Journal of Engineering and Applied Sciences 7 (1): 86-89, 2012
ISSN: 1816-949X

Pairwise Semi Star Generalized Homeomorphisms

K. Kannan, D. Narasimhan and K. Chandrasekhara Rao
Department of Mathematics, Srinivasa Ramanujan Centre,
SASTRA University, 612001 Kumbakonam, India

Abstract: Homeomorphism helps us to determine the properties of complicated spaces easily. In this sequel, the aim of this short communication is introduce and study two new types of homeomorphisms, namely; pairwise s*g-homeomorphism and pairwise s*gc-homeomorphism in bitopological spaces. Here researchers proved that the set of all pairwise s*gc-homeomorphisms forms a group which contains the set of all pairwise homeomorphisms as a subgroup. Furthermore, the relationships between newly introduced and existing homeomorphisms are established.

Key words: Pairwise s*g-homeomorphism, pairwise s*gc-homeomorphism, pairwise g-homeomorphism, pairwise sg-homeomorphism, pairwise gs-homeomorphism, sequel, India

INTRODUCTION

Kelly (1963) initiated the study of bitopological space. Thereafter, several researches (Bose, 1995; Rao and Kannan, 2005; Rao *et al*., 2007; Fukutake, 1986, 1989; Kannan *et al*., 2010a, b; Ravi and Thivagar, 2006; Maheshwari and Prasad, 1977; John and Sundaram, 2004) has been made for converting topological concepts to the bitopological spaces.

Homeomorphism is one of the important concepts in topological spaces. It helps us to determine the properties of complicated spaces easily. Semi homeomorphisms were introduced by Biswas (1969) and Crossley and Hildebrand (1971).

Generalized homeomorphisms and gc-homeomorphisms in terms of preserving generalized closed sets (Levine, 1970) were first introduced by Maki *et al*. (1991).

Every homeomorphism is a generalized homeomorphism but not vice versa (Maki *et al*., 1993b). The two concepts coincide when both the domain and the range satisfy the weak separation axiom $T_{1/2}$. The class of gc-homeomorphism is properly placed between the classes of homeomorphism and g-homeomorphism (Maki *et al*., 1991). Semi continuous mappings (Biswas, 1970; Levine, 1963) and sg-continuous mappings (Caldas, 1995) were studied. Sg-homeomorphisms and gs-homeomorphisms (Maki *et al*., 1995) and sg-closed maps and gs-closed maps (Maki *et al*., 1993a) were introduced by Bhattacharya and Lahiri (1987) with the help of sg-closed sets and gs-closed sets. Rao and Joseph (2000) introduced the concepts of semi star generalized closed sets and semi star generalized homeomorphisms (Rao and Joseph, 2007) in topological spaces. The aim of this short communication is introduce and study two new types of homeomorphisms, namely; pairwise s*g-homeomorphism and pairwise s*gc-homeomorphism in bitopological spaces. Here researchers proved that the set of all pairwise s*gc-homeomorphisms forms a group which contains the set of all pairwise homeomorphisms as a subgroup. Furthermore, the relationships between newly introduced and existing homeomorphisms are established.

PRELIMINARIES

Let (X, τ_1, τ_2), (Y, σ_1, σ_2) or simply X, Y denote a bitopological space. For any subset $A \subseteq X$, τ_i-int(A) and τ_i-cl(A) denote the interior and closure of a set A with respect to the topology τ_i, respectively. A^c denotes the complement of A in X unless explicitly stated.

A set A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-g closed {resp. $\tau_1\tau_2$-sg closed, $\tau_1\tau_2$-gs closed, $\tau_1\tau_2$-s*g closed} if τ_2-cl(A) {resp. τ_2-scl(A), τ_2-scl(A), τ_2-cl (A)}$\subseteq$U whenever $A \subseteq U$ and U is τ_1-open {resp. τ_1-semi open, τ_1-open, τ_1-semi open}. A function f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is pairwise g-continuous {resp. pairwise sg-continuous, pairwise gs-continuous, pairwise s*g-continuous} if $f^{-1}(U)$ is $\tau_i\tau_j$-g closed {resp. $\tau_i\tau_j$-sg closed, $\tau_i\tau_j$-gs closed, $\tau_i\tau_j$-s*g closed} for each σ_j-closed set U in Y, $i \neq j$ and i, j = 1, 2.

