Journal of Engineering and Applied Sciences 5 (6): 420-426, 2010
ISSN: 1816-949X

Wind Speed Distributions and Power Densities of Some Cities in Northern Nigeria

J. Aidan and J.C. Ododo
Department of Physics, Federal University of Technology Yola, Adamawa State, Nigeria

Abstract: The wind speed data of 8 cities in Northern Nigeria have been fitted to four distribution functions (normal, Weibull, Rayleigh and gamma). The goodness-of-fit at the 5% significance level has been determined by using chi-square, Kolmogorov-Smirnov and Anderson-Darling tests. For four of the cities (Bida, Minna, Yelwa and Yola) which have the lowest altitudes (<260 m) the gamma distribution is found to give the best fit while for the other cities (Gusau, Kaduna, Maiduguri and Zaria) either the Weibull or normal distribution gives the best fit. By considering the predicted wind speed variations at various heights above the ground, it is seen that the potential for utility-scale wind power generation at a height of about 80 m is very satisfactory especially for Gusau, Kaduna, Maiduguri and Zaria.

Key words: Wind speed, distribution function, goodness-of-fit, power density, utility-scale, Nigeria

INTRODUCTION

An estimated 1-3% of the energy from the sun that reaches the earth is converted into wind energy through convection and Coriolis forces. Having a cubic relation with the power, the wind speed is the most critical parameter needed to appraise the power potential of a candidate site. The wind is never steady at any site. It is influenced by the weather system, the local land terrain and the height above the ground surface. Therefore, the annual mean wind speed needs to be averaged over at least 10 years so as to raise the confidence in assessing the energy-capture potential of a site. Moreover because windiness varies, an average value for a given location does not alone indicate the amount of energy a wind turbine could produce there. To assess, the climatology of wind energy at any location, probability distribution functions are often needed to fit the observed wind speed data (Patel, 1999).

In this study, the Weibull, gamma, Rayleigh and the normal distribution functions were fitted onto constructed frequency diagrams of the wind speed data of 8 stations (Table 1) and validated both with standard statistical goodness-of-fit tests. Also, the corresponding power density potentials of the sites were determined.

MATERIALS AND METHODS

The mesa-scale 3 h records of monthly average wind speeds (measured in knots) at a height of 3 m with their stations coordinates are collected from the Meteorological Department, Climate Investigation Unit of the Federal Ministry of Aviation, Lagos, Nigeria. The station coordinates and dates of records are shown in Table 1. The actual wind speed data have been restructured into histograms which are shown in Fig. 1 and 2.

Table 1: Station coordinates and dates of records

Station	Alt., (hm)	Lat., (°N)	Long., (°E)	Dates of records
Bida	144	09.100	06.017	1978-86
Gusau	464	12.167	06.700	1978-88
Kaduna	645	10.360	07.270	1979-88
Maiduguri	354	11.850	13.083	1979-88
Minna	259	09.617	06.467	1978-87
Yelwa	244	10.333	04.750	1978-88
Yola	186	09.233	12.467	1978-88
Zaria	656	11.130	07.680	1978-88

Distribution functions: The 4 distribution functions used are briefly described below:

The normal distribution function: The density function of the normal distribution function f (v) is given as:

$$n(v)=\frac{1}{\sqrt{2\pi}\,\sigma}\exp\left[-\tfrac{1}{2}\left(\frac{v-\mu}{\sigma}\right)^2\right],0\leq\ v\leq\infty \tag{1}$$

where, μ and σ are respectively the mean and standard deviation of v and are the parameters of the distribution.

The weibull distribution function: The formula for the probability density function h (v) for the 2-parameter Weibull distribution is (Weibull, 1951):

Corresponding Author: J. Aidan, Department of Physics, Federal University of Technology Yola, Adamawa State, Nigeria

