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# Global Hybrid Fuzzy Controller for a Flexible Single-Link Manipulator

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**Abstract:** This study presents a development of a global fuzzy control scheme for the vibration suppression and the input tracking of a flexible single-link manipulator based on fuzzy logic theory and commutation strategy. A flexible single-link with a horizontal planar vibration is considered and then the dynamic model is derived using Lagrange's equations and finite element method. To design the proposed global fuzzy controller, 2 separated mamdani fuzzy controllers are determined for the hub angle and the end-point residual loops. The control signal is the result of an auto-commutative takagi-sugeno fuzzy switcher that selects the adequate command to become the hub torque input. Simulation results of the global fuzzy controller are discussed in terms of level of vibration reduction, speed of response and trajectory tracking.

**Key words:** Flexible manipulator, dynamic model, mamdani fuzzy controller, sugeno fuzzy switcher, adequate command, switcher

# INTRODUCTION

Robotic manipulators are generally built using heavy material in a manner to maximize stiffness in an attempt to minimize system vibration and achieve good positional accuracy. As a consequence such robots are usually heavy with respect to the operating payload. The operation speed of the robot manipulation is limited so, the actuators size is increased boosting energy consumption and increasing the overall cost. Moreover, the robot has a low payload to robot weight ratio. In order to solve these problems, robotic manipulators are designed to be lightweight. Conversely, flexible manipulators exhibit many advantages over their rigid counterparts: they require less material are lighter in weight have higher manipulation speed, lower power consumption, require smaller actuators are more maneuverable and transportable are safer to operate due to reduced inertia have enhanced back-drive ability due to elimination of gearing have less overall cost and higher payload to robot weight ratio. However, the control of flexible manipulators to maintain accurate positioning is an extremely challenging problem. Due to the flexible nature and distributed characteristics of the system, the dynamics are highly nonlinear and complex. Problems arise due to precise positioning requirement, vibration due to system flexibility, the difficulty in obtaining accurate

model and non-minimum phase characteristics of the system. Furthermore, considerable researches have been carried out on the control of flexible structures over past 2 decades. The control strategies for flexible robot manipulator systems can be classified as feed-forward and feedback control schemes. Feed-forward techniques for vibration suppression do not require any additional sensors or actuators and do not account for changes in the system once the input is developed (Mohamed and Tokhi, 2004). However, feedback control techniques use measurement and estimations of the system states to reduce vibration. Feedback controllers are designed to be robust to parameter uncertainty. Over the years, a wide range of control laws as PD, nonlinear, linear-quadratic, adaptive and Linear Quadratic Gaussian (LQG) have been synthesized for flexible manipulators (Wolovich, 1994; Wang and Gao, 2003; Moberg, 2010). In the same way, a number of research investigations exploit soft computing approaches such as fuzzy logic and neural network techniques to increase level of control reliability and effectiveness (Passino and Yurkovich, 1998; De Abreu and Ribeiro, 2002; Zain et al., 2004; Subudhi and Morris, 2009). Thus, the motivation of the present research is to develop a control approach based on mamdani and takagi-sugeno fuzzy systems to determine a global controller used for the vibration reduction and the input tracking of a flexible single-link manipulator.

# MATERIALS AND METHODS

The flexible manipulator system: A schematic representation of the single-link flexible manipulator system is shown in Fig. 1 where a control torque  $\tau(t)$  is applied at the hub by an actuator motor with E, I,  $\rho$ , L and IH represent Young's modulus, second moment of area, mass density per unit volume, length and hub inertia moment, respectively (Mohamed and Tokhi, 2004; Zain et al., 2004). The angular displacement of the link in the  $X_0OY_0$  coordinates is denoted as  $\theta(t)$ . w (x, t)represents the elastic deflection of the manipulator at a distance x from the hub, measured along the OX axis. X<sub>0</sub>OY<sub>0</sub> and XOY represent the stationary and moving frames, respectively. The height (width) of the link is assumed to be much greater than its depth, thus allowing the manipulator to vibrate dominantly in the horizontal direction (X<sub>0</sub>OY<sub>0</sub> plane). To avoid difficulties arising from time varying lengths, the length of the manipulator is assumed to be constant. Moreover, the shear deformation, rotary inertia and the effect of axial force are ignored. For an angular displacement  $\theta$  and an elastic deflection w, the total displacement y (x, t) of a point along the manipulator at a distance x from the hub can be described as a function of both the rigid body motion  $\theta$  (t) and the elastic deflection w (x, t) (Meirovitch, 1982):

