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# Study of Transient Transverse Vibration for a Pipe

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**Abstract:** A Fluid Structure Interaction (FSI) is the interaction of some movable or deformable structure with an internal or surrounding fluid flow, FSI are essential consideration in the design of many engineering systems, e.g., aircrafts, nuclear and piping system. The governing equation of transverse vibration for pipe conveying fluid was derived from momentum equation for moving control volume and the continuity equations, the coupling between the fluid and pipe forces was considered. The equation derived was compared with those from the previous researchers for validation of the proposed equation. The FSI water hammer equation coupled with proposed equation was presented and the system was solved to find the transient pressure and the displacement associated with it. The Method of Characteristic (MOC) and Finite Difference Method (FDM) were used in the solution. In this study, a numerical application was done for the proposed equation, the present results reveal acceptable with previous researchers' results. The pipe length between the supports has a significant affect on the pipe displacement. The displacement to the pipe length ratio when succeeds (0.5%) or more has effect on the fluid pressure and velocity.

**Key words:** FSI, transverse displacement, transient pressure, MOC, momentum equation, vibration

## INTRODUCTION

Unsteady flow induced vibration often occurs due to pump and valve operations in pipeline systems and even in human circulation is of concern to the hydropower, petroleum industries and biomedical engineering research. In this study, we focus on the vibration of hydraulic pipe occurs from water hammer phenomenon. Many researchers deal with the formulation of water hammer equations and the solution of these equations, they used MOC and FDM explicit and implicit (Afshar and Rohani, 2008; Kwon and Lee, 2008). From the literature review in the pipe conveying fluid, there are many researchers studied various cases. The MOC with no interpolations, no adjustments (of wave speeds) and no approximations was used for solving the four equations model describes the axial vibration of liquid-filled pipes. It is valid for linear, non-dispersive, non-dissipative, hyperbolic systems with linear (or quadratic) time-dependent boundary conditions. It gives exact solutions without the errors of the conventional approaches (Tijsseling, 2003). Hamilton principle with the assumption that gravity, pressure and fluid friction effects and restoring flexural forces are neglected was used to model the transverse vibrations of highly tensioned fixed-fixed and fixed-sliding supported pipes with vanishing flexural stiffness and transporting fluid with time-dependent velocity (Halio and Boyaci, 2000). The vibration of a straight pipe conveying fluid when both ends are fixed analyzed by using the Euler-Bernoulli beam theory and the general lagrange strain from the extended Hamilton principle, the non-linear equations of motion for the longitudinal and transverse displacements are derived where the longitudinal and transverse displacements are coupled with each other. With the discredited equations obtained by the Galerkin method, the natural frequencies and the dynamic responses are computed (Lee and Chung, 2002). The equations of motion for straight fluid-filled pipes are approximated to be similar to those for a Timoshenko beam on a Winkler foundation.

The derivation is also based on the assumption of long axial wavelengths, the compressibility of the fluid is neglected and the internal fluid loading on the pipe is approximated as an increase in the radial inertia (Finnveden, 1997). The Finite Element Method (FEM) and Transfer matrix method were used for solving many combinations of the coupled pipe fluid equations for the dynamic pressure and displacement in longitudinal and transverse directions in different models (Birgerssona *et al*., 2004; Kochupillai1 *et al*., 2004; Lia *et al*., 2002; Hansson and Sandberg, 1998; Lin and Tsai, 1996). The equation of motion for cantilever pipe