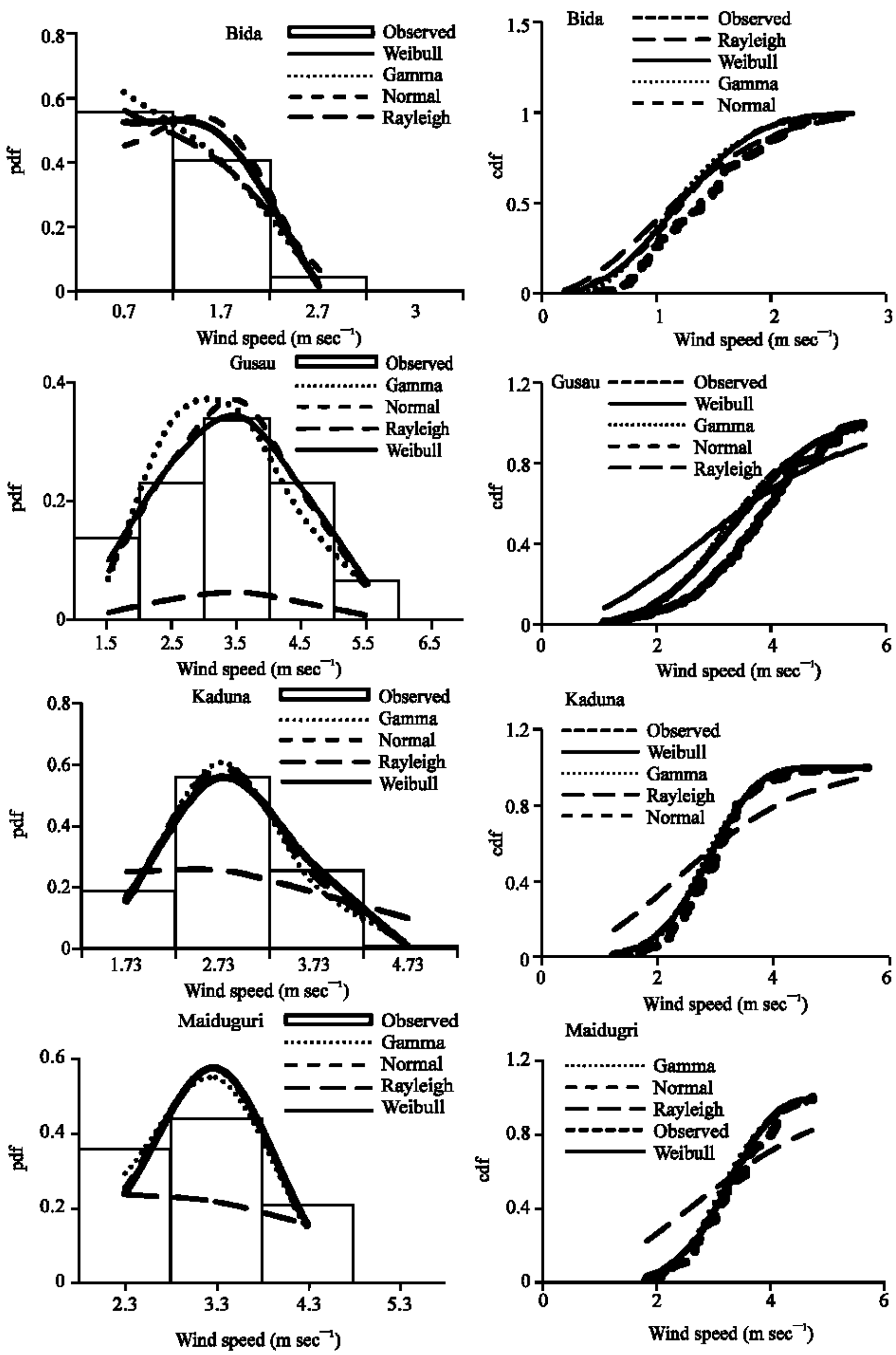

Fig. 1: Fitted probability density functions (pdf) and cumulative density functions (cdf) for Bida, Gusau, Kaduna and Maiduguri

$$h(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^{k}\right], v \geq 0; k, c > 0 \qquad (2)$$

Where:

k = Determines the shape of the Weibull distribution

c = The scale parameter (Justus *et al.*, 1976; Justus, 1978)

$$k = \left(\frac{\sigma}{\mu}\right)^{-1.086} \qquad (3)$$

When k = 2, Eq. 1 reduces to the Rayleigh distribution function:

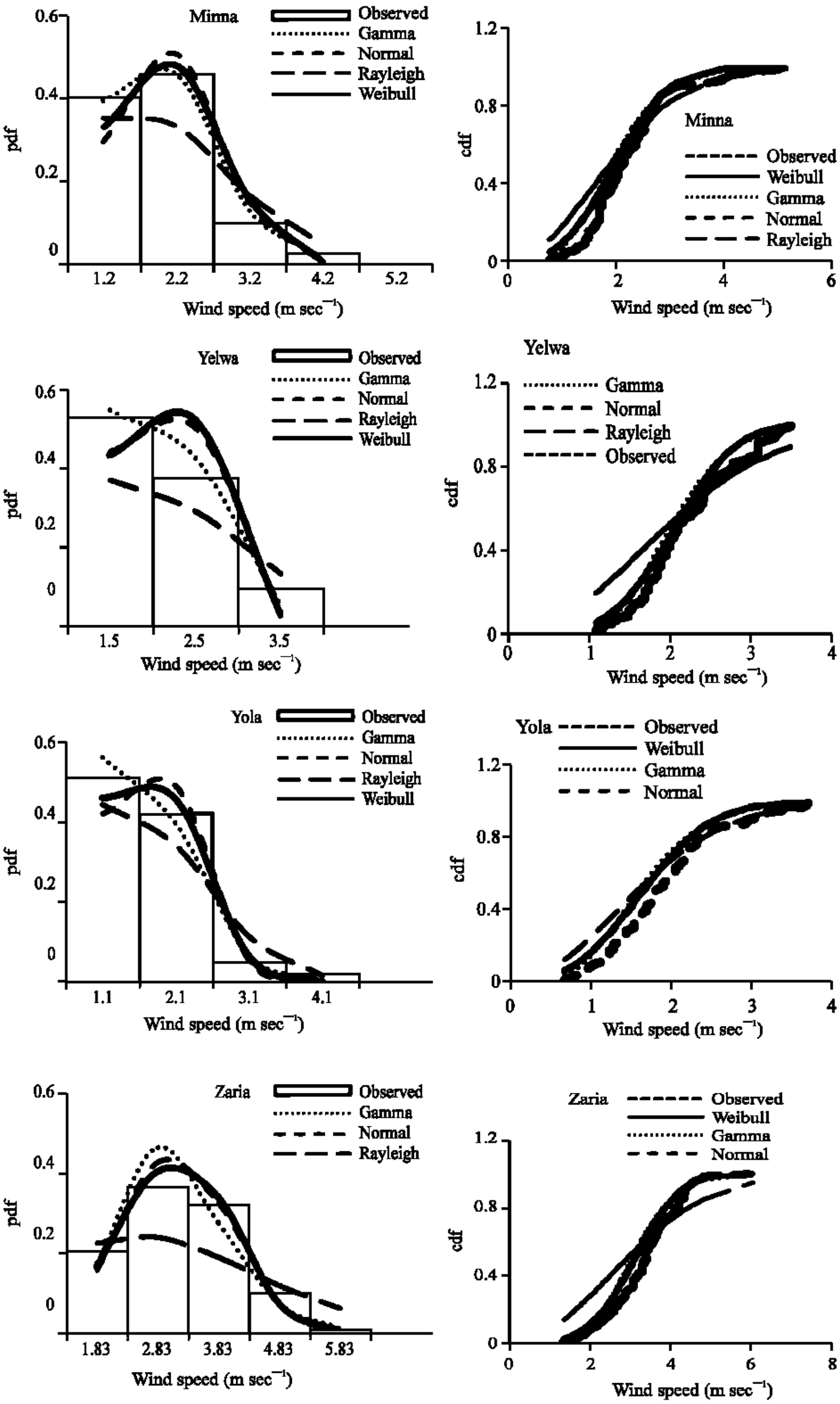

Fig. 2: Fitted probability density functions (pdf) and cumulative density functions (cdf) for Minna. Yelwa, Yola and Zaria

$$R(v) = \frac{2v}{c^2} \exp\left[-\left(\frac{v}{c}\right)^2\right] v \geq 0 \ , c > 0 \qquad (4)$$

The gamma distribution function: The probability density function of the 2-parameter gamma distribution function is defined as:

$$g(\nu) = \frac{\nu^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} e^{-\frac{\nu}{\beta}} \nu, \alpha, \beta > 0 \quad (5)$$

where, $\Gamma(\beta)$ is the gamma function.

Goodness-of-fit tests: The Chi-square (χ^2), Kolmogorov-Smirnov (KS) and the Anderson-Darling (AD) goodness-of-fit tests are used to validate the fitted distribution functions at 5% significance level (i.e., 0.05). It should be noted that a distribution function that is acceptable at one significance level may be unacceptable at another significance level. Therefore, such tests remain useful for determining the relative goodness-of-fit of two or more theoretical distributions. The details for these goodness-of-fit tests can be read elsewhere (Aifredo and Wilson, 1975; Romeu, 2003a, b; Law and Kelton, 1991).