A function f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is pairwise g-irresolute {resp. pairwise sg-irresolute, pairwise gs-irresolute, pairwise s*g-irresolute} if $f^{-1}(U)$ is $\tau_i\tau_j$-g closed

Corresponding Author: K. Kannan, Department of Mathematics, Srinivasa Ramanujan Centre, SASTRA University, 612001 Kumbakonam, India

{resp. $\tau_i\tau_j$-sg closed, $\tau_i\tau_j$-gs closed, $\tau_i\tau_j$-s*g closed} for each $\sigma_i\sigma_j$-g closed {resp. $\sigma_i\sigma_j$-sg closed, $\sigma_i\sigma_j$-gs closed, $\sigma_i\sigma_j$-s*g closed} set U in Y, $i \neq j$ and i, j = 1, 2. A function f: $(X, \tau_1, \tau_2)\to(Y, \sigma_1, \sigma_2)$ is pairwise s*g-closed {resp. pairwise s*g-open} if f(U) is $\sigma_i\sigma_j$-s*g closed {resp. $\sigma_i\sigma_j$-s*g open} for each τ_j-closed {resp. τ_j-open} set U in X, $i \neq j$ and i, j = 1, 2. A bijection function f: $(X, \tau_1, \tau_2)\to(Y, \sigma_1, \sigma_2)$ is pairwise homeomorphism {resp. pairwise g-homeomorphism, pairwise sg-homeomorphism, pairwise gs-homeomorphism} if both f and f^{-1} are pairwise continuous {resp. pairwise g-continuous, pairwise sg-continuous, pairwise gs-continuous}. A bijection function f: $(X, \tau_1, \tau_2)\to(Y, \sigma_1, \sigma_2)$ is pairwise gc-homeomorphism {resp. pairwise sgc-homeomorphism, pairwise gsc-homeomorphism} if both f and f^{-1} are pairwise g-irresolute {resp. pairwise sg-irresolute, pairwise gs-irresolute}.

A bitopological space (X, τ_1, τ_2) is called pairwise $T_{1/2}$ -space {resp. pairwise T_b-space if every $\tau_1\tau_2$-g closed set {resp. $\tau_1\tau_2$-gs closed set} is τ_2-closed and every $\tau_2\tau_1$-g closed set {resp. $\tau_2\tau_1$-gs closed set} is τ_1-closed.

PAIRWISE SEMI STAR GENERALIZED HOMEOMORPHISMS

Definition: A bijection f: $(X, \tau_1, \tau_2)\to(Y, \sigma_1, \sigma_2)$ is pairwise semi star generalized homeomorphism (pairwise s*g-homeomorphism) if f is both pairwise s*g-continuous and pairwise s*g-open.

Example 1: Let X = Y = {a, b, c}, τ_1 = {ϕ, X {a} }, τ_2 = {ϕ, X , {b , c}}, σ_1 = {ϕ, Y {a}}, σ_2 = {ϕ, Y {a}, {b, c}}. Let f: $(X, \tau_1, \tau_2)\to(Y, \sigma_1, \sigma_2)$ be a identity mapping. Then f is pairwise s*g-homeomorphism.

Theorem:

- Every pairwise homeomorphism is pairwise s*g-homeomorphism
- Every pairwise s*g-homeomorphism is pairwise g-homeomorphism
- Every pairwise s*g-homeomorphism is pairwise sg-homeomorphism
- Every pairwise s*g-homeomorphism is pairwise gs-homeomorphism

Proof: Obvious from definitions. But the converses of the assertions of the above theorem are not true in general as can be seen from the following examples.

Example 2: In example 1, f is pairwise s*g-homeomorphism. But it is not pairwise continuous and hence, it is not pairwise homeomorphism.

Example 3: Let X = Y = {a , b, c}, τ_1 = {ϕ, X {a}}, τ_2 = {ϕ, X {b, c}}, σ_1 = {ϕ, Y {a}}, σ_2 = {ϕ, Y {a} , {b , c}}. Let f: $(X, \tau_1, \tau_2)\to(Y, \sigma_1, \sigma_2)$ be a function defined by f (a) = b, f (b) = a, f (c) = c. Then f is pairwise g-homeomorphism, pairwise gs-homeomorphism but not pairwise s*g-homeomorphism.

