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Adaptive Anti-Synchronization of Uncertain Tigan and Li Systems

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Abstract: In this study, researchers apply adaptive control method to derive new results for the anti-synchronization of identical Tigan Systems (2008), identical Li Systems (2009) and non-identical Tigan and Li Systems. In adaptive anti-synchronization of identical chaotic systems, the parameters of the master and slave systems are unknown and researchers devise feedback control law using the estimates of the system parameters. In adaptive anti-synchronization of non-identical chaotic systems, the parameters of the master system are known but the parameters of the slave system are unknown and researchers devise feedback control law using the estimates of the parameters of the slave system. The adaptive synchronization results derived in this study for the uncertain Tigan and Li Systems are established using Lyapunov Stability Theory. Since, the Lyapunov exponents are not required for these calculations, the adaptive control method is very effective and convenient to achieve anti-synchronization of identical and non-identical Tigan and Li Systems. Numerical simulations are shown to demonstrate the effectiveness of the adaptive anti-synchronization schemes for the uncertain chaotic systems addressed in this study.

Key words: Anti-synchronization, adaptive control, Tigan System, Li System, master system, India

INTRODUCTION

Chaotic systems are non-linear dynamical systems that are highly sensitive to initial conditions. This sensitivity is popularly known as the butterfly effect (Alligood *et al*., 1997). Since, the pioneering research by Pecora and Carroll (1990), chaos synchronization and anti-synchronization problems have been studied extensively and intensively in the literature (Pecora and Carroll, 1990; Lakshmanan and Murali, 1996; Han *et al*., 1995; Blasius *et al*., 1999; Cuomo and Oppenheim, 1993; Kocarev and Parlitz, 1995; Tao, 1999; Ott *et al*., 1990; Ho and Hung, 2002; Huang *et al*., 2004; Chen, 2005; Sundarapandian and Karthikeyan, 2011a, b; Lu *et al*., 2004; Chen and Lu, 2002; Park and Kwon, 2003). Chaos theory has been applied to a variety of fields such as physical systems (Lakshmanan and Murali, 1996), chemical systems (Han *et al*., 1995), ecological system (Blasius *et al*., 1999), secure communications (Cuomo and Oppenheim, 1993; Kocarev and Parlitz, 1995; Tao, 1999), etc. In the last two decades, various schemes have been successively applied for chaos synchronization such as PC Method (Pecora and Carroll, 1990), OGY Method (Ott *et al*., 1990), active control method (Ho and Hung, 2002; Huang *et al*., 2004; Chen, 2005; Sundarapandian and Karthikeyan, 2011a, b), adaptive control method (Lu *et al*., 2004; Chen and Lu, 2002), time-delay feedback method (Park and Kwon, 2003), backstepping design method (Yu and Zhang, 2006), sampled-data feedback synchronization method (Zhao and Lu, 2008) and sliding mode control method (Konishi *et al*., 1998; Sundarapandian and Sivaperumal, 2011a, b), etc.

In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the master or drive system and another chaotic system is called the Slave or Response System then the goal of anti-synchronization is to use the output of the master system to control the slave system so that, the states of the slave system have the same amplitude but opposite signs as the states of the master system asymptotically. In this study, researchers discuss the anti-synchronization of identical hyperchaotic Tigan Systems (Tigan and Opris, 2008), identical Li Systems (Li *et al*., 2009) and non-identical Tigan and Li Systems. The synchronization results are established using Lyapunov Stability Theory.

In adaptive synchronization of identical chaotic systems, the parameters of the master and slave systems are unknown and researchers devise feedback control laws using the estimates of the system parameters. In adaptive synchronization of non-identical chaotic systems, the parameters of the master system are known

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but the parameters of the slave system are unknown and researchers devise feedback control laws using the estimates of the parameters of the slave system.