Wind power density: The root-mean-cube wind power density is given as:

$$P_{rmc} = \frac{1}{2} C_p P \nu_{rmc}^3 \quad (6)$$

Where:

- C_p = The rotor efficiency (in this research, it is taken as 0.5)
- P = The air density corresponding to turbine height h above sea level and is given asp = $p_c e^{-ah}$ (a = 9.744×10^{-5} m^{-1}, p_c = 1.225 kgm^3) and ν_{rmc} is the root-mean-cube wind velocity

For a given site's specific distribution function, f (ν) the root-mean-cube wind velocity, ν_{rmc} is given as:

$$\nu_{rmc} = \left(\int_0^{\infty} \nu^a f(\nu) d\nu \right)^{\frac{1}{3}} \quad (7)$$

Hence, the specific site's root-mean-cube power density can then be rewritten as:

For f (ν) Weibull fitted:

$$P_{rmc} = \frac{1}{2} p C_{pc}{}^3 \Gamma\left(1 + \frac{3}{k}\right) \quad (8)$$

For f (ν) gamma fitted:

$$P_{rmc} = \frac{1}{2} p C_p \beta^3 [\alpha(\alpha+2)(\alpha+1)] \quad (9)$$

For f (ν) normally fitted:

$$P_{rmc} = \frac{1}{2} C_p p \left\{ \frac{1}{\sqrt{2\pi}} (2\sigma^2 + 3\sigma\mu^2) + \frac{1}{2}(\mu^2 + \mu\sigma^2) \right\} \quad (10)$$

The mean wind speed, ν_{mean} for n observation is given as:

$$\nu_{mean} = \frac{1}{n} \sum_{1=1}^{n} \nu i, i = 1, 2, \ldots\ldots, n \quad (11)$$

RESULTS AND DISCUSSION

The estimates of the parameters of the distribution functions determined from the observed wind speed data for all stations are shown in Table 2. Figure 1 and 2 show the fitted probability density functions (pdf) onto histograms constructed from the observed wind speed data for all the stations together with their corresponding cumulative density functions (cdf). The goodness-of-fit tests that determine which of the distribution functions best fit the data are shown in Table 3. The blank spaces in the table indicate unavailability of critical values and/or test statistics for the given conditions or distribution functions. The χ^2-test, for instance, requires at least one degree of freedom to determine distribution acceptance.

However in the absence of this, the best distribution function can still be determined on the relative comparison of their x^2 values. For every station, priority is given first to the distribution function (s) that is/are accepted by the AD test because of its sensitivity before the KS and then the χ^2-tests.

It is seen from Table 3 that Bida, Minna, Yelwa and Yola can be best represented by the gamma distribution function whereas Gusau, Kaduna, Maiduguri and Zaria are better fitted by either the normal or Weibull distribution function. It is to be noted that those stations which are fitted by gamma

Table 2: Parameters of the distribution functions

	Gamma distribution		Normal distribution		Rayleigh distribution	Weibull distribution	
Stations	α	β	μ	σ	(c)	k	c
Bida	5.981	0.206	1.235	0.505	1.393	2.641	1.389
Gusau	9.775	0.344	3.361	1.075	3.792	3.448	3.738
Kaduna	17.631	0.160	2.827	0.673	3.190	4.750	3.088
Maiduguri	21.559	0.147	3.174	0.684	3.582	5.299	3.446
Minna	6.981	0.288	2.009	0.760	2.266	2.872	2.253
Yelwa	11.971	0.171	2.044	0.591	2.307	3.850	2.260
Yola	6.487	0.257	1.665	0.654	1.879	2.760	1.871
Zaria	12.349	0.252	3.116	0.887	3.516	3.915	3.442