Example 4: Let X = Y = {a, b, c}, τ_1 = {ϕ, X {a}, {b}, {c}, {a, b}, {a, c}, {b, c}}, τ_2 = {ϕ, X, {a}}, σ_1 = {ϕ, Y {b}}, σ_2 = {ϕ, Y {a, c}}. Let f: $(X, \tau_1, \tau_2)\to(Y, \sigma_1, \sigma_2)$ be a function defined by f (a) = b, f (b) = a, f (c) = c. Then f is pairwise sg-homeomorphism but not pairwise s*g-homeomorphism.

However, every pairwise s*g-homeomorphism from a pairwise $T_{1/2}$-space onto itself is a pairwise homeomorphism and every pairwise g-homeomorphism from a pairwise $T_{1/2}$-space onto itself is a pairwise s*g-homeomorphism. Also, every pairwise sg-homeomorphism {resp. pairwise gs-homeomorphism} from a pairwise T_b-space onto itself is a pairwise s*g-homeomorphism. Concerning the compositions of the maps, the composition of two pairwise s*g-homeomorphisms is not a pairwise s*g-homeomorphism. Also, one can easily verify that pairwise s*g-homeomorphisms and pairwise gc-homeomorphisms {resp. pairwise sgc-homeomorphisms and pairwise gsc-homeomorphisms} are independent each other.

But if X and Y are pairwise $T_{1/2}$-spaces then the function f: $(X, \tau_1, \tau_2)\to(Y, \sigma_1, \sigma_2)$ is pairwise gc-homeomorphism if and only if f is pairwise s*g-homeomorphism. Also if X and Y are pairwise T_b-spaces then the map f: $(X, \tau_1, \tau_2)\to(Y, \sigma_1, \sigma_2)$ is pairwise s*g-homeomorphism if and only if f is pairwise sgc-homeomorphism {resp. pairwise gsc-homeomorphism}. Moreover for any bijection f: $(X, \tau_1, \tau_2)\to(Y, \sigma_1, \sigma_2)$, the following are equivalent:

- f^{-1}: $(Y, \sigma_1, \sigma_2)\to(X, \tau_1, \tau_2)$ is pairwise s*g-continuous
- f is pairwise s*g-open
- f is pairwise s*g-closed

Also for any bijection and pairwise s*g-continuous function f: $(X, \tau_1, \tau_2)\to(Y, \sigma_1, \sigma_2)$, the following are equivalent:

- f is pairwise s*g-open
- f is pairwise s*g-homeomorphism
- f is pairwise s*g-closed

Definition: A bijection f: $(X, \tau_1, \tau_2)\to(Y, \sigma_1, \sigma_2)$ is pairwise s*gc-homeomorphism if f and f^{-1} are pairwise s*g-irresolute.

Example 5: Let X = Y = {a, b, c}, τ_1 = {ϕ, X {a}}}, τ_2 = {ϕ, X {b, c}}, σ_1 = {ϕ, Y {b}}, σ_2 = {ϕ, Y {a, c}}. Let f: (X, τ_1, τ_2)→(Y, σ_1, σ_2) be a function defined by f (a) = b, f (b) = a, f (c) = c. Then f is pairwise s*gc-homeomorphism.

Theorem: Let f: (X, τ_1, τ_2)→(Y, σ_1, σ_2) be a map. Then the following are true:

- Every pairwise homeomorphism is pairwise s*gc-homeomorphism
- Every pairwise s*gc-homeomorphism is pairwise g-homeomorphism
- Every pairwise s*gc-homeomorphism is pairwise sg-homeomorphism
- Every pairwise s*gc-homeomorphism is pairwise gs-homeomorphism
- Every pairwise s*gc-homeomorphism is pairwise s*g-homeomorphism

Proof: Obvious from definitions. But the converses of the assertions of the above theorem are not true in general as can be seen from the following examples.

Example 6: In this example, f is pairwise g-homeomorphism, pairwise gs-homeomorphism but not pairwise s*gc-homeomorphism. In example 1, f is pairwise s*g-homeomorphism. But it is not pairwise s*gc-homeomorphism. In example 4, f is pairwise sg-homeomorphism. But it is not pairwise s*gc-homeomorphism.

