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Stochastic Approximation Simulation for Highway Vertical Curves Elements

I. Akiije
Department of Civil and Environmental Engineering, University of Lagos, Lagos, Nigeria

Abstract: The goal of this study is to examine a stochastic approximation technique for simulation of highway vertical curve elements by using Microsoft Excel 2010 program. The program is general purpose application software and in graphical user interface. Stochastic approximation simulation method established here is approached thus for reducing modeling-development time and towards getting inexpensive design methodology. The vertical curve elements of highway for investigation in this study are parabolic curves. The basic concept presented here is how to move progressively toward realisation of the best-value stochastic approximation algorithm for the development of vertical curves while providing adequate sight distance leading to the ability to see ahead on highway for safe, comfort and efficiency operation of a vehicle. Stochastic approximation simulation work is described in this study as a result of greater emphasis on automation modeling via Information Communication Technology (ICT) to improve productivity and quality of analysis and design of complex system such as highway. It is pertinent to note that alternative proposed vertical curves during design stage can be compared through stochastic approximation simulation modeling analysis methodology based upon an authentic policy for geometric design of highways and streets. Hence, a successful optimal system to meet specified requirements conforming to able policy on geometric design of highways and street has been established in this study via Microsoft Excel 2010 similarly like conventional hand method.

Key words: Stochastic, simulation, modeling, analysis, safe, comfort, vertical curve, Nigeria

INTRODUCTION

Background: Stochastic approximation is a method of stochastic-optimization including technique such as gradient search (Weisstein, 2010) that is useful for designing a vertical curve for highway geometric design. Stochastic-optimization refers to the minimization (or maximization) of a function (a vertical curve) in the presence of stochastic approximation in the optimization process.

A parabolic curve is used exclusively in connecting profile grade tangents according to Wright and Dixon (2003) and the paradigm is useful in this study to automate crest and sag vertical curves by using Microsoft Excel 2010 which is general purpose application software and it is in graphical user interface. When a vertical curve connects a positive grade with a negative grade, it is referred to as a crest curve and for a vertical curve to connect a negative grade with positive grade is termed a sag curve.

Processing and evaluation of highway vertical curve elements by simulation approach is currently an active area of research (Garber and Hoel, 2010). Since, the early days of simulation people have constantly looked for new and better ways to model a system as well as novel ways to use existing computer hardware and software simulation. A simulation model (gradient or curve) can be stochastic if contains probabilistic or random (subjective) parameter according to Law and Kelton (2007) and Institute of Transportation Engineers (ITE, 2008). Highway vertical curve elements are usually modelled as having at least some arbitrary input components as asserted by AASHTO (2004) (America Association of State Highway and Transportation Officials), Amekudi and Meyer (2006) and FHWA (2006) (Federal Highway Authority) together with Wooddrige *et al.* (2003). This paradigm has paved way to stochastic approximation simulation modeling for highway vertical curve that is being examined in this study.

Stochastic is often used as counterpart of the word deterministic which means that random phenomena are not involve (Vincenzo, 2010). The subjective input components for vertical curve design include design speed, gradients, vertical curvature, length of curve, station locations for vertical point of curvature, intersection and tangent lengths.

Most real-world systems are not amenable to compact analytical tools and such systems must be studied by means of simulation (Law and Kelton, 2007; Akiije, 2009). In a simulation practice, the use of Microsoft (MS) Excel package is unique to evaluate mathematical modeling numerically and graphically when data are gathered in order to investigate the desired true characteristics of the model. To understand and cope with change in a stochastic environment, the frontiers of knowledge are being rapidly expanded as a result of new development in computers and information technology (Ibidapo-Obe, 1996).

The aim of this study, therefore is to advance stochastic approximation modeling approach to simulate vertical curves elements in highway geometric design that are dependable and consistent to help reduce accidents on roads and other problems related to driver expectance through the use of Microsoft Excel 2010. Specific objectives of this study include:

- To investigate mathematical and statistical models of highway vertical curves elements
- To investigate the stochastic approximation modeling and simulation for highway vertical curves elements
- To carry out the computation related to vertical curves elements by numerically exercising the required elemental models to see how they affect the output measures of performance of the inputs in the geometric design of highways

Justification for this study is that in the course of the stochastic approximation modeling while contemplating through analysis and design subjectively, a careful simulation of vertical curve elements could shed vivid light on proposed highway safety and cost-effectiveness.