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conveying fluid was modeled based on non-linear Bernoulli-Euler type beam with assumption that transverse shear deformation and rotator-inertia are neglected where the equations of motion directly result in a finite set of ordinary differential equations (Michael, 2008). A rigorous derivation of one-dimensional equations describing fluid-structure interaction mechanisms in the axial/radial vibration of liquid-filled pipes has been done thereby taking the thickness of the pipe wall into account through the averaging of hoop and radial stresses. FSI coupled wave speeds have been formulated and investigated (Tijsseling, 2007). A new matrix method was used for calculating critical flow velocity of curved pipes conveying fluid which have arbitrary centerline shape and spring supports (Huang *et al.*, 2002). The parameter of the pipe and the fluid on the frequencies at up and down stream of the pipe by the shell theory were studied (Zhang, 2001). The non-linear equations of motion of a flexible pipe conveying unsteadily flowing fluid are derived from the continuity and momentum equations of unsteady flow. These partial differential equations are fully coupled through equilibrium of contact forces, the normal compatibility of velocity at the fluid-pipe interfaces and the conservation of mass and momentum of the transient fluid (Gorman *et al.*, 1999). From literature reviewed, the equations governing the vibration of the pipe derived with various assumptions depends on the operation conditions of the research. In this study, there is a model of equations to study the pipe vibration in a very simple assumption with acceptable result with previous models.

## THE EQUATIONS OF MOTION

The equation of motion of supported ends pipe conveying pulsating fluid for a pipe of flexural rigidity EI, length l, cross-sectional area $A_P$ and mass per unit length mp, conveying fluid of mass per unit length mf with velocity U varying with time and cross-sectional area A. t is the time and x is the coordinate along the centerline of the pipe. The displacement w with negligible longitudinal displacement. The non-inertial momentum equation for moving control volume can be given by:

$$\sum F_{sys} - \int a_{CV}\rho dV = \frac{\partial}{\partial t}\int_{CV} U_r \rho \, dV + \int_{CS} U_r \rho \, (U_r . dA) \quad (1)$$

Where:

$F_{sys}$ = The external forces acting on the control volume

$a_{CV}$ = The acceleration of the control volume relative to inertial axes

$\rho$ = The fluid density

V = The volume

$U_r$ = The control volume relative velocity measured relative to moving control volume

Elements (dx) of the pipe and the enclosed fluid was considered as shown in Fig. 1, subjected to small transverse motion w(x, t). From Fig. 1a for the fluid element, the external force balances in the x-z plane in x and z direction, respectively yield:

$$-[(AP\cos\theta)' + N\sin\theta + fS\cos\theta + m_f g\sin\theta]dx = \sum F_x \quad (2)$$

$$-[(AP\sin\theta)' - N\cos\theta + fS\sin\theta + m_f g\cos\theta]dx = \sum F_z \quad (3)$$

Where:

fs = Shear stresses on the internal surface of the pipe

N = Transverse force per unit length between pipe wall and fluid

S = Inner perimeter

Similarly for the pipe element (Fig. 1b):

$$[(T\cos\theta)' - (Q\sin\theta)' + (N\sin\theta) + fS\cos\theta - m_p g\sin\theta]dx = 0 \quad (4)$$

$$(Q\cos\theta)' + (T\sin\theta)' - N\cos\theta + fS\sin\theta - m_p g\cos\theta = m_p\ddot{w} \quad (5)$$

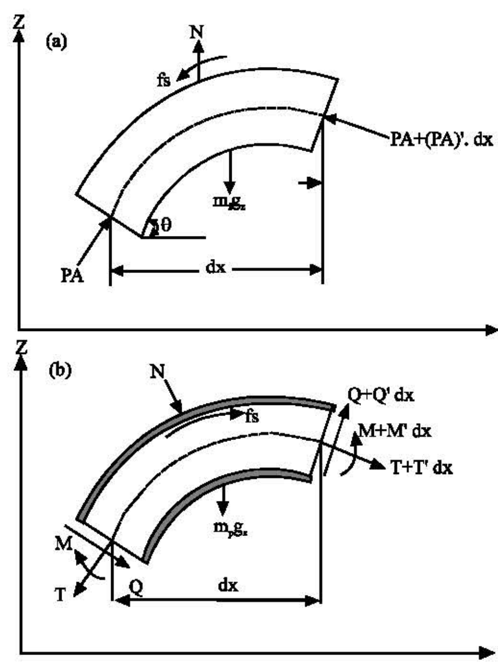


Fig. 1: a) The fluid element forces; b) The pipe element forces

The transverse shear force in the pipe given by:

$$Q = -EI(w''' - 2w'w''^2 - 0.5w'^2w''') \tag{6}$$

For Eq. 1, the 1st term on the left was represented by Eq. 3 in the z direction. For the 2nd term on the left of Eq. 1 with some manipulations:

$$\int a_{CV}\rho dV = m_f a_{CV} dx = m_f(\ddot{w} + U\dot{w}')dx \tag{7}$$

For the 1st term on the right side of Eq. 1:

$$\frac{\partial}{\partial t}\int_{CV} U_r \rho \; dV = m_f \; (\dot{U}w' + U\dot{w}')dx \tag{8}$$