ADAPTIVE ANTI-SYNCHRONIZATION OF IDENTICAL TIGAN CHAOTIC SYSTEMS

Theoretical results: In this study, researchers discuss the adaptive synchronization of identical Tigan Systems (Tigan and Opris, 2008) when the parameters of the master and slave systems are unknown. As the master system, researchers consider the Tigan dynamics described by:

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1)\\ \dot{x}_2 &= (c-a)x_1 - ax_1x_3\\ \dot{x}_3 &= -bx_3 + x_1x_2\end{aligned} \tag{1}$$

where, x_1-x_4 are the state variables and a-c are unknown parameters of the system. As the slave system, researchers consider the controlled Tigan dynamics described by:

$$\begin{aligned}\dot{y}_1 &= a(y_2 - y_1) + u_1\\ \dot{y}_2 &= (c-a)y_1 - ay_1y_3 + u_2\\ \dot{y}_3 &= -by_3 + y_1y_2 + u_3\end{aligned} \tag{2}$$

where, y_1-y_3 are the state variables and u_1-u_3 are the non-linear controls to be designed. The Tigan Systems (Eq. 1 and 2) are chaotic when the parameter values are chosen as:

$$a = 2.1,\quad b = 0.6 \text{ and } c = 30$$

The state orbits of the Tigan System are shown in Fig. 1. The anti-synchronization error is defined as:

$$e_i = y_i + x_i \quad (i = 1, 2, 3) \tag{3}$$

A simple calculation gives the error dynamics as:

$$\begin{aligned}\dot{e}_1 &= a(e_2 - e_1) + u_1\\ \dot{e}_2 &= (c-a)e_1 - ay_1y_3 - ax_1x_3 + u_2\\ \dot{e}_3 &= -be_3 + y_1y_2 + x_1x_2 + u_3\end{aligned} \tag{4}$$

Let us now define the adaptive functions $u_1(t)$-$u_3(t)$ as:

$$\begin{aligned}u_1(t) &= -\hat{a}(e_2 - e_1) - k_1e_1\\ u_2(t) &= -(\hat{c} - \hat{a})e_1 + \hat{a}y_1y_3 + \hat{a}y_1y_3 - k_2e_2\\ u_3(t) &= -\hat{b}e_3 - y_1y_2 - x_1x_2 - k_3e_3\end{aligned} \tag{5}$$

where, $\hat{a}-\hat{c}$ are estimates of a-c, respectively and k_i (i = 1-3) are positive constants. Substituting Eq. 5 into 4, the error dynamics simplifies to:

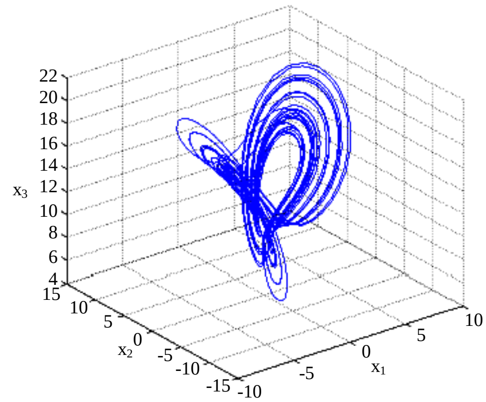

Fig. 1: State orbits of the Tigan Chaotic System

$$\begin{aligned}\dot{e}_1 &= (a - \hat{a})(e_2 - e_1) - k_1e_1\\ \dot{e}_2 &= (c - \hat{c})e_1 - (a - \hat{a})(e_1 + y_1y_3 + x_1x_3) - k_2e_2\\ \dot{e}_3 &= -(b - \hat{b})e_3 - k_3e_3\end{aligned} \tag{6}$$

Let us now define the parameter estimation error as:

$$e_a = a - \hat{a},\quad e_b = b - \hat{b},\quad e_c = c - \hat{c} \tag{7}$$

Substituting Eq. 7 into 6, researchers obtain the error dynamics as:

$$\begin{aligned}\dot{e}_1 &= e_a(e_2 - e_1) - k_1e_1\\ \dot{e}_2 &= e_ce_1 - e_a(e_1 + y_1y_3 + x_1x_3) - k_2e_2\\ \dot{e}_3 &= -e_be_3 - k_3e_3\end{aligned} \tag{8}$$

