup>-1</sup>,  $M = 1.57 \times 10^{-5}$  kmol,  $k_1 = 0.00008$  k<sup>-1</sup>,  $\beta = 1.5-2.5$ ,  $\gamma_0 = 1.4$  and  $N = 3000$  rpm,  $R = 8.314$  Jmol<sup>-1</sup> k<sup>-1</sup> (Ust, 2004; Ge *et al.*, 2005; Ebrahimi, 2010; Chen *et al.*, 2006; Lin *et al.*, 1999; Ebrahimi, 2011; Ceviz and Kaymaz, 2005).

Using the above constants, the thermal efficiency versus compression ratio characteristic, the ECOP versus compression ratio characteristic and the ECOP versus the power output characteristic can be plotted as in Fig. 2-7. Figure 2-7 show the effects of the variable specific heat ratio of the working fluid on the performance of the cycle with heat resistance and frictional losses. Compression ratio ( $r_c$ ), pressure ratio ( $r_p$ ) and cut-off ratio ( $\beta$ ) are the principal engine design parameters. Therefore, investigation of effects of these parameters is important.

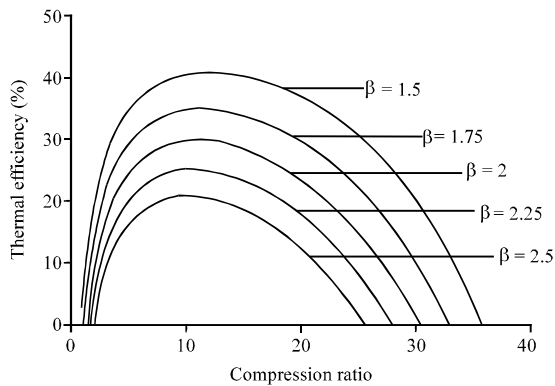


Fig. 2: Effect of cut-off ratio on  $\eta$ - $r_c$  characteristic for  $r_p = 1.1$

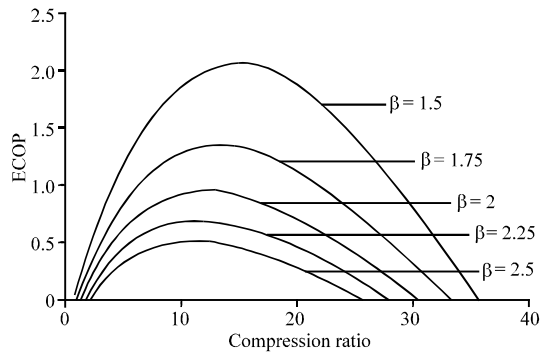


Fig. 3: Effect of cut-off ratio on ECOP- $r_c$  characteristic for  $r_p = 1.1$

Thus in Fig. 2-7 effects of compression, pressure and cut-off ratio are investigated. They reflect the performance characteristics of an irreversible dual cycle engine.

Figure 2-4 show the influence of the cut-off ratio on the dual cycle performance. It can be shown that the cut-off ratio plays a significant role on the dual cycle performance. The point of maximum thermal efficiency decreases with increasing cut-off ratio: When  $\beta$  increases

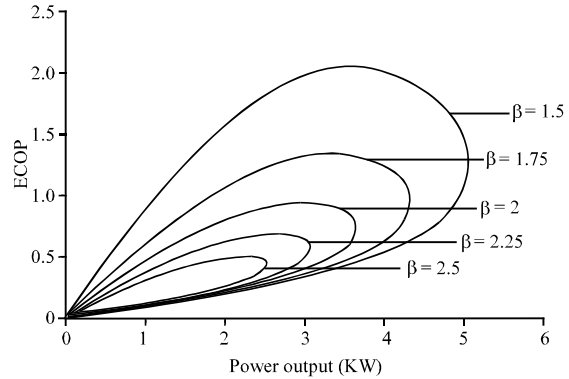


Fig. 4: Effect of cut-off ratio on ECOP- $P_{\text{output}}$  characteristic for  $r_p = 1.1$

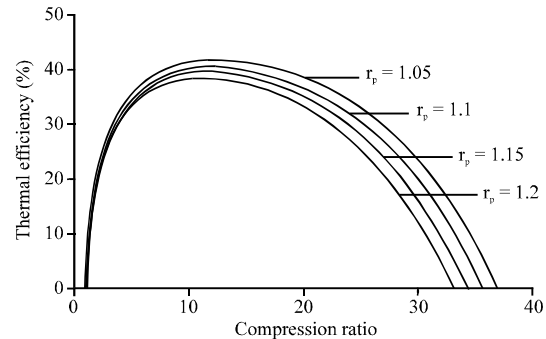


Fig. 5: Effect of pressure ratio on  $\eta$ - $r_c$  characteristic for  $\beta = 1.5$