For the 2nd term on the right side of Eq. 1:

$$\int_{CS} U_r \rho \; (U_r.dA) = m_f(U^2w'' + UU'w')dx \tag{9}$$

Equation 1 in the z direction becomes:

$$\sum F_z = [\ddot{w} + 2U\dot{w}' + \dot{U}w' + U^2w'' + UU'w']m_f dx \tag{10}$$

By the use of Eq. 3 with Eq. 10, we found:

$$\begin{aligned}&[(AP\sin\theta)' - N\cos\theta + fS\sin\theta + m_f g\cos\theta]dx + \\ &[\ddot{w} + 2U\dot{w}' + \dot{U}w' + U^2w'' + UU'w']m_f dx = 0\end{aligned} \tag{11}$$

From Eq. 6 to 5 then in Eq. 11 with some manipulation give:

$$\begin{aligned}&m\ddot{w} + (APw')' + [2U\dot{w}' + \dot{U}w' + U^2w'' + UU'w']m_f + \\ &EIw'''' + mg_z - EI[5w'w''w''' + 2w''^3 + 0.5w'^2w''''] = 0\end{aligned} \tag{12}$$

Where:

$$m = m_f + m_p \tag{13}$$

With same manner, the Eq. 1 in the x direction becomes:

$$\sum F_x = (\dot{U} + UU')m_f dx \tag{14}$$

From Eq. 2 in to Eq .14:

$$\begin{aligned}&(AP\cos\theta)' + N\sin\theta + fS\cos\theta + \\ &m_f(\dot{U} + UU' + g\sin\theta) = 0\end{aligned} \tag{15}$$

The two fluid momentum equations; Eq. 11 and 15 can be combined by eliminating the N term. With neglect to higher order terms the fluid momentum equation can be:

$$\begin{aligned}&(AP)' + fS + m_f(\dot{U} + UU' + 2gw') + \\ &[\ddot{w} + 2U\dot{w}' + \dot{U}w' + U^2w'' + UU'w']w'm_f = 0\end{aligned} \tag{16}$$

The shear stress in Eq. 16 can be represented by Darcy-Weisbach equation:

$$fS = m_f(f/2D)(U|U|) \tag{17}$$

Substitution from Eq. 17 into Eq. 16 gives:

$$\begin{aligned}&(AP)' + m_f((f/2D)(U|U|) + \dot{U} + UU' + 2gw') + \\ &[\ddot{w} + 2U\dot{w}' + \dot{U}w' + U^2w'' + UU'w']w'm_f = 0\end{aligned} \tag{18}$$

Now, the continuity equation for the control volume given by (Wylie and Streeter, 1983):

$$\frac{\dot{\rho}}{\rho} + \frac{\dot{A}}{A} + U' = 0 \tag{19}$$

Also, the fluid bulk modulus of elasticity and the time rate of change of the cross-sectional area of a control volume are given with the pipe pre-tension, respectively by (Wylie and Streeter, 1983):

$$\frac{\dot{\rho}}{\rho} = \frac{\dot{P}}{K} \tag{20}$$

$$\frac{\dot{A}}{A} = \frac{D_p}{Eh}(\dot{P} - \frac{\upsilon}{2A}\dot{T}) \tag{21}$$

The tensile force to axial displacement was given to be:

$$T = T_0 + EA_p(u' + \frac{1}{2}(w')^2) \tag{22}$$

Where:

- $u'$ = Longitudinal displacement
- $T_0$ = Initial tension
- $A_P$ = The pipe cross-sectional area
- $K$ = Fluid bulk modulus of elasticity
- $D_P$ = Internal diameter of the pipe
- $h$ = Pipe walls thickness
- $\nu$ = Poisson ratio