For the derivation of the update law for adjusting the estimates of the parameters, the Lyapunov approach is used. Researchers consider the quadratic Lyapunov function defined by:

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2\right) \tag{9}$$

which is a positive definite function on R^6. There is also note that:

$$\dot{e}_a = -\dot{\hat{a}},\quad \dot{e}_b = -\dot{\hat{b}},\quad \dot{e}_c = -\dot{\hat{c}} \tag{10}$$

Differentiating Eq. 9 along the trajectories of Eq. 8 and noting Eq. 10, researchers find that:

$$\begin{aligned}\dot{V} = {} & -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_a\left[-e_1^2 - e_2(y_1y_3 + x_1x_3) - \dot{\hat{a}}\right] + \\ & e_b\left[-e_3^2 - \dot{\hat{b}}\right] + e_c\left[e_1e_2 - \dot{\hat{c}}\right]\end{aligned} \tag{11}$$

In view of Eq. 11, the estimated parameters are updated by the following law:

$$\dot{\hat{a}} = -e_1^2 - e_2(y_1y_3 + x_1x_3) + k_4e_a$$
$$\dot{\hat{b}} = -e_3^2 + k_5e_b \qquad (12)$$
$$\dot{\hat{c}} = e_1e_2 + k_6e_c$$

where, k_4-k_6 are positive constants. Substituting Eq. 12 into 11, researchers obtain:

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_a^2 - k_5e_b^2 - k_6e_c^2 \qquad (13)$$

which is a negative definite function on R^6. Thus by Lyapunov Stability Theory (Hahn, 1967), it is immediate that the anti-synchronization error e_i (i = 1-3) and the parameter estimation error e_a-e_c decay to zero exponentially with time. Hence, researchers have proved the following result.

Theorem 1: The identical uncertain Tigan Systems (Eq. 1 and 2) are globally and exponentially anti-synchronized by the adaptive control law (Eq. 5) where the update law for the parameter estimates is given by Eq. 12 and k_i (i = 1, ..., 6) are positive constants.

Numerical results: For the numerical simulations, the 4th-order Runge-Kutta Method with time-step $h = 10^{-6}$ is used to solve the two systems of differential Eq. 1 and 2 with the adaptive non-linear controller (Eq. 5). Researchers take:

$$k_i = 2 \quad \text{for} \quad i = 1, 2, ..., 6$$

The parameters of the Tigan Systems are chosen so that, the systems are chaotic, i.e. :

$$a = 2.1, \quad b = 0.6 \quad \text{and} \quad c = 30$$

The initial values of the parameter estimates are taken as:

$$\hat{a}(0) = 1, \quad \hat{b}(0) = 2 \quad \text{and} \quad \hat{c}(0) = 5$$

The initial values of the master system (Eq. 1) are chosen as:

$$x_1(0)=21, \quad x_2(0)=15, \quad x_3(0)=30$$

The initial values of the slave system (Eq. 2) are chosen as:

$$y_1(0)=12, \quad y_2(0)=20, \quad y_3(0)=10$$

Figure 2 shows anti-synchronization of the Tigan Systems (Eq. 1 and 2). Figure 3 shows that the estimated values of the parameters viz., $\hat{a} - \hat{c}$ converge to the system parameters a = 2.1, b = 0.6 and c = 30, respectively.