$$y(x,t) = x\theta(t) + w(x,t)$$
 (1)

Thus by allowing, the manipulator to be dominantly flexible in the horizontal direction, the elastic deflection of the manipulator can be assumed to be confined to the horizontal plane only. Kinetic energy of the flexible manipulator depends of hub rotation, modes rotation in the  $X_0\mathrm{O}Y_0$  and  $X\mathrm{O}Y$  frames and has the flowing expression:

$$T = \frac{1}{2} I_{H} \dot{\theta}^{2} + \frac{1}{2} (\dot{q} + L\dot{\theta})^{T} M (\dot{q} + L\dot{\theta})$$
 (2)

Potential energy just depending of link flexibility has the form:

$$V = \frac{1}{2} q^{T} K q \tag{3}$$

After applying Lagrange's equations, the dynamic model can be written as:

$$\begin{split} &(I_H + L^T M L) \ddot{\theta} + L^T M \ddot{q} = \tau \\ &M \ddot{q} + L^T M \ddot{\theta} + K q = 0 \end{split} \tag{4}$$

where, M, K and L are the mass matrix, the stiffness matrix and the length array, respectively and q is the elastic odes vector.

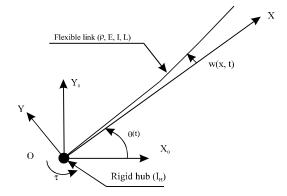


Fig. 1: Flexible manipulator scheme

**Dynamic behavior:** The dynamic equations can be presented in a sate-space form as:

$$\dot{\mathbf{v}} = \mathbf{A}\mathbf{v} + \mathbf{B}\mathbf{u}, \quad \mathbf{y} = \mathbf{C}\mathbf{v} + \mathbf{D}\mathbf{u}$$
 (5)

where, the state-space matrices are:

$$A = \begin{pmatrix} 0 & I \\ -M^{-1}K & 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ M^{-1} \end{pmatrix}, C = \begin{pmatrix} I & 0 \end{pmatrix}, D = \begin{pmatrix} 0 \end{pmatrix}$$

The state and control vectors are given by:

$$\begin{aligned} \boldsymbol{v}^{\scriptscriptstyle T} = & \begin{pmatrix} \boldsymbol{\theta} & \boldsymbol{q}_{\scriptscriptstyle 1} & \boldsymbol{q}_{\scriptscriptstyle 2} & \cdots & \dot{\boldsymbol{\theta}} & \dot{\boldsymbol{q}}_{\scriptscriptstyle 1} & \dot{\boldsymbol{q}}_{\scriptscriptstyle 2} & \cdots \end{pmatrix} \\ \boldsymbol{u}^{\scriptscriptstyle T} = & \begin{pmatrix} \boldsymbol{\tau} & \boldsymbol{0} & \cdots \end{pmatrix} \end{aligned}$$