By substituting Eq. 20-22 into Eq. 19, the following equation can be derived:

$$\dot{P} + UP' - 2\nu\rho a^2(w'\dot{w}' + Uw'w'') + \rho a^2 U' = 0 \quad (23)$$

$$a^2 = (K/\rho)/(1 + KD_p/Eh) \quad (24)$$

Equation 18 and 23 are fully coupled with the vibration of the pipe in the lateral directions. For the pipe conveying fluid, Eq. 18 and 23 become identical to the equations of continuity and momentum in the classical water hammer theory used by Wylie and Streeter (1983). Thus, far three fully coupled pipe dynamic Eq. 12, 18 and 23 were derived in terms of three dependent variables: transverse displacement, fluid velocity and fluid pressure.

## DISCUSSION

To validate the equation modeled in this study, we rewrite Paidoussis equation's (Lee and Chung, 2002) with same notation of this study to be:

$$\begin{aligned} & m\ddot{w} + [\dot{U}w' + 2U\dot{w}' + U^2 w'']m_f - EA[\frac{3}{2}w'^2 w''] + \\ & EI[w'''' - 2w'^2 w'''' - 8w'w''w''' - 2w''^2] = P \end{aligned} \quad (25)$$

Also, Lee equation (Lee and Chung, 2002) to be:

$$m\ddot{w} + [\dot{U}w' + 2U\dot{w}' + U^2 w'']m_f + EI[w''''] = P \quad (26)$$

The equation presented in this study, Eq. 12 compared with those equations from the literature review, Eq. 25 and 26.

All the equations have the same linear terms and the difference in the non linear terms. In Eq. 12 and 25, the non-linear terms depends on the transverse displacement compared to Eq. 26. In Eq. 12, the pressure forces is considered to be function of distance along the pipe but in Eq. 25 and 26, the pressure is constant along the pipe.

## NUMERICAL APPLICATIONS

A Finite Difference Method (FDM) is applied directly to solve the partial differential Eq. 12 for the dynamic response of the fluid-pipe system. The boundary conditions to complement the equation of motion, Eq. 12 for a pipe simply supported at both ends are:

$$\begin{aligned} & w(0, t) = w''(0, t) = 0 \\ & w(L, t) = w''(L, t) = 0 \end{aligned} \quad (27)$$

The fluid-pipe system is discredited into n short segments. The initial velocity is assumed to be steady. The fluid transient forces exerted on the pipe to cause the pipe displacement was covered by the coupled equations of water hammer with fluid structure interaction with the assumption that the displacement in longitudinal direction of the pipe was neglected.

The Method of Characteristic (MOC) was applied on Eq. 18 and 23 and then the FDM used to solve the equation to estimate the transient pressure which will be used to solve Eq. 12. The application of MOC on Eq. 18 and 23 give the flowing two equations:

$$\begin{aligned} & C^+ : (1 + w'^2)\frac{dU}{dt} + \frac{1}{a\rho}\frac{dP}{dt} + \frac{f}{2D}U|U| + \\ & 2gw' - a(2\nu(w'\dot{w}' + Uw'w'') + w'^2 U') + \\ & (\ddot{w} + 2U\dot{w}' + U^2 w'')w' = 0 \\ & \frac{dx}{dt} = U + a \end{aligned} \quad (28)$$

$$\begin{aligned} & C^- : (1 + w'^2)\frac{dU}{dt} - \frac{1}{a\rho}\frac{dP}{dt} + \frac{f}{2D}U|U| + \\ & 2gw' + a(2\nu(w'\dot{w}' + Uw'w'') + w'^2 U') + \\ & (\ddot{w} + 2U\dot{w}' + U^2 w'')w' = 0 \\ & \frac{dx}{dt} = U - a \end{aligned} \quad (29)$$

The equations derived for the water hammer seen to be more complete in the terms describing the FSI coupling. A computer code was written to calculate the dynamic response of a pipe in a system consist of reservoir, pipe and valve; the pipe simply supported at both ends and conveying fluid with initially study velocity to observe the effect of the sudden valve closure on the pipe structure, the ratio of the thickness to the diameter of the pipe, the ratio of the diameter to the pipe length and the verification of the present Eq. 12 with equation of Gorman *et al.* (1999).