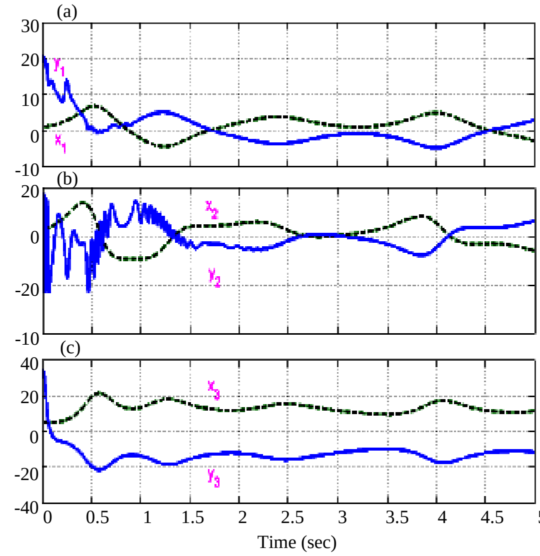

Fig. 2: Anti-synchronization of identical Tigan Systems

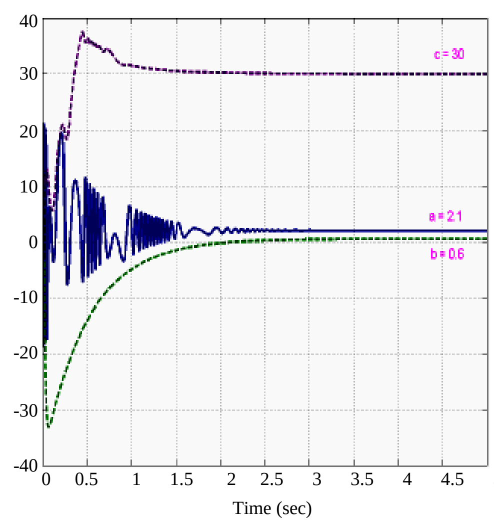

Fig. 3: Parameter estimates $\hat{a}(t) - \hat{c}(t)$

ADAPTIVE ANTI-SYNCHRONIZATION OF IDENTICAL LI CHAOTIC SYSTEMS

Theoretical results: In this study, researchers discuss the adaptive synchronization of identical Li Systems (Li *et al*., 2009) when the parameters of the master and slave systems are unknown. As the master system, researchers consider the Li dynamics described by:

$$\dot{x}_1 = \alpha(x_2 - x_1), \; \dot{x}_2 = x_1x_3 - x_2, \; \dot{x}_3 = \beta - x_1x_2 - \gamma x_3 \qquad (14)$$

where, x_1-x_4 are the state variables and α, β and γ are unknown parameters of the system. As the slave system, researchers consider the controlled Li dynamics described by:

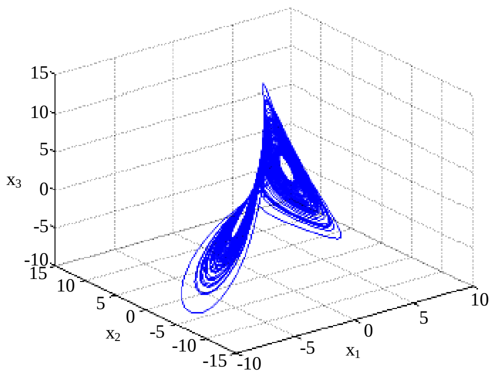

Fig. 4: State orbits of the Li Chaotic System

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= y_1 y_3 - y_2 + u_2 \\ \dot{y}_3 &= \beta - y_1 y_2 - \gamma y_3 + u_3 \end{aligned} \tag{15}$$

where, y_1-y_3 are the state variables and u_1-u_3 are the non-linear controls to be designed. The Li Systems (Eq. 1 and 2) are chaotic when the parameter values are chosen as:

$$\alpha = 5, \quad \beta = 16 \text{ and } \gamma = 1$$

The state orbits of the Li system are shown in Fig. 4. The anti-synchronization error is defined as:

$$e_i = y_i + x_i \quad (i = 1, 2, 3) \tag{16}$$

A simple calculation gives the error dynamics as:

$$\begin{aligned} \dot{e}_1 &= \alpha(e_2 - e_1) + u_1 \\ \dot{e}_2 &= -e_2 + y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -\gamma e_3 - y_1 y_2 - x_1 x_2 + 2\beta + u_3 \end{aligned} \tag{17}$$