However, every pairwise s*gc-homeomorphism from a pairwise $T_{1/2}$-space onto itself is a pairwise homeomorphism every pairwise g-homeomorphism from a pairwise $T_{1/2}$-space onto itself is a pairwise s*gc-homeomorphism and every pairwise s*g-homeomorphism from a pairwise $T_{1/2}$-space onto itself is a s*gc-pairwise homeomorphism. Also, every pairwise sg-homeomorphism {resp. pairwise gs-homeomorphism} from a pairwise T_b-space onto itself is a pairwise s*gc-homeomorphism. Concerning the compositions of the maps, the composition of two pairwise s*gc-homeomorphisms is always a pairwise s*gc-homeomorphism. Also, one can easily verify that pairwise s*gc-homeomorphisms and pairwise gc-homeomorphisms {resp. pairwise sgc-homeomorphisms and pairwise gsc-homeomorphisms} are independent each other.

But if X and Y are pairwise $T_{1/2}$-spaces then the function f: (X, τ_1, τ_2)→(Y, σ_1, σ_2) is pairwise gc-homeomorphism if and only if f is pairwise s*gc-homeomorphism. Also if X and Y are pairwise T_b-spaces then the map f: (X, τ_1, τ_2)→(Y, σ_1, σ_2) is pairwise s*gc-homeomorphism if and only if f is pairwise sgc-homeomorphism {resp. pairwise gsc-homeomorphism}. In order to state the algebraic structure of the set of all pairwise s*g-homeomorphisms and pairwise s*gc-homeomorphisms, we introduce the following notations:

- a) ph(X, τ_1, τ_2) = {f: f: (X, τ_1, τ_2)→(Y, σ_1, σ_2) is pairwise homeomorphism}
- b) ps*gch(X, τ_1, τ_2) = {f: f: (X, τ_1, τ_2)→(Y, σ_1, σ_2) is pairwise s*gc-homeomorphism}
- c) ps*gh(X, τ_1, τ_2) = {f: f: (X, τ_1, τ_2)→(Y, σ_1, σ_2) is pairwise s*g-homeomorphism}
- d) pgh(X, τ1, τ_2) = {f: f: (X, τ_1, τ_2)→(Y, σ_1, σ_2) is pairwise g-homeomorphism}
- e) psgh(X, τ_1, τ_2) = {f: f: (X, τ_1, τ_2)→(Y, σ_1, σ_2) is pairwise sg-homeomorphism}
- f) pgsh(X, τ_1, τ_2) = {f: f: (X, τ_1, τ_2)→(Y, σ_1, σ_2) is pairwise gs-homeomorphism}

Theorem:

- a) ph(X, τ_1, τ_2)⊆ps*gch(X, τ_1, τ_2)⊆ps*gh(X, τ_1, τ_2)⊆ pgsh(X, τ_1, τ_2)
- b) ps*gh(X, τ_1, τ_2)⊆psgh(X, τ_1, τ_2)
- c) ps*gh(X, τ_1, τ_2)⊆pgh(X, τ_1, τ_2)
- d) the set ps*gch(X, τ_1, τ_2) is a group which contains ph(X, τ_1, τ_2) as a subgroup

Proof: a-c are obvious and d for f, g∈ps*gch(X, τ_1, τ_2), we define a binary operation μ: ps*gch(X, τ_1, τ_2)× ps*gch(X, τ_1, τ_2)→ps*gch(X, τ_1, τ_2) such that μ(f, g) = the composition of f and g, namely; g, o, f. Since, the composition of two pairwise s*gc-homeomorphisms is always a pairwise s*gc-homeomorphism, it is obvious that the closure and associative properties are true under μ. The proof of the existence of identity and inverse property is left to the reader. Since, ph(X, τ_1, τ_2)⊆ ps*gch(X, τ_1, τ_2) and ph(X, τ_1, τ_2) is also a group with μ, ph(X, τ_1, τ_2) is a sub group of ps*gch(X, τ_1, τ_2).

Theorem: Let f: (X, τ_1, τ_2)→(Y, σ_1, σ_2) is pairwise s*gc-homeomorphism. Then, it induces an isomorphism from the group ps*gch(X, τ_1, τ_2) onto ps*gch(Y, σ_1, σ_2).