The parameter used in the numerical calculation as follow; the initial pressure is 10 Mpa. The initial velocity is 5 m sec$^{-1}$. h/D = 0.05 and 0.01. D/L = 0.05 and 0.025, L represents the test section of the pipe. The remaining of the pipe length was chosen to combatable with wave velocity to allow the resonance to occur. For the reality, the velocity and the pressure are coupled directly in the calculation with displacement equation, the initial excitation from the equilibrium was calculated based on pipe deflection. For the numerical stability the pipe grid and time interval were chosen according to equation of Watters:

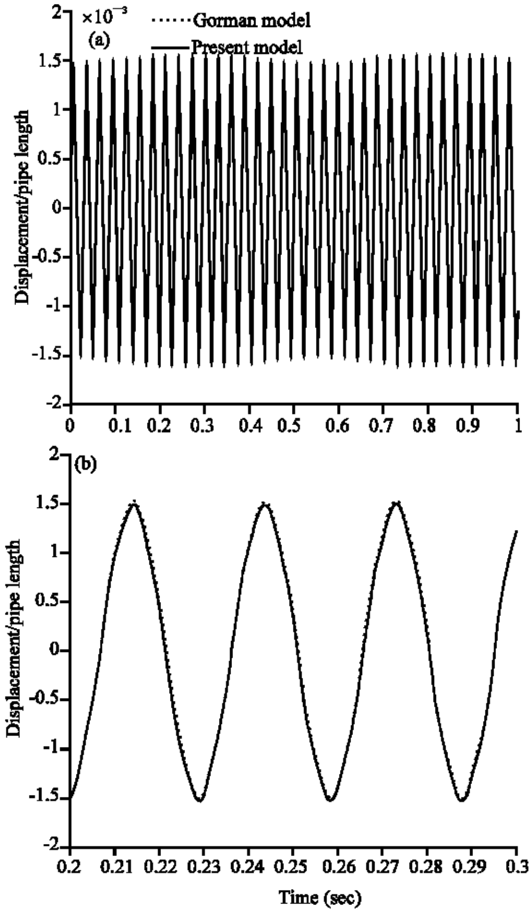


Fig. 2: a) Displacement comparison (h/D = 0.05, D/L = 0.05, time = 0-1 sec); b) Displacement comparison (h/D = 0.05, D/L = 0.05, time = 0.02-0.03 sec)

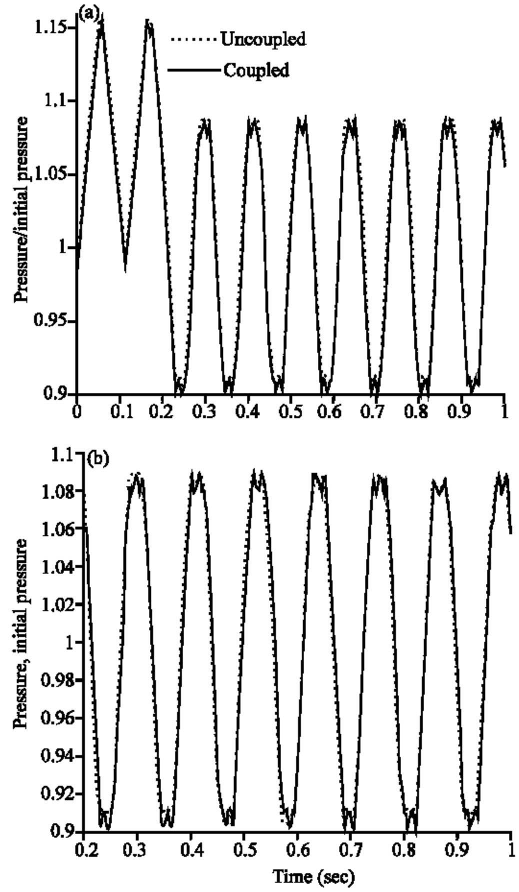


Fig. 3: a) Pressure history comparison (h/D = 0.05 D/L = 0.05, time = 0-1 sec), b) Pressure history comparison (h/D = 0.05 D/L = 0.05, time = 0.2-1 sec)