Table 3: Goodness-of-fit test results for all stations

		χ^2-test		K-S test		Anderson-Darling test		
Station/n	pdf	Test value	Decision rule	Test value	Decision rule	Test value	Critical value	Decision rule
Bida n = 94	Weibull	4.786	-	0.0937	Accept	0.0291	0.0500	Reject
	gamma	0.753	-	0.0731	Accept	-	-	-
	Rayleigh	0.735	-	0.1526	Reject	-	-	-
	normal	11.833	-	0.1157	Accept	1.1497	0.7459	Reject
	C. value	DoF=0	-	0.1403	-	-	-	-
Gusau n = 87	Weibull	2.704	Accept	0.0530	Accept	0.6414	0.0500	Accept
	gamma	10.527	Reject	0.0953	Accept	-	-	-
	Rayleigh	7.952	Reject	0.1503	Reject	-	-	-
	normal	4.523	Accept	0.0504	Accept	0.2906	0.7454	Accept
	C. value	5.99 at DoF = 2	-	0.1458	-	-	-	-
Kaduna n = 107	Weibull	24.983	Reject	0.0922	Accept	0.0569	0.0500	Accept
	gamma	1.110	Accept	0.0829	Accept	-	-	-
	Rayleigh	38.281	Reject	0.2371	Reject	-	-	-
	normal	5.604	Reject	0.0812	Accept	0.3954	0.7466	Accept
	C. value	3.84 at DoF = 1	-	0.1315	-	-	-	-
Maiduguri n = 98`	Weibull	9.811	Reject	0.0911	Accept	0.0408	0.0500	Reject
	gamma	6.076	Reject	0.0725	Accept	-	-	-
	Rayleigh	3.108	Accept	0.2540	Reject	-	-	-
	normal	8.567	Reject	0.0809	Accept	0.5113	0.7461	Accept
	C. value	3.84 at DoF = 1	-	0.1374	-	-	-	-
Minna n = 70	Weibull	4.731	Reject	0.0884	Accept	0.0047	0.0500	Reject
	gamma	0.769	Accept	0.0840	Accept	-	-	-
	Rayleigh	5.429	Reject	0.1751	Reject	-	-	-
	normal	7.384	Reject	0.1050	Accept	0.9663	0.7437	Reject
	C. value	3.84 at DoF = 1	-	0.1626	-	-	-	-
Yelwa n = 93	Weibull	21.669	-	0.1102	Accept	0.0041	0.0500	Reject
	gamma	5.024	-	0.0642	Accept	-	-	-
	Rayleigh	3.450	-	0.2019	Reject	-	-	-
	normal	15.814	-	0.1017	Accept	-3.030	0.7458	Reject
	C. value	DoF=0	-	0.1410	-	-	-	-
Yola n = 102	Weibull	37.538	Reject	0.0636	Accept	0.0487	0.0500	Reject
	gamma	3.980	Reject	0.0395	Accept	-	-	-
	Rayleigh	5.644	Reject	0.1269	Accept	-	-	-
	normal	62.901	Reject	0.0747	Accept	1.0310	0.7464	Reject
	C. value	3.84 at DoF = 1	-	0.1347	-	-	-	-
Zaria n = 128	Weibull	4.204	Accept	0.0449	Accept	0.4523	0.0500	Accept
	gamma	19.746	Reject	0.0905	Accept	-	-	-
	Rayleigh	26.076	Reject	0.1703	Reject	-	-	-
	normal	7.756	Reject	0.0492	Accept	0.4136	0.7475	Accept
	C. value	5.99 at Dof = 1	-	0.1202	-	-	-	-

distribution function are generally of lower wind speed distributions (i.e., in the range of 1 and 2 m sec^{-1}) and their pdf graphs especially at higher wind speeds closely approach the exponential distribution function implying a rapidly changing wind speed sites. Such rapidly changing wind speeds may be unsuitable for the installation of wind power turbines.

Table 4 gives the predicted mean and root-mean cube (rmc) wind speeds in m sec^{-1} and the extractible wind power densities in Wm^{-2} derived from the station's best fitted distribution function at a C_p = 0.5 for all the stations at various heights. The minimum and the maximum wind speed, excluding gust and the associated values of the distribution parameters at the predicted heights are also presented. It could be seen that v_{rmc} are generally >v_{mean} for all the stations. This means that a representation of a site's wind potential in terms of an average wind speed is an under estimation of the site's actual potential. Gusau, Kaduna, Maiduguri and Zaria have shown greater extractible wind power densities than the others. The values of the extractible wind power densities obtained from these sites show that a medium or a larger size wind turbines installed especially at heights of 60 and 80 m above the ground could generate sufficiently high wind power if not for electricity, at least for water pumping. Figures 3a-c shows the variations of predicted values, using the Hellman power law (Musgrove, 1987) (with a friction coefficient of 0.2), of the monthly mean wind speeds for all the stations at heights of 30, 50 and 80 m, respectively. The 4 m sec^{-1} straight line that cut across the variations in the figures represent the cut-in wind speed required by most modern wind power turbines. At these heights, Gusau, Maiduguri and Zaria are the most suitable site for wind power generation with at least 8 month wind availability except that the capacity factor may be very low as the maximum wind speeds for these sites are less than the rated wind speeds (i.e., 10-15 m sec^{-1}) for most turbines.