The significance of this study is that the simulated elemental curves with adequate design parameters are genuine parts of a highway that are to serve as guides for the processing of the highway geometry alignments during construction and as they would be when the road work is completed.

Stochastic approximation modeling methodology of highway vertical curve elements is quite independent of the software and hardware used for it is the rudiment of the conceptual framework of geometric design of highways that matters most. This is because geometric design of highways could be approached by any one out of the conventional hand method, general purpose application program or specific purpose application program.

Conceptual framework: The design of the parabolic vertical curves of highways depends primarily on the design speed selected for the highway (Garber and Hoel, 2010). Design standards may vary with the type of terrain (level, rolling and hilly), anticipated traffic to be served and whether the highway is to be in an urban or rural area (Wright and Dixon, 2003). However, mechanism to the solution of indeterminacy to vertical curve length is made in relationship to each of the parabolic curve. In other words, for a parabolic curve in highway geometric design the length is the same whether measured along the tangents, the chord, the horizontal or the curve itself (Uren and Price, 2010).

The cognizance of this mechanism and the use ICT approach leads to stochastic approximation process of modeling to analysis the development of standard vertical curves in this study in MS Excel program environment for optimal achievement.

Meanwhile, available specific purpose programs go obsolete quickly despite being under licence, costly and they are yet to realise the target of the parabolic curve design mechanism optimally.

Figure 1 shows the 4 types of standard vertical parabolic curves models that are classified as Crest and Sag vertical curves (AASHTO, 2004). Vertical curves may be calculated in relationship to Fig. 2 which is an approximation of a circular arc to a simple parabola as follows:

$$A = g_2 - g_1 \quad (1)$$

$$K = \frac{L}{A} \quad (2)$$

$$e = \frac{(G_1 - G_2)L}{8} = \frac{AL}{800} = \frac{A^2K}{800} \quad (3)$$

For any point p on curve:

$$y = \frac{(G_1 - G_2)x^2}{2L} = \frac{Ax^2}{200L} = \frac{x^2}{200K} \quad (4)$$

$$E_x = E_{VPC} + G_1x + \frac{(G_2 - G_1)x^2}{2L} \quad (5)$$

Figure 2 is showing typical vertical curve type I. In Fig. 1 and 2, together with Eq. 1-5: G_1 and G_2 are grades of initial and final tangents, respectively; A is the percentile algebraic difference; L is the length of vertical curve; PVC is the point of vertical curve; PVI is the point of vertical intersection; PVT is the point of vertical tangent; g_1 is the grade of initial tangent in percent; g_2 is the grade of final tangent in percent; K is the vertical curve length coefficient as determined for stopping sight distance; x is

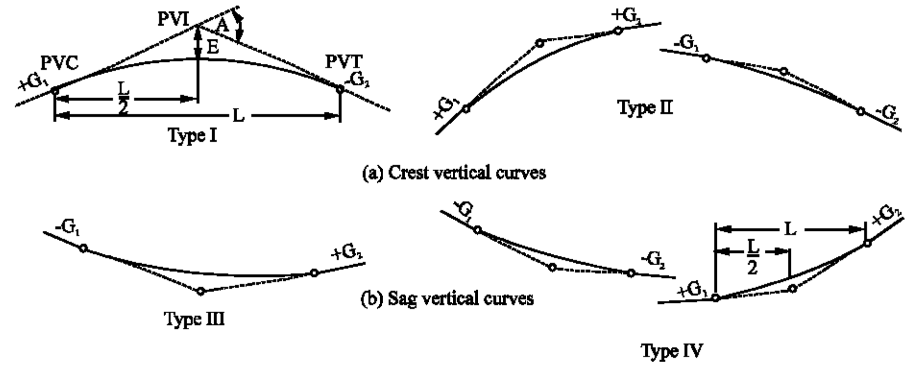


Fig. 1: Type of vertical curves (AASHTO, 2004)

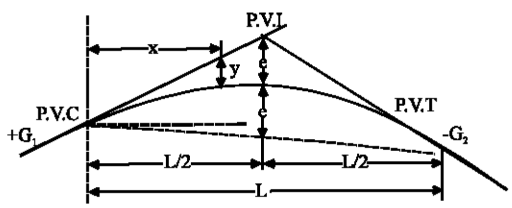


Fig. 2: Typical vertical curve (Wright and Dixon, 2003)