Let us now define the adaptive functions $u_1(t)$-$u_3(t)$ as:

$$\begin{aligned} u_1(t) &= -\hat{\alpha}(e_2 - e_1) - k_1 e_1 \\ u_2(t) &= e_2 - y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3(t) &= \hat{\gamma} e_3 + y_1 y_2 + x_1 x_2 - 2\beta - k_3 e_3 \end{aligned} \tag{18}$$

where, $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ are estimates of α, β and γ, respectively and k_i (i = 1, 2, 3) are positive constants. Substituting Eq. 18 into 17, the error dynamics simplifies to:

$$\begin{aligned} \dot{e}_1 &= (\alpha - \hat{\alpha})(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -(\gamma - \hat{\gamma}) e_3 + 2(\beta - \hat{\beta}) - k_3 e_3 \end{aligned} \tag{19}$$

Let us now define the parameter estimation error as:

$$e_\alpha = \alpha - \hat{\alpha}, \quad e_\beta = \beta - \hat{\beta}, \quad e_\gamma = \gamma - \hat{\gamma} \tag{20}$$

Substituting Eq. 20 into 19, it is obtained the error dynamics as:

$$\begin{aligned} \dot{e}_1 &= e_\alpha(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -e_\gamma e_3 + 2e_\beta - k_3 e_3 \end{aligned} \tag{21}$$

For the derivation of the update law for adjusting the estimates of the parameters, the Lyapunov approach is used. Researchers consider the quadratic Lyapunov function defined by:

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_\alpha^2 + e_\beta^2 + e_\gamma^2\right) \tag{22}$$

which is a positive definite function on R^6. It is also noted that:

$$\dot{e}_\alpha = -\dot{\hat{\alpha}}, \quad \dot{e}_\beta = -\dot{\hat{\beta}}, \quad \dot{e}_\gamma = -\dot{\hat{\gamma}} \tag{23}$$

Differentiating Eq. 22 along the trajectories of Eq. 21 and noting Eq. 23, researchers find that:

$$\begin{aligned} \dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_\alpha\left[e_1(e_2 - e_1) - \dot{\hat{\alpha}}\right] + \\ &e_\beta\left[2e_3 - \dot{\hat{\beta}}\right] + e_\gamma\left[-e_3^2 - \dot{\hat{\gamma}}\right] \end{aligned} \tag{24}$$

In view of Eq. 11, the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{\alpha}} &= e_1(e_2 - e_1) + k_4 e_\alpha \\ \dot{\hat{\beta}} &= 2e_3 + k_5 e_\beta \\ \dot{\hat{\gamma}} &= -e_3^2 + k_6 e_\gamma \end{aligned} \tag{25}$$

where, k_4-k_6 are positive constants. Substituting Eq. 12 into 11, researchers obtain:

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_\alpha^2 - k_5 e_\beta^2 - k_6 e_\gamma^2 \tag{26}$$

which is a negative definite function on R^6. Thus by Lyapunov Stability Theory (Hahn, 1967), it is immediate that the anti-synchronization error e_i (i = 1-3) and the parameter estimation error e_α, e_β and e_γ decay to zero exponentially with time. Hence, there have proved the following result.

Theorem 2: The identical uncertain Li Systems Eq. 14 and 15 are globally and exponentially anti-synchronized by the adaptive control law (Eq. 18) where the update law for the parameter estimates is given by (Eq. 25) and k_i (i = 1, .., 6) are positive constants.

Numerical results: For the numerical simulations, the 4th-order Runge-Kutta Method with time-step $h = 10^{-6}$ is used to solve the two systems of differential Eq. 14 and 15 with the adaptive non-linear controller (Eq. 18). Researchers take $k_i = 2$ for i = 1, 2, ..., 6. The parameters of the Li Systems are chosen so that, the systems are chaotic, i.e., $\alpha = 5$, $\beta = 16$ and $\gamma = 1$. The initial values of the parameter estimates are taken as:

$$\hat{\alpha}(0) = 4,\ \hat{\beta}(0) = 5,\ \hat{\gamma}(0) = 12$$

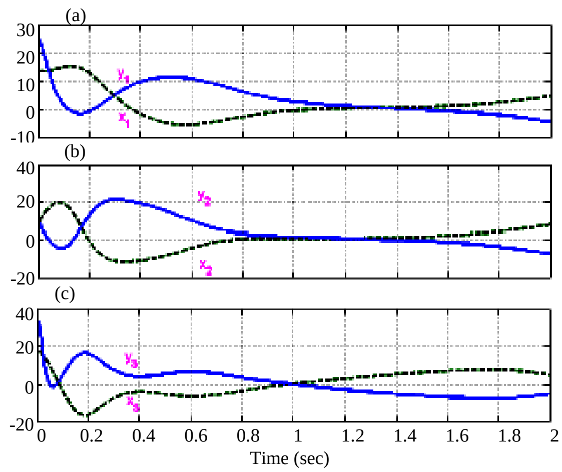

Fig. 5: Anti-synchronization of identical Li Systems

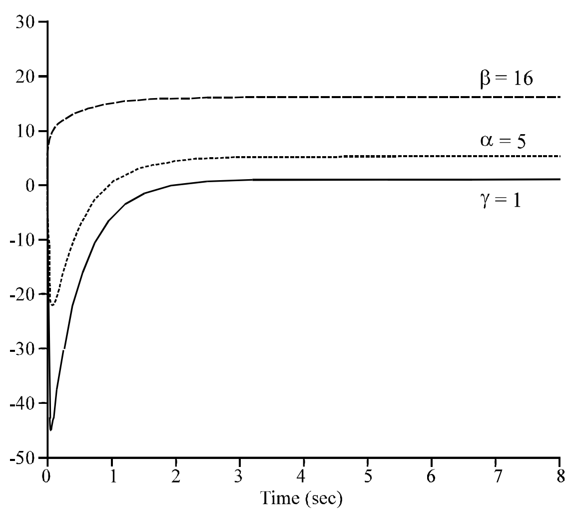

Fig. 6: Parameter estimates $\hat{\alpha}(t), \hat{\beta}(t), \hat{\gamma}(t)$

The initial values of the master system (Eq. 14) are chosen as:

$$x_1(0)=14,\ x_2(0)=8,\ x_3(0)=19$$

The initial values of the slave system (Eq. 15) are chosen as:

$$y_1(0)=25,\ y_2(0)=10,\ y_3(0)=32$$

Figure 5 shows anti-synchronization of the Li Systems (Eq. 14 and 15). Figure 6 shows that the estimated values of the parameters viz., $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ converge to the system parameters $\alpha = 5$, $\beta = 16$ and $\gamma = 1$, respectively.

ADAPTIVE ANTI-SYNCHRONIZATION OF TIGAN AND LI CHAOTIC SYSTEMS

Theoretical results: In this study, researchers discuss the adaptive anti-synchronization of non-identical Tigan and Li Systems. Here, there is considerd the Tigan System (Tigan and Opris, 2008) as the master system whose parameters are known. We consider the controlled Li System (Li *et al*., 2009) as the slave system whose parameters are unknown. As the master system, we consider the Tigan dynamics described by:

$$\dot{x}_1 = a(x_2 - x_1),\ \dot{x}_2 = (c-a)x_1 - ax_1x_3,\ \dot{x}_3 = -bx_3 + x_1x_2 \quad (27)$$

where, x_1-x_3 are the state variables and a-c are unknown parameters of the system. As the slave system, we consider the controlled Li dynamics described by:

$$\begin{aligned}\dot{y}_1 &= \alpha(y_2 - y_1) + u_1\\ \dot{y}_2 &= y_1y_3 - y_2 + u_2\\ \dot{y}_3 &= \beta - y_1y_2 - \gamma y_3 + u_3\end{aligned} \quad (28)$$

where, y_1-y_3 are the state variables, α, β and γ are unknown parameters of the system and u_1-u_3 are the non-linear controllers to be designed. The anti-synchronization error is defined as:

$$e_i = y_i + x_i,\quad (i = 1, 2, 3) \quad (29)$$

A simple calculation gives the error dynamics as:

$$\begin{aligned}\dot{e}_1 &= \alpha(y_2 - y_1) + a(x_2 - x_1) + u_1\\ \dot{e}_2 &= -y_2 + (c-a)x_1 + y_1y_3 - ax_1x_3 + u_2\\ \dot{e}_3 &= \beta - \gamma y_3 - bx_3 - y_1y_2 + x_1x_2 + u_3\end{aligned} \quad (30)$$