Table 4: The predicted mean, mode and rmc wind speed (m sec^{-1}) and extractible wind power densities (Wm^{-2}) at various heights for all stations at $C_p = 0.5$ and ρ at corresponding heights

Stations	h (m)	$\frac{\beta}{c\sqrt{\sigma}}$	$v_{min.}$	$v_{max.}$	v_{mean}	v_{rmc}	P_{mean}	P_{rmc}
Bida (gamma)	30	0.3271	0.33	4.32	1.96	2.27	2.26	3.51
	50	0.3623	0.36	4.79	2.17	2.51	3.06	4.76
	60	0.3758	0.37	4.96	2.25	2.61	3.41	5.31
	70	0.3875	0.39	5.12	2.32	2.69	3.73	5.82
	80	0.3980	0.40	5.26	2.38	2.76	4.04	6.30
Gusau (Weibull)	30	5.9245	1.71	8.89	5.33	5.83	44.10	57.76
	50	6.5618	1.89	9.84	5.90	6.45	59.80	78.32
	60	6.8055	1.96	10.21	6.12	6.69	66.64	87.29
	70	7.0186	2.03	10.53	6.31	6.90	73.03	95.66
	80	7.2086	2.08	10.81	6.48	7.09	79.05	103.54
Kaduna (Weibull)	30	4.8940	1.96	8.89	4.48	4.72	25.78	30.16
	50	5.4204	2.17	9.84	4.96	5.23	34.96	40.90
	60	5.6217	2.25	10.21	5.15	5.42	38.97	45.58
	70	5.7977	2.32	10.52	5.31	5.59	42.70	49.95
	80	5.9546	2.38	10.81	5.45	5.74	46.21	54.06
Maiduguri (normal)	30	1.0834	2.85	7.50	5.03	5.48	37.55	48.56
	50	1.2000	3.16	8.31	5.57	6.07	50.92	65.85
	60	1.2445	3.28	8.62	5.78	6.30	56.75	73.39
	70	1.2835	3.38	8.89	5.96	6.49	62.19	80.42
	80	1.3182	3.47	9.13	6.12	6.67	67.31	87.04
Minna (gamma)	30	0.4560	1.22	8.15	3.18	3.62	9.60	14.12
	50	0.5050	1.35	9.03	3.53	4.01	13.02	19.15
	60	0.5238	1.40	9.36	3.66	4.16	14.51	21.34
	70	0.5407	1.45	9.66	3.77	4.29	15.90	23.39
	80	0.5548	1.49	9.92	3.87	4.40	17.21	25.31
Yelwa (gamma)	30	0.2706	1.71	5.54	3.24	3.50	10.14	12.82
	50	0.2998	1.90	6.14	3.59	3.88	13.75	17.39
	60	0.3109	1.97	6.37	3.72	4.02	15.32	19.38
	70	0.3206	2.03	6.57	3.84	4.15	16.79	21.24
	80	0.3293	2.08	6.75	3.94	4.26	18.18	22.99
Yola (gamma)	30	0.4067	1.06	5.87	2.64	3.03	5.51	8.32
	50	0.4504	1.17	6.50	2.92	3.35	7.47	11.28
	60	0.4672	1.22	6.74	3.03	3.48	8.32	12.57
	70	0.4818	1.26	6.95	3.13	3.59	9.12	13.77
	80	0.4948	1.29	7.14	3.21	3.68	9.87	14.90
Zaria (Weibull)	30	5.4547	2.21	9.62	4.94	5.31	34.49	42.89
	50	6.0414	2.35	10.65	5.46	5.88	46.77	58.17
	60	6.2658	2.43	11.05	5.67	6.10	52.12	64.83
	70	6.4619	2.51	11.40	5.85	6.29	57.12	71.04
	80	6.6368	2.58	11.71	6.01	6.46	61.82	76.89