Solving the state-space matrices gives the vector of states v that is the hub angle, the elastic modes and their velocities. The derived dynamic model is non-minimum phase not strictly proper and unstable. Also, the model has zeros very close to the imaginary axis; deteriorates the time domain performance of the closed-loop system. In order to simulate the flexible manipulator system, an aluminum type flexible link of dimensions L = 0.61 m and  $S = 3 \times 10^{-5} \text{ m}^2 \text{ with } E = 200 \times 10^9 \text{ N m}^{-2}, I = 2.5 \times 10^{-12} \text{ m}^4,$  $I_H = 4.3 \times 10^{-3} \text{ kg m}^2 \text{ and } \rho = 7.8 \times 10^3 \text{ kg m}^{-3} \text{ is considered.}$ The link is discretized into 2 elements. Generally, linear models of flexible structures used in design of controllers are derived under restrictive assumptions which are often not valid for large motions that occur during slewing maneuvers. Hence, considerable uncertainty in the linear model exists. Another feature characteristic of lightly damped systems is the occurrence of poles ( $\pm 159.67$ j,  $\pm 37$ j, 0, 0) and zeros (±158j, ±25j) very close to the imaginary axis that gives rise to ill-conditioned systems. The state-space matrices arising out of such systems have largely separated singular values, posing considerable computational difficulty in controller design. In the spectral density given by Fig. 2, the vibration frequencies

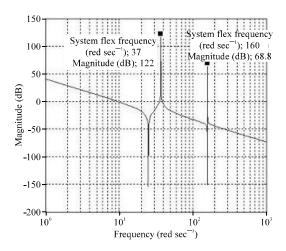


Fig. 2: Open-loop frequency response

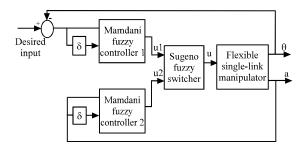


Fig. 3: Structure of the globel fuzzy controller for the flexibel manipulator

of the system are obtained as 37 and 160 rad sec<sup>-1</sup>, i.e., 5.9 and 25.46 Hz and the magnitude of frequency response for the 2 resonance modes are 122 and 68.8 dB.

Global fuzzy control: The concept of fuzzy theory was introduced by Zadeh in 1965 and used to describe dynamic systems that are too complex and/or too ill-defined to synthesize controllers using conventional mathematical modeling techniques. Mamdani applied the fuzzy set theory for developing fuzzy controllers for controlling dynamic systems and since, then many more researches have developed fuzzy controllers for various applications. Sugeno and Takagi proposed another kind of fuzzy systems which uses linear functions in rule consequent and the centroid defuzzifier applied mainly in modeling approaches (Passino and Yurkovich, 1998; De Abreu and Ribeiro, 2002; Subudhi and Morris, 2009).

The main difference between Mamdani and Sugeno fuzzy systems lies in the consequent of the fuzzy rules: the former uses fuzzy sets whereas the latter employs linear functions of input variable. Both types of fuzzy systems have been used as effective tools to solve various practical problems mainly in the

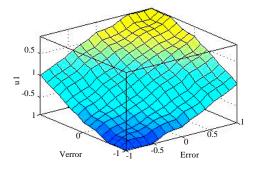


Fig. 4: Control surface of the Mamdani controller

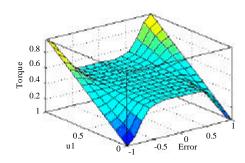


Fig. 5: Control surface of the Sugeno switcher

fields of modeling and control. The main control objective in the case is to reach the desired position of the hub angle with zero end-point vibration. The structure of the proposed global fuzzy controller is shown in Fig. 3.

In this control design, the use of 2 uncoupled direct Mamdani fuzzy controllers is investigated for the flexible robot. Each controller has 2 inputs and one output as shown in Fig. 3. For the first Mamdani fuzzy controller, the inputs are the hub angle error and its variation for the second one, the inputs are end-point acceleration and its variation.