Let us now define the adaptive functions $u_1(t)$-$u_4(t)$ as:

$$\begin{aligned} u_1(t) &= -\hat{\alpha}(y_2 - y_1) - a(x_2 - x_1) - k_1 e_1 \\ u_2(t) &= y_2 - (c-a)x_1 - y_1 y_3 + a x_1 x_3 - k_2 e_2 \\ u_3(t) &= -\hat{\beta} + \hat{\gamma} y_3 + b x_3 + y_1 y_2 - x_1 x_2 - k_3 e_3 \end{aligned} \quad (31)$$

where, $\hat{\alpha}$ $\hat{\beta}$ and $\hat{\gamma}$ are estimates of α, β and γ, respectively and k_1-k_3 are positive constants. Substituting Eq. 31 into 30, the error dynamics simplifies to:

$$\begin{aligned} \dot{e}_1 &= (\alpha - \hat{\alpha})(y_2 - y_1) + k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= (\beta - \hat{\beta}) - (\gamma - \hat{\gamma}) y_3 - k_3 e_3 \end{aligned} \quad (32)$$

Let us now define the parameter estimation error as:

$$e_\alpha = \alpha - \hat{\alpha}, \quad e_\beta = \beta - \hat{\beta}, \quad e_\gamma = \gamma - \hat{\gamma} \quad (33)$$

Substituting Eq. 33 into 32, there is obtained the error dynamics as:

$$\begin{aligned} \dot{e}_1 &= e_\alpha (y_2 - y_1) - k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= e_\beta - e_\gamma y_3 - k_3 e_3 \end{aligned} \quad (34)$$

For the derivation of the update law for adjusting the estimates of the parameters, the Lyapunov approach is used. We consider the quadratic Lyapunov function defined by:

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_\alpha^2 + e_\beta^2 + e_\gamma^2\right) \quad (35)$$

which is a positive definite function on R^6. We also note that:

$$\dot{e}_\alpha = -\dot{\hat{\alpha}}, \quad \dot{e}_\beta = -\dot{\hat{\beta}}, \quad \dot{e}_\gamma = -\dot{\hat{\gamma}} \quad (36)$$

Differentiating Eq. 35 along the trajectories of Eq. 34 and noting Eq. 36, we find that:

$$\begin{aligned} \dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_\alpha \left[e_1 (y_2 - y_1) - \dot{\hat{\alpha}} \right] + \\ &e_\beta \left[e_3 - \dot{\hat{\beta}} \right] + e_\gamma \left[-e_3 y_3 - \dot{\hat{\gamma}} \right] \end{aligned} \quad (37)$$

In view of Eq. 37, the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{\alpha}} &= e_1 (y_2 - y_1) + k_4 e_\alpha \\ \dot{\hat{\beta}} &= e_3 + k_5 e_\beta \\ \dot{\hat{\gamma}} &= -e_3 y_3 + k_6 e_\gamma \end{aligned} \quad (38)$$

where, k_5-k_8 are positive constants. Substituting Eq. 38 into 37, we obtain:

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_\alpha^2 - k_5 e_\beta^2 - k_6 e_\gamma^2 \quad (39)$$

which is a negative definite function on R^6. Thus by Lyapunov Stability Theory (Li *et al*., 2009), it is immediate that the anti-synchronization error i_e (i = 1, 2, 3) and the parameter estimation error decay to zero exponentially with time. Hence, we have proved the following result.

Theorem 3: The Tigan System (Eq. 27) with known parameters and Li System (Eq. 28) with unknown parameters are globally and exponentially anti-synchronized by the adaptive control law (Eq. 31) where the update law for the parameter estimates is given by Eq. 38 and k_i (i = 1, ..., 6) are positive constants.