$$\Delta t = \frac{\Delta L}{\max|a + V|} \qquad (30)$$

Figure 2-10 show the data at x = 0.5 L, Fig. 2, 5 and 8 show the displacement of the pipe under transient condition when the valve was closed rapidly for the comparison of the two models, we see that the tow models have the same result in Fig. 2 but in Fig. 3, a little differences appear.

In Fig. 8, the differences appear to show the responsibility of the present model to the dynamic pressure is more than the Gorman model. Figure 2 and 5 show the effect of the pipe thickness on the displacement; the result shows the decrease in the h/D ratio five times increase the displacement about three times. Figure 2 and 8 show the effect of the pipe length on the displacement; the result shows the decrease in the D/L ratio two times

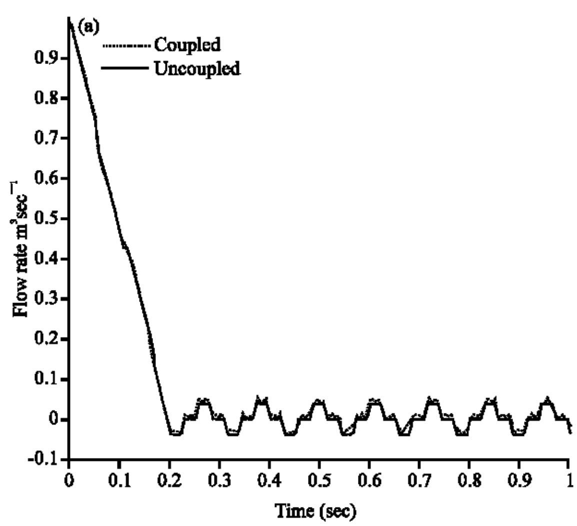


Fig. 4: Continue

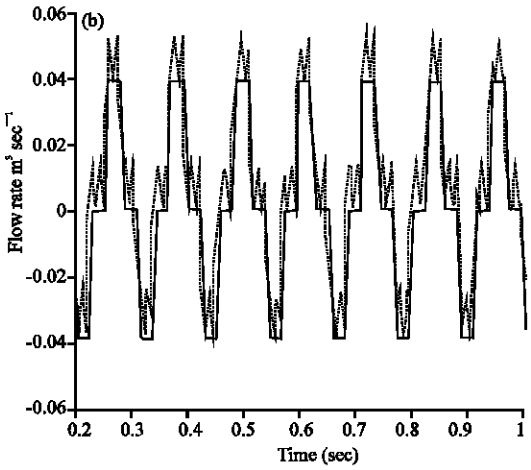


Fig. 4: a) Flow rate history comparison (h/D = 0.05 D/L = 0.05, time = 0-1 sec); b) flow rate history comparison (h/D = 0.05 D/L = 0.05, time = 0.2 -1 sec)

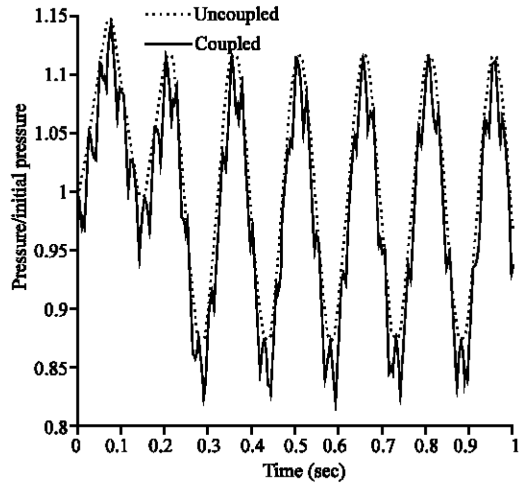


Fig. 6: Pressure historu comaprison (h/D = 0.01, D/L = 0.05)