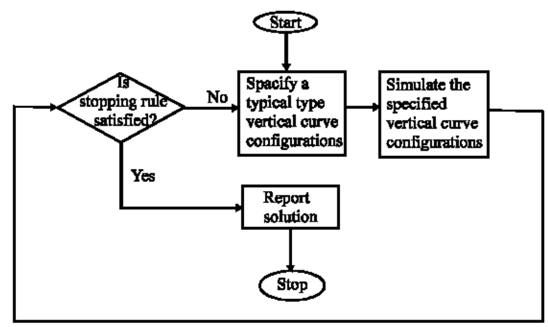


Fig. 3: Optimisation simulation flow chart

the horizontal distance to point on curve, measured from VPC; E_x is the elevation of point on curve located at distance x from VPC; X_m is the location of min/max point on curve, measured from VPC; E_m is the elevation of min/max point on curve at distance X_m from VPC; e = external distance = middle ordinate; y = offset of curve from initial grade line.

According to AASHTO (2004) desirable maximum gradient is 3% for motorway; 4% for all-purpose dual carriageway; 6% for all-purpose single carriageway and for effective drainage a minimum gradient of 0.5% is to be maintained wherever possible.

Excel application program, as presented for use in this study is software that performs simulation by modeling calculations and analysing information in addition to visualising data and graphics as spreadsheet at optimal level. A spreadsheet is a document that stores data in a grid of horizontal rows and vertical columns.

The most commonly used spreadsheet application is Microsoft Excel but several other spreadsheet programs are available including Lotus 1-3 for Windows IBM, Apple Works and Numbers for Mac OS X., Tech Terms.com (2010).

Excel application program is amenable to stochastic approximation simulation for highway vertical curves elements.

MATERIALS AND METHODS

The stochastic approximation simulation methodology introduced in this study is by using spreadsheet program for the verification of the validation of the presented four types of vertical curves in Fig. 1 by modeling approach (Adedimila and Akiije, 2006). This methodology generally involves simulating sequence of a selected vertical curve out of the six configurations as shown in Fig. 1.

In the stochastic approximation process VPI, VPC, VPT, the two intercepting gradients are specified, sight distance and K-values are defined and the reduced levels along the vertical curve are calculated. The process will result to simulated configurations that will suggest promising new directions to search through the space within the right of way of possible input-factor combinations subjectively that will lead to better configurations of a selected vertical curve. The subjective methodology can be made possible while utilising optimisation simulation flow chart setup of Fig. 3. This is indeed a convenient way to make the stochastic approximation process workable to simulate an optimum-seeking sequence of vertical curve configurations in the context of the spreadsheet Table 1.

Table 1: Modeling module for the optimisation of type I crest vertical curve

A-F label	G-I modeling	J unit	K-P label	Q-V modeling	W unit
Vertical point of intersection, VPI station	680	m	Station location for the VPC = VPI Station -L/2 =	= G13-G19/2	m
Elevation for the VPI	93.6	m	Station location for the VPT = VPC Station + L=	= Q13+G19	m
Grade of tangent 1,	1.5	%	Elevation for the VPC, EVPC = EVPI - g_1 (L/2)=	= G14-G15*G19/2/100	m
Grade of tangent 2,	-1	%	Elevation for the VPT, EVPT = EVPI + g_2 (L/2)=	= G14+G16*G19/2/100	m
Design speed	100	km h^{-1}	Location of high point, $x_m = g_1 L/(g_2-g_1)$	= ABS(G15*G19/ (G16-G15))	m
Minimum Vertical curvature, K, is 55	64	-	High point station = VPC station + x_m	= Q13+Q17	m
Length of curve, L = KA = $K(g_2-g_1)$	=ABS(G18 *(G16-(G15)))	m	Elevation of the high point , Ex = EVPC + $g_1x_m/100$ + $(g_2-g_1)\ g_1x_m^2/200$ L	= Q15+G15*Q17/100+ (G16-G15)*Q17^2/200/G19	m