The input and output universes of discourse of the fuzzy controllers are normalized on the range [-1, 1]. The use of scaling gains is fuzzified inputs and outputs have triangular membership functions. The rule base array is composed of  $5\times 5$  array as we have 5 fuzzy sets on the input universes of discourse. The rule base principle is given as:

- If error is negative then torque signal is negative
- If error is positive then torque signal is positive
- If error is zero then torque signal is zero

The signal command applied to the flexible manipulator is the result of a commutation between the 2 mamdani fuzzy controllers doing by the sugeno fuzzy switcher that has 3 inputs: the hub angle error and the

2 command signals deduced from the mamdani systems and one output; the resulted command. The commutation is based on the following rule base:

- If hub angle error is large then torque signal = u1
- If hub angle error is small then torque signal = u2

where, u1 is thehub angle command and u2 is the end-point command. The control surface for the fuzzy controllers in Fig. 4 and 5 shows the fuzzified outputs plotted against its 2 fuzzified inputs to notice differences between nonlinear and linear consequences.

# RESULTS AND DISCUSSION

In order to evaluate the performance of the global fuzzy controller, a block scheme is organized in a Matlab-Simulink file, fuzzy controllers are constructed in a separated graphical interfaces and then introduced in the precedent file. Desired input angle is chosen a constant equal to 90° (1.57 rad). Total 3 responses of hub angle,

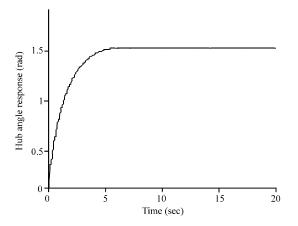


Fig. 6: Hub angle response

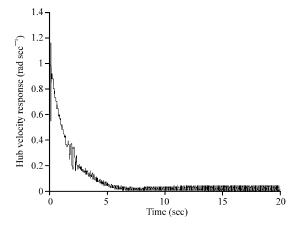


Fig. 7: Hub velocity response

hub velocity and end-point residual are respectively shown in Fig. 6-8. Criteria used to evaluate the performances of the control scheme are level of vibration reduction and time response specifications. So, parameters evaluated are settling time, rise time and the magnitude of vibration. Referring to simulation results, the rise time is obtained as 3 sec and the settling time as 4 sec for the hub angle response that reaches desired position without overshoot. The vibration amplitude is found to

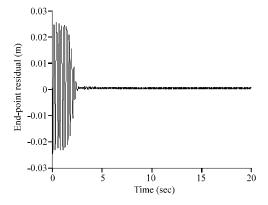


Fig. 8: End-point residual response

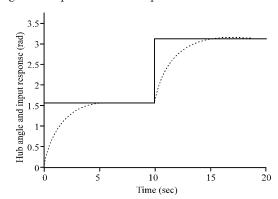


Fig. 9: Hub angle input tracking response

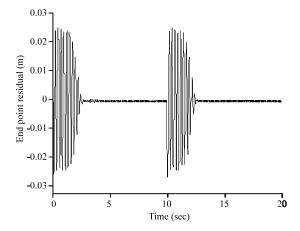


Fig. 10: End-point residual response

oscillate between -0.025 m in min and +0.025 m in max. So, the system is stable and accurate and the end-point vibration is reduced clearly and canceled rapidly. To study input tracking, another desired position input is applied with 2 amplitudes between 0 and 10 sec, we have 90° (1.57 rad) and since, 10 sec, it becomes 180° (3.14 rad). The simulation results are shown in Fig. 9 and 10. A good input tracking is observed in the hub angle response with an important vibration reduction in spite of the rapid change of the desired input.

#### CONCLUSION

This study has developed a global hybrid fuzzy control scheme proposed for a flexible single-link manipulator to reduce end-point vibration and to realize a good input tracking. A flexible single-link manipulator is first considered. The dynamic model is developed using Lagrange's equations associated to finite element method, an open-loop study is then done to deduce system characteristics. Mamdani and Takagi-Sugeno fuzzy systems are utilized to construct the global fuzzy controller which is contained 2 Mamdani controllers for the hub angle (rigid body) and the end-point residual (elastic modes) loops and a Takagi-Sugeno fuzzy switcher to commute between fuzzy controllers according to an adequate expertise which allows to achieve good performance in vibration reduction and input tracking.

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