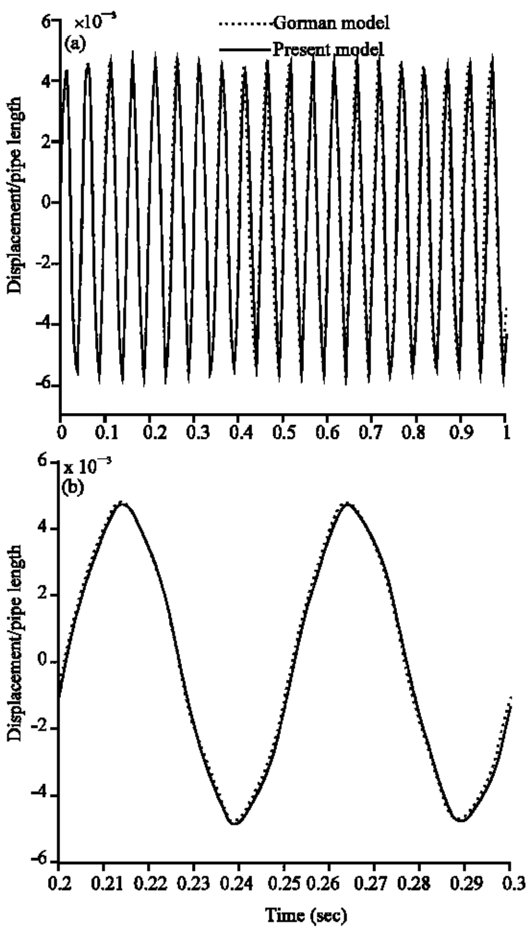


Fig. 5: a) Displacement comparison (h/D = 0.01 D/L = 0.05, time = 0-1 sec); b) Displacement comparison (h/D = 0.01 D/L = 0.05, time = 0.2-0.3 sec)

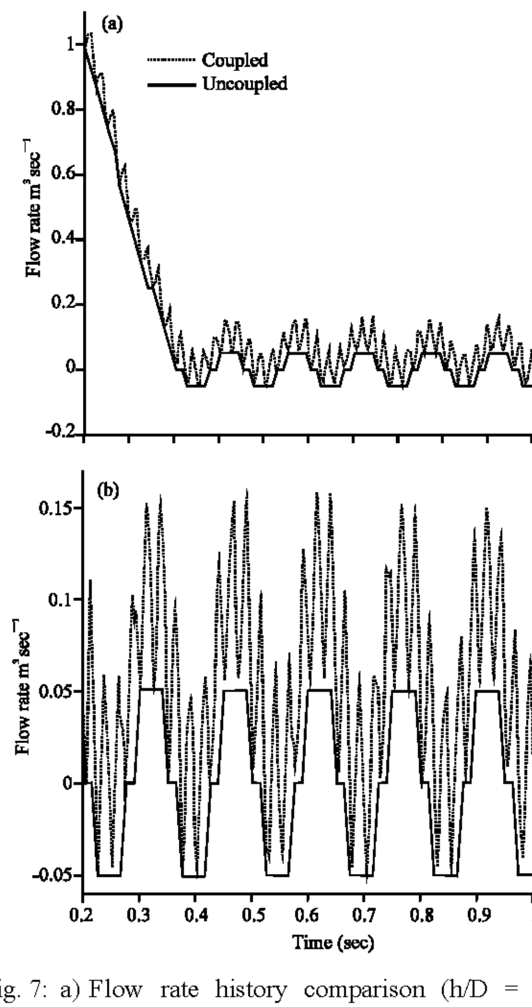


Fig. 7: a) Flow rate history comparison (h/D = 0.01 D/L = 0.05, Time = 0-1 sec) ;b) Flow rate history comparison (h/D = 0.01 D/L = 0.05, Time = 0.2-1 sec)