Proof: The pairwise homomorphism f_*: ps*gch (X, τ_1, τ_2)→ ps*gch (X, τ_1, τ_2) is induced from f by f*(h) = fohof^{-1} for every h∈ps*gch(X, τ_1, τ_2). The f* is a bijection and also is an pairwise isomorphism by usual argument.

CONCLUSION

Thus, the study of two new types of homeomorphisms, namely; pairwise s*g-homeomorphism and pairwise s*gc-homeomorphism in bitopological spaces are done in this study. Bitopological spaces have some applications in the study of digraphs and hence, this study of these two homeomorphisms may yield some improvements and preserving theorems in this area. The further research in this direction is undergoing.

REFERENCES

Bhattacharya, P. and B.K. Lahiri, 1987. Semi-generalized closed sets in topology. Indian J. Math., 29: 375-382.

Biswas, N., 1969. On some mappings in topological spaces bull. Calcutta 61: 127-135.

Biswas, N., 1970. On Characterization of semi-continuous function. Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., 48: 399-402.

Bose, S., 1995. Semi open sets, semi continuity and semi open mappings in bitopological spaces. Bull. Cal. Math. Soc., 73: 237-246.

Caldas, M.C., 1995. Semi-generalized continuous maps in topological spaces. Portugal. Math., 52: 399-407.

Crossley, S.G. and S.K. Hildebrand, 1971. Semi-closure. Texas J. Sci., 22: 99-112.

Fukutake, T., 1986. On generalized closed sets in bitopological spaces. Bull. Fukuoka Univ. Ed. Part III, 35: 19-28.

Fukutake, T., 1989. Semi open sets on bitopological spaces. Bull. Fukuoka Uni. Educ., 38: 1-7.

John, M.S. and P. Sundaram, 2004. g-closed sets in bitopological spaces. Indian J. Pure Applied Math., 35: 71-80.

Kannan, K., D. Narasimhan, K.C. Rao and M. Sundararaman, 2010a. \tau_1\tau_2-s^*-Semi star generalized closed sets in bitopological spaces. J. Adv. Res. Pure Math., 2: 34-47.

Kannan, K. D. Narasimhan and K.C. Rao, 2010b. On semi star generalized closed sets in bitopological spaces. Bol. Soc. Paranaensede Mat., 28: 29-40.

Kelly, J.C., 1963. Bitopological spaces. Proc. London Math. Soc., 13: 71-89.

Levine, N., 1970. Generalized closed sets in topology. Rend. Circ. Mat. Palermo, 19: 89-96.

Levine, N., 1963. Semi-open sets and semi-continuity in topological spaces. Amer. Math. Monthly, 70: 36-41.

Maheshwari, S.N. and R. Prasad, 1977. Semiopen sets and semi continuous functions in bitopological spaces. Math. Note, 26: 29-37.

Maki, H., P. Sundaram and K. Balachandran, 1991. On generalized homeomorphisms in topological spaces. Bull. Fukuoka Univ. Ed. III, 40: 13-21.

Maki, H., R. Devi and K. Balachandran, 1993a. Semi-generalized closed maps and generalized semi-closed maps. Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 14: 41-54.

Maki, H., R. Devi and K. Balachandran, 1993b. Generalized _-closed sets in topology. Bull. Fukuoka Univ. Ed. Part III, 42: 13-21.

Maki, H., R. Devi and K. Balachandran, 1995. Semi-generalized homeomorphisms and generalized semi-homeomorphisms in topological spaces. Indian J. Pure Appl. Math., 26: 271-284.

Rao, K.C. and K. Joseph, 2000. Semi star generalized closed sets in topological spaces. Bull. Pure Applied Sci., 19: 281-290.

Rao, K.C. and K. Joseph, 2007. Semi star generalized homeomorphisms in topological spaces. Applied Sci. Periodical, 3: 176-187.

Rao, K.C. and K. Kannan, 2005. Semi star generalized closed sets and semi star generalized open sets in bitopological spaces. Varahimir J. Math., 5: 473-485.

Rao, K.C., K. Kannan and D. Narasimhan, 2007. Characterizations of \tau_1\tau_2-s^*g losed sets. Acta Ciencia Indica, 3: 807-810.

Ravi, O. and M.L. Thivagar, 2006. A bitopological (1, 2)*semi-generalised continuous maps. Bull. Malays. Math. Sci. Soc., 29: 79-88.