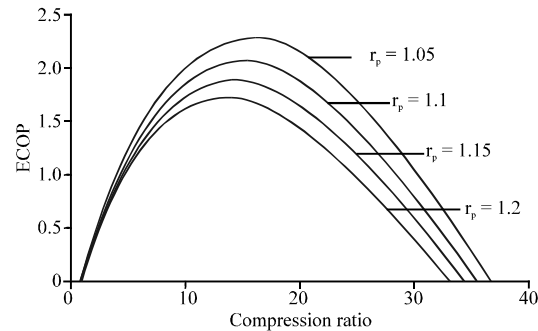


Fig. 6: Effect of pressure ratio on ECOP- $r_c$  characteristic for  $\beta = 1.5$

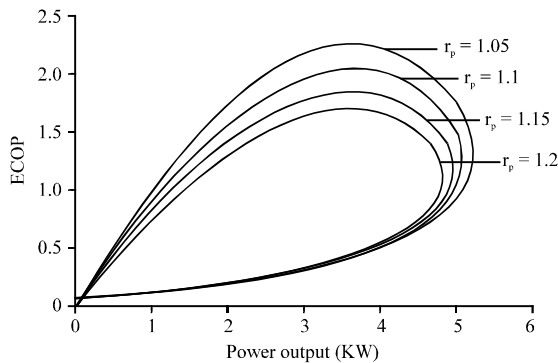


Fig. 7: Effect of pressure ratio on ECOP- $P_{\text{output}}$  characteristic for  $\beta = 1.5$

from 1.5-2.5, the points of maximum thermal efficiency of dual cycle will decrease from 40.65-20.79% (48.86%). Also, the ECOP of dual cycle decreases sharply with increase of  $\beta$ , when  $\beta$  increases from 1.5-2.5, the points of ECOP of dual cycle will decrease from 2.06-0.506 (75.43%). The influence of the cut-off ratio on the ECOP versus power output is displayed in Fig. 4. As can be seen from Fig. 4, the ECOP versus power output is a loop shaped. It can be seen that the ECOP at maximum power output improves with decreasing cut-off ratio from 2.5-1.5. However, researchers must be determining optimal compression ratio based on thermal efficiency and ECOP. For example, if  $\beta = 1.5$ , the points of maximum thermal efficiency and ECOP occurs at compression ratio of 12 and 15, respectively. Also, the results show that the point of maximum thermal efficiency and ECOP occurs at the lower compression ratio with increase of cut-off ratio.

Figure 5-7 show the influence of the pressure ratio on the dual cycle performance. It can be shown that the pressure ratio plays a significant role on the dual cycle performance, too. The maximum thermal efficiency decreases with increasing pressure ratio: When  $r_p$  increases from 1.05-1.2, the points of maximum thermal efficiency of dual cycle will decrease from 41.75-38.46% (7.88%). Also, the ECOP of dual cycle decreases sharply with increase of  $\beta$ : When  $\beta$  increases from 1.05-1.2, the points of ECOP of dual cycle will decrease from 2.276-1.71 (24.67%). The influence of the pressure ratio on the ECOP versus power output is shown in Fig. 7. As can be seen Fig. 7, the ECOP versus power output is a loop shaped. It can be seen that the ECOP at maximum power output improves with decreasing pressure ratio from 1.2-1.05. However, we must be determining optimal compression ratio based on thermal efficiency and ECOP. For example, if  $r_p = 1.05$ , the points of maximum thermal efficiency and ECOP occurs at compression ratio of 12.5 and 15.5, respectively. Also, the results show that the

point of maximum thermal efficiency and ECOP occurs at the lower compression ratio with increase of pressure ratio.

## CONCLUSION

This study is aimed at investigating the effects of compression, cut-off and pressure ratios on the dual cycle's performance based on an ecological coefficient of performance. By using finite-time thermodynamics theory, the characteristic curves of the thermal efficiency versus compression ratio, ECOP versus compression ratio and the ECOP versus power output are obtained. In the model, the linear relation between the specific heats ratio of working fluid and its temperature, the frictional loss computed according to the mean velocity of piston and heat transfer loss are considered. The general conclusions drawn from the results of this research are as follows:

- The cut-off and pressure ratios play a significant role on the dual cycle performance
- The points of maximum ECOP and thermal efficiency of dual cycle will decrease with an increase of cut-off and pressure ratios
- The point of maximum thermal efficiency and ECOP occurs at the lower compression ratio with increases of cut-off and pressure ratios
- The results of this investigation are of importance when considering the designs of dual engines

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