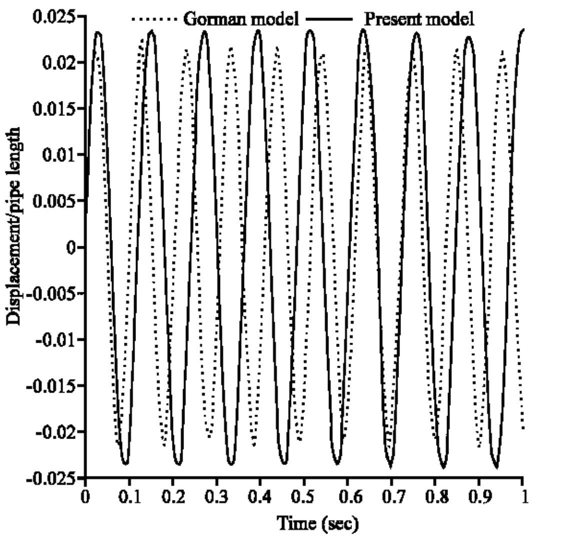


Fig. 8: Displacement comparison (h/D = 0.05, D/L = 0.025)

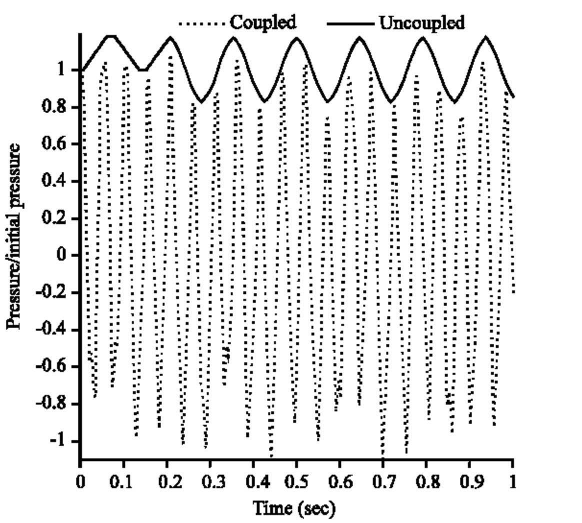


Fig. 9: Pressure history comparison (h/D = 0.05, D/L = 0.025)

increase the displacement >150 times. Figure 3 and 6 show the effect of the displacement on the pressure history.

The effect become more significant with decrease of the ratio h/D also from this, we find that the ratio h/D affect the amount of the transient pressure in Fig. 3, the pressure decrease after one cycle but in Fig. 6, the pressure fluctuate with transient increase in the pressure.

Figure 4 and 7 show the effect of the displacement on the velocity profile; the velocity means value remain constant for h/D = 0.05 but have a little change for h/D = 0.01. In Fig. 9 and 10, we found that the effect of the FSI is very clear on the pressure and the velocity profile.

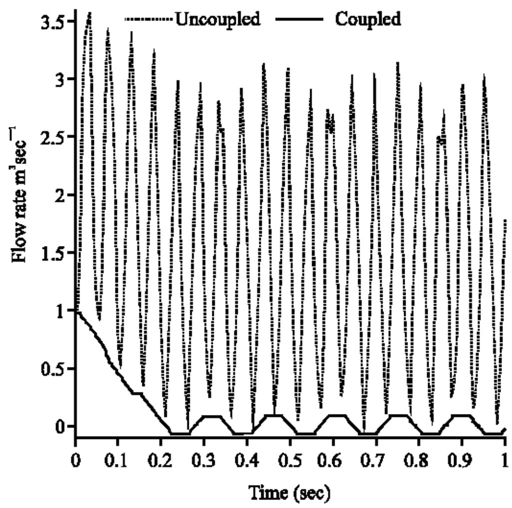


Fig. 10: Flow rare history comparison (h/D = 0.05, D/L = 0.025)

## CONCLUSION

Study on the pipe vibration from transient pressure was done to cover the derivation of the pipe displacement equations which coupled with the fluid forces and equations of the classical water hammer coupled with pipe forces. The MOC and FDM were used to solve the equations for the present model to verify the present model. Also, the effect of the pipe thickness and the pipe length were studied. The model presented in this study for transverse vibration has a good response to the dynamic forces applied from the pipe. By the current model, the displacement, the pressure and the velocity at any point in the pipe can be easily calculated for unsteady internal flow under transient condition.

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