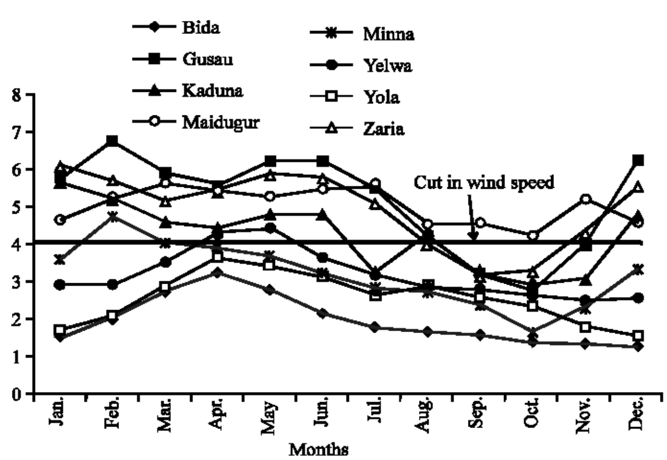

Fig. 3: Continued

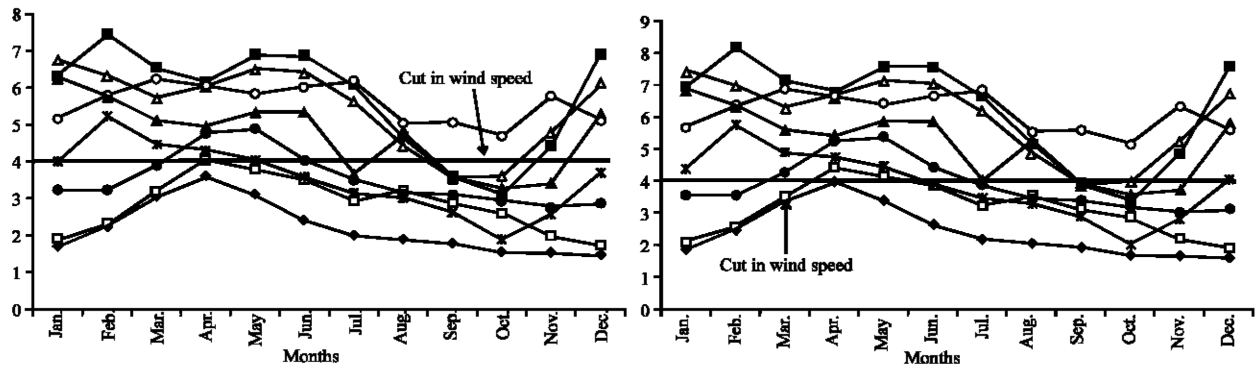

Fig. 3: Monthly variations of predicted monthly mean wind speed for all stations at a height of a) 30 m; b) 50 m and c) 30 m

However, at heights ≥80 m wind power generation from a majority of these sites may be economically satisfactory.

CONCLUSION

The distribution function that best fits the wind speed data of eight cities in Northern Nigeria has been determined. At a 5% significance level the speed data for Bida, Minna, Yelwa and Yola are best, represented by the gamma distribution function while those of Gusau, Kaduna, Maiduguri and Zaria are best fitted by either the Weibull or normal distribution function. The potential for utility-scale wind power generation at a height of about 80 m may be satisfactory especially for Gusau, Kaduna, Maiduguri and Zaria which incidentally have the highest altitudes (>350 m).

REFERENCES

Aifredo, H.S. and H.T. Wilson, 1975. Probability Concepts in Engineering Planning and Design. John Wiley and Sons Inc., New York, pp: 277-279.

Justus, C., 1978. Wind and Systems Performance. Franklin Institute Press, Philadelphia, USA.

Justus, W.R., W.R. Harngrave and A. Yalcin, 1976. Nationwide assessment of output from wind power generators. J. Appl. Meteor., 15: 673-678.

Law, A.M. and W.D. Kelton, 1991. Simulation Modeling and Analysis. McGraw-Hill Inc., New York.

Musgrove, P.J., 1987. Wind energy conservation. Recent progress and future prospects. Solar Wind Technol., 4: 37-44.

Patel, M.R., 1999. Wind and Solar Power System. CRC press, London, pp: 4-91.

Romeu, J.L., 2003a. Anderson-darling: A goodness-of-fit test for small samples assumptions. START, Vol. 10, No. 5.

Romeu, J.L., 2003b. Kolmogorov-smirnov: A goodness-of-fit test for small samples. RAC START, Vol. 10, No. 6.

Weibull, W., 1951. A statistical distribution function of wide applicability. J. Applied Mech., 18: 253-253.