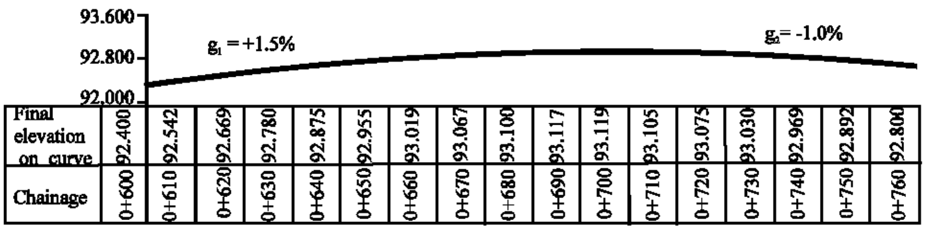


Fig. 4: Type 1 (positve and negative gardients) crest vertical curves simulation

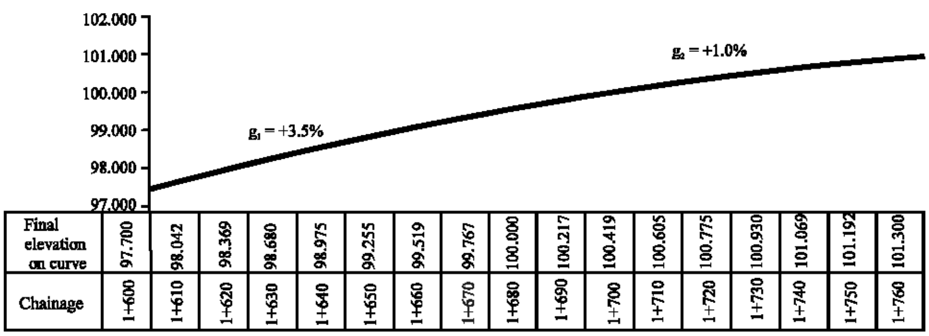


Fig. 5: Type 2 (both gardients positve) crest vertical curves simulation

RESULTS AND DISCUSSION

Stochastic approximation simulation methodology has being successfully employed in this research which generated highway vertical curves elements of different configurations similarly like conventional hand approach while using Microsoft Excel 2010, a general purpose application program. Statistically, Eq. 1, 2 are linear regression logical models while Eq. 3-5 are nonlinear regression models to define the vertical parabolic curves subjectively before arriving at a logical conclusion. These equations have been used successfully to determine modeling module for the optimisation of Types I-IV as labelled by AASHTO (2004) while employing the vertical curve variables subjectively within the given boundary conditions of a right of way Table 2. Table 3 and 4 are showing the simulation run of a crest vertical curve Type I that is equally applicable to other Types II-IV in conformity to AASHTO (2004) and Federal Ministry of Works (FMW, 2007) Highway Standards.

The success of the stochastic approximation simulation methodology developed here is being reported upon as a prodigy approach to highway geometric design. With the use of the statistical equations that are related to vertical curves design, it has been made possible to produce stochastic approximation simulation models for the graphical expression of the four types with six representations of vertical curves as presented by AASHTO (2004) as in Fig. 1 completely while using the Microsoft Excel program. The six representations are in Fig. 4-9 slated for crest and sag vertical curves as carried out in this study that are stated as:

Table 2: Final elevation on curve modeling module for the simulation of type I

A-F label	G	H	I	J	K	L
	Modeling					
Final elevation on curve, (elevation on tan - y)	= G33+G32	= H33+H32	= I33+H32	= J33+J32	= K33+K32	= L33+L32
Offset of curve from initial grade line, $y = (g_2 - g_1)\ x^2/200/L$	= ($G16-$G15) *G34^2/200/$G19	= ($G16-$G15) *H34^2/200/$G19	= ($G16-$G15) *I34^2/200/$G19	= ($G16-$G15) *J34^2/200/$G19	= ($G16-$G15) *K34^2/200/$G19	= ($G16-$G15) *L34^2/200/$G19
Elevation on initial tangent ($EVPC + G_1x$)	= $Q15+$G15 *G34/100	= $Q15+$G15 *H34/100	= $Q15+$G15 *I34/100	= $Q15+$G15 *J34/100	= $Q15+$G15 *K34/100	= $Q15+$G15 *L34/100
Horizontal distance to point on curve, measured from VPC, x	0	= H35-$G35	= I35-$G35	= J35-$G35	= K35-$G35	= L35-$G35
Chainage	0+600	0+610	0+620	0+630	0+640	0+650