Numerical results: For the numerical simulations, the 4th-order Runge-Kutta Method with time-step $h = 10^{-6}$ is used to solve the two systems of differential Eq. 27 and 28 with the adaptive non-linear controller (Eq. 31). Researchers take:

$$k_i = 2 \text{ for } i = 1, 2, ..., 6$$

The parameters of the Tigan System (Eq. 27) are chosen as:

$$a = 2.1, \; b = 0.6 \text{ and } c = 30$$

The parameters of the Li System (Eq. 28) are chosen as:

$$\alpha = 5, \; \beta = 16 \text{ and } \gamma = 1$$

The initial values of the parameter estimates are taken as:

$$\hat{\alpha}(0) = 12, \; \hat{\beta}(0) = 8, \; \hat{\gamma}(0) = 20$$

The initial values of the master system (Eq. 27) are chosen as:

$$x_1(0) = 12, \; x_2(0) = 8, \; x_3(0) = 10$$

The initial values of the slave system (Eq. 28) are chosen as:

$$y_1(0) = 24, \; y_2(0) = 18, \; y_3(0) = 20$$

Figure 7 shows anti-synchronization of the Tigan System (Eq. 27) and Li System (Eq. 28). Figure 8 shows

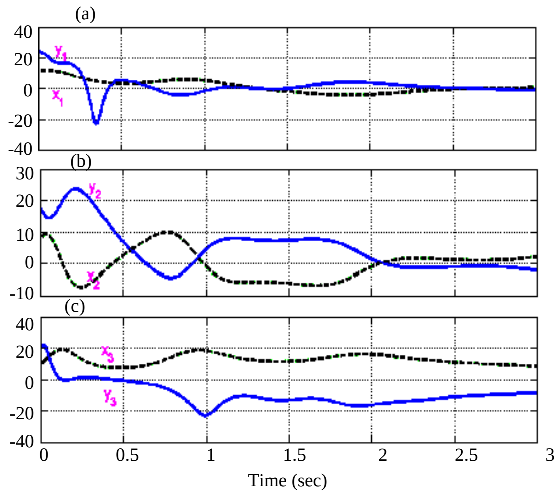

Fig. 7: Anti-synchronization of Tigan and Li Systems

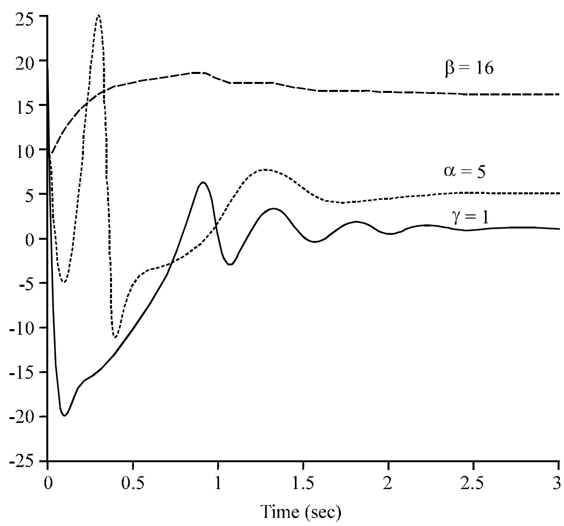

Fig. 8: Parameter estimates $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$

that the estimated values of the parameters viz., $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ converge to the system parameters $\alpha = 5$, $\beta = 16$ and $\gamma = 1$, respectively.

CONCLUSION

In this study, we have applied adaptive control method for the global chaos anti-synchronization of identical Tigan Systems (2008), identical Li Systems (2009) and non-identical Tigan System with known parameters and Li System with unknown parameters. The adaptive anti-synchronization results derived in this study are established using Lyapunov Stability Theory.

Since, the Lyapunov exponents are not required for these calculations, the adaptive non-linear control method is very effective and convenient to achieve global chaos anti-synchronization for the uncertain chaotic systems discussed in this study. Numerical simulations are also shown for the anti-synchronization of identical and non-identical uncertain Tigan and Li Chaotic Systems to demonstrate the effectiveness of the adaptive anti-synchronization schemes derived in this study.

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