A-F label	M	N	O	P	Q	R
	Modeling					
Final elevation on curve, (elevation on tan - y)	= M33+M32	= N33+N32	= O33+O32	= P33+P32	= Q33+Q32	= R33+R32
Offset of curve from initial grade line, $y = (g_2 - g_1)\ x^2/200/L$	= ($G16-$G15)* M34^2/200/$G19	= ($G16-$G15) *N34^2/200/$G19	= ($G16-$G15) *O34^2/200/$G19	= ($G16-$G15) *P34^2/200/$G19	= ($G16-$G15) *Q34^2/200/$G19	= ($G16-$G15) *R34^2/200/$G19
Elevation on initial tangent ($EVPC + G_1x$)	= $Q15+$G15 *M34/100	= $Q15+$G15 *N34/100	= $Q15+$G15 *O34/100	= $Q15+$G15 *P34/100	= $Q15+$G15 *Q34/100	= $Q15+$G15 *R34/100
Horizontal distance to point on curve, measured from VPC, x	= M35-$G35	= N35-$G35	= O35-$G35	= P35-$G35	= Q35-$G35	= R35-$G35
Chainage	0+660	0+670	0+680	0+690	0+700	0+710

A-F label	S	T	U	V	W
	Modeling				
Final elevation on curve, (elevation on tan - y)	= S33+S32	= T33+T32	= U33+U32	= V33+V32	= W33+W32
Offset of curve from initial grade line, $y = (g_2 - g_1)\ x^2/200/L$	= ($G16-$G15) *S34^2/200/$G19	= ($G16-$G15) *T34^2/200/$G19	= ($G16-$G15) *U34^2/200/$G19	= ($G16-$G15) *V34^2/200/$G19	= ($G16-$G15) *W34^2/200/$G19
Elevation on initial tangent ($EVPC + G_1x$)	= $Q15+$G15 *S34/100	= $Q15+$G15 *T34/100	= $Q15+$G15 *U34/100	= $Q15+$G15 *V34/100	= $Q15+$G15 *W34/100
Horizontal distance to point on curve, measured from VPC, x	= S35-$G35	= T35-$G35	= U35-$G35	= V35-$G35	= W35-$G35
Chainage	0+720	0+730	0+740	0+750	0+760

Table 3: Simulation module for the optimisation of type I crest vertical curve

A-F label	G-I modeling	J unit	K-P label	Q-R modeling	W unit
Vertical point of intersection, VPI station	680	m	Station location for the VPC = VPI Station - L/2 =	600	m
Elevation for the VPI	93.6	m	Station location for the VPT = VPC Station + L=	760	m
Grade of tangent 1, g_1	1.5	%	Elevation for the VPC, $EVPC = EVPI - g_1(L/2) =$	92.4	m
Grade of tangent 2, g_2	-1	%	Elevation for the VPT, $EVPT = EVPI + g_2(L/2) =$	92.8	m
Design speed	100	km h^{-1}	Location of high point, $x_m = g_1\ L/(g_2 - g_1)$	96	m
Minimum Vertical curvature, K is 55	64	-	High point station = VPC station + x_m	696	m
Length of curve, $L = KA = K(g_2\text{-}g_1)$	160	m	Elevation of the high point , Ex = $EVPC + g_1x_m/100 + (g_2 - g_1)\ g_1x_m^2/200\ L$	93.12	m

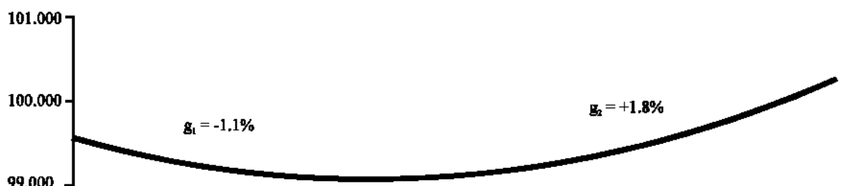
Final elevation on curve	89.630	89.444	89.245	89.034	88.810	88.574	88.325	88.064	87.790	87.504	87.205	86.894	86.570	86.234	85.885	85.524	85.150
Chainage	2+720	2+730	2+740	2+750	2+760	2+770	2+780	2+790	2+800	2+810	2+820	2+830	2+840	2+850	2+860	2+870	2+880

Fig. 6: Type 2 (both gardients negitive) crest vertical curves simulation

Table 4: Final elevation on curve simulation module of type I crest vertical curve

A-F label	G	H	I	J	K	L	M	N	O
	Modeling								
Final elevation on curve, (elevation on tan - y)	92.400	92.542	92.669	92.780	92.875	92.955	93.019	93.067	93.100
Offset of curve from initial grade line, $y = (g_2 - g_1)\ x^2/200/L$	0.000	-0.008	-0.031	-0.070	-0.125	-0.195	-0.281	-0.383	-0.500
Elevation on initial tangent (EVPC + G_1x)	92.4	92.6	92.7	92.9	93	93.2	93.3	93.5	93.6
Horizontal distance to point on curve, measured from VPC, x	0	10	20	30	40	50	60	70	80
Chainage	0+600	0+610	0+620	0+630	0+640	0+650	0+660	0+670	0+680

A-F label	P	Q	R	S		T	U	V	W
	Modeling								
Final elevation on curve, (elevation on tan - y)	93.117	93.119	93.105	93.075		93.030	92.969	92.892	92.800
Offset of curve from initial grade line, $y = (g_2 - g_1)\ x^2/200/L$	-0.633	-0.781	-0.945	-1.125		-1.320	-1.531	-1.758	-2.000
Elevation on initial tangent (EVPC + G_1x)	93.8	93.9	94.1	94.2		94.4	94.5	94.7	94.8
Horizontal distance to point on curve,	90	100	110	120		130	140	150	160
Chainage	0+690	0+700	0+710	0+720		0+730	0+740	0+750	0+760



Final elevation on curve	99.630	99.529	99.446	99.382	99.335	99.307	99.296	99.304	99.330	99.374	99.436	99.517	99.615	99.732	9.866	100.019	100.190
Chainage	3+600	3+610	3+620	3+630	3+640	3+650	3+660	3+670	3+680	3+690	3+700	3+710	3+720	3+730	3+740	3+750	3+760

Fig. 7: Type3 (negitive and positive gardients) sag vertical curves simulation

Final elevation on curve	95.620	95.239	94.876	94.532	94.205	93.897	93.606	93.334	93.080	92.844	92.626	92.427	92.245	92.082	91.936	91.809	91.700
Chainage	4+510	4+520	4+530	4+540	4+550	4+560	4+570	4+580	4+590	4+600	4+610	4+620	4+630	4+640	4+650	4+660	4+670

Fig. 8: Type 4 (both gardients negitive) sag vertical curves simulation

Final elevation on curve	98.362	98.455	98.574	98.720	98.892	99.091	99.315	99.566	99.844	100.147	100.477	100.834	101.216	101.625	102.061	102.522	103.010
Chainage	5+600	5+610	5+620	5+630	5+640	5+650	5+660	5+670	5+680	5+690	5+700	5+710	5+720	5+730	5+740	5+750	5+760

Fig. 9: Type 4 (both gardients positive) sag vertical curves simulation

- Crest vertical curves for

Type I are g_1 = +1.5% and g_2 = -1%

Type II (all positive gradients) are:

g_1 = +3.5%

g_2 = +1%

Type II (all negative gradients) are:

g_1 = -1.8%

g_2 = -3.8%

- Sag vertical curves for

Type III are:

g_1 = -1.1%

g_2 = 1.8%

Type IV (both negative gradients) are:

g_1 = -3.9%

g_2 = -1.0%

Type IV (both positive gradients) are:

g_1 = +0.8%

g_2 = +0.5%

CONCLUSION

This study has vividly come out with the Optimisation Simulation Flow Chart and two Modeling Modules in Microsoft Excel 2010 office program for use in support of stochastic approximation simulation of highway geometric vertical curves. The objective-function values that include g_1, g_2, K, A, L and different station locations for VPT, VPC and VPT are variables that have been used subjectively via stochastic approximation to simulate reduced levels of vertical curves for highway vertical alignments easily and rapidly while utilising Microsoft Excel program.

Microsoft Excel is readily available and cheaper software than any other program for use in this facet. With this methodology, it is easily possible while employing stochastic approximation method advanced in this study to generate the simulation of the six vertical curves representations as presented by AASHTO (2004) in an electronics office similarly like that of the conventional hand method while carrying out geometric design of highway. It is pertinent to note that stochastic approximation process is sequel to subjective trials and it is a means to deterministic process to evolve the same output from the finally arrived vertical curve variables for the required solution to complex problems such as in defining highway vertical curves as shown in Fig. 4-9. This methodology is highly recommended to researchers, professional engineers and students as a cheaper and cost effective methodology of highway geometric design on comparison with the use of specific purpose application programs and conventional